光画像工学
Optical imaging and image processing
(III)

1.6 2D Discrete Fourier transform

2D DFT

$$F[k,l] = \boldsymbol{DFT}\{f[m,n]\} = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f[m,n]\exp\{-j2\pi(\frac{mk}{M}+\frac{nl}{N})\}$$

Inverse 2D DFT

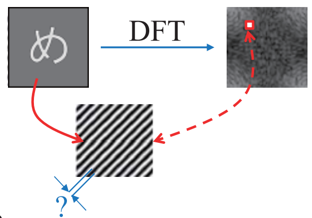
$$f[m,n] = \boldsymbol{DFT}^{-1}\{F[k,l]\} = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} F[k,l]\exp\{j2\pi(\frac{mk}{M}+\frac{nl}{N})\}$$

2D Fourier transform in continuous space

$$F(u,v) = \iint f(x,y)\exp\{-j2\pi(ux+vy)\}dxdy$$

Inverse 2D Fourier transform in continuous space

$$f(x,y) = \iint F(u,v)\exp\{j2\pi(ux+vy)\}dudv$$



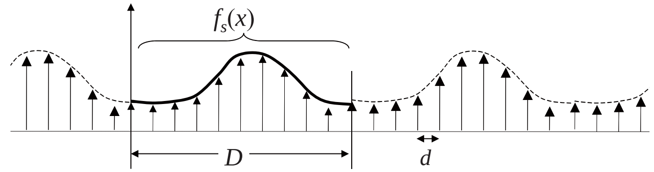
What we need to know is;

The relationship between
the frequency obtained by DFT
and
the spatial frequency on an object
e.g., [cycles/mm], [lines/mm], [cycles/deg], …

DFT assumes sampled periodic signals.

Spatial domain:

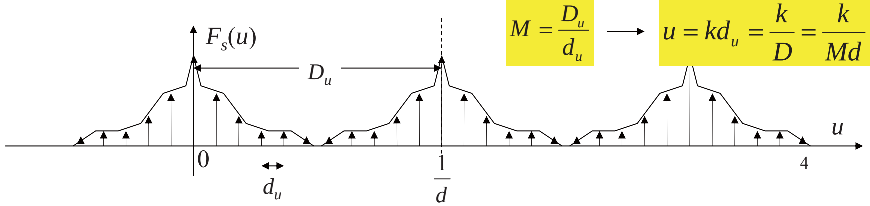
Sampling pitch (intervals of delta functions) = d,
Image size (period) = D $\longrightarrow$ $M = \frac{D}{d}$ $\longrightarrow$ $x = md$



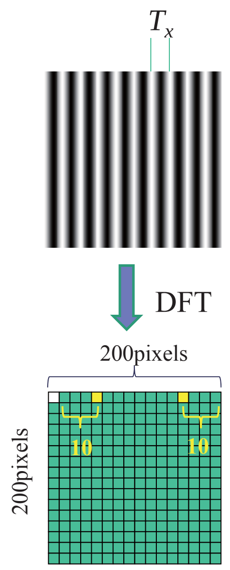
Fourier domain (Frequency domain):

Sampling pitch (intervals of delta functions) = d_u,
Period in frequency domain = D_u $\longrightarrow$ $D_u = \frac{1}{d}$, $d_u = \frac{1}{D} = \frac{1}{Md}$

$M = \frac{D_u}{d_u}$ $\longrightarrow$ $u = kd_u = \frac{k}{D} = \frac{k}{Md}$



Example



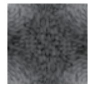
# of pixels = 200x200 pixels
Sampling interval = d = 0.5mm
Image size = D = 0.5x200 = 100 [mm]
Periodic pattern, Period = T_x = 10 [mm]
Spatial frequency = $u_x = 1/T_x$ = 0.1 [cycles/mm]

$$\boldsymbol{F}\{ \cos 2\pi u_x x \} = \{ \delta(u - u_x) + \delta(u + u_x) \} / 2$$

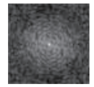
Sampling interval in DFT space = d_u
= $1/D$ = 0.01 [cycles/mm]
$u_x = k_x d_u = 0.1 \rightarrow k_x = 0.1 / 0.01 = 10$

The periodicity in DFT

(a) Original image

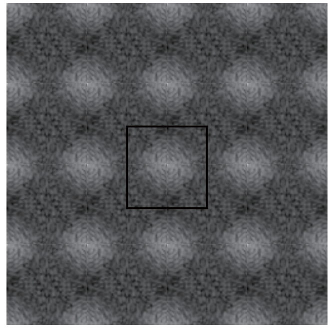


(b) DFT of (a)



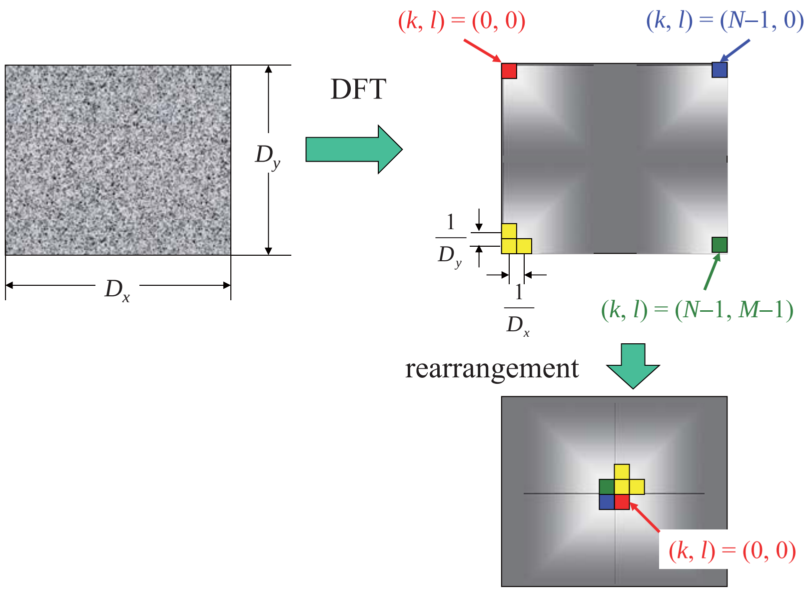
(c) zero Frequency centered.

(d) DFT is considered as the Fourier transform of the periodic function like this figure.



(e) DFT of (d). The frequency spectra are also periodic. The square region surrounded by □ corresponds to (c).

For example, for an image in $N\times M$ pixels, we have $N\times M$ Fourier coefficients; $F[k, l]$



1.7 Fourier analysis of linear shift-invariant imaging system

- 2-D linear system in continuous space

$$g(x,y) = \iint h(x,y;x',y') f(x',y') dx' dy'$$

- Shift-invariant (space-invariant)

$$g(x,y) = \iint h(x-x', y-y') f(x',y') dx' dy'$$
$$= f(x,y) * h(x,y)$$

$\rightarrow$ Convolution

$h(x, y)$: Impulse response, point spread function (PSF)　インパルス応答 点像分布関数

- 2-D linear shift-invariant imaging system with additive noise

$$g(x,y) = \iint_{-\infty}^{\infty} h(x-x', y-y') f(x' y') dx' dy' + n(x,y)$$

- Fourier transform of 2-D shift-invariant imaging system

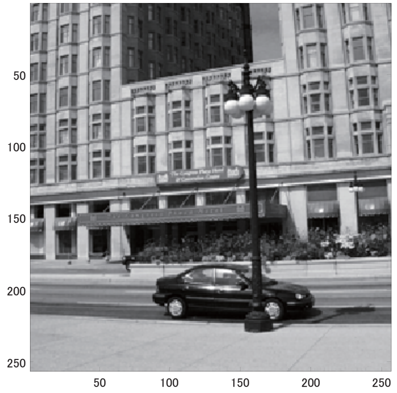
$$G(u,v) = H(u,v) F(u,v)$$
$$G(u,v) = H(u,v) F(u,v) + N(u,v)$$

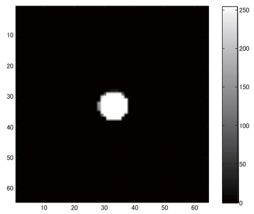
$H(u,v)$: Transfer function　伝達関数、周波数特性

2-D Linear, shift-invariant system in discrete space

Discrete convolution 離散たたみ込み

$$g[m,n] = \boldsymbol{S}\{f[m,n]\} = \sum_{m',n'} h[m-m',n-n']\,f[m',n']$$
$$= h[m,n] * f[m,n]$$





$h_D[m,n]$

$M_h \times N_h$ pixels

Discrete signals within a finite region

$f_D[m,n]$

$M \times N$ pixels

```
imagesc( img );
imagesc( ci );
```


Discrete convolution by using DFT

Circulant convolution

$$F_D[k,l] = \boldsymbol{DFT}\{f_D[m,n]\}$$
$$H_D[k,l] = \boldsymbol{DFT}\{h_D[m,n]\}$$

} Discrete signals within a finite interval

$$g_D[m,n] = \boldsymbol{DFT}^{-1}\{G_D[k,l]\} = \boldsymbol{DFT}^{-1}\{H_D[k,l]\ F_D[k,l]\}$$

Consider

$$f_p[m,n] = f_D[m-kM, n-lN]$$
$$h_p[m,n] = h_D[m-kM, n-lN]$$
$$g_p[m,n] = g_D[m-kM, n-lN]$$

} Periodic functions

$0 \le m' \le M-1 \qquad 0 \le m-m' \le M_h - 1$

$0 \le n' \le N-1 \qquad 0 \le n-n' \le N_h - 1$

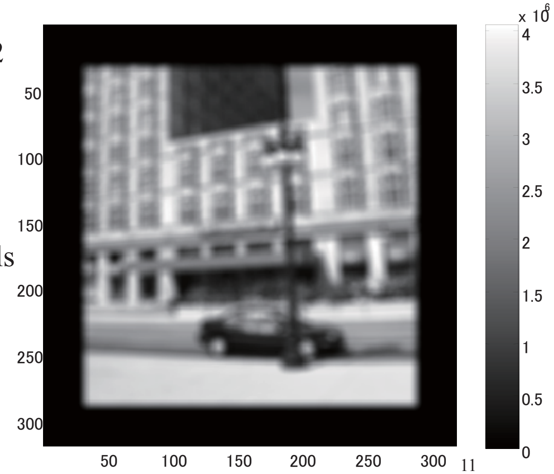
⇒ $0 \le m \le M + M_h - 2$

$0 \le n \le N + N_h - 2$

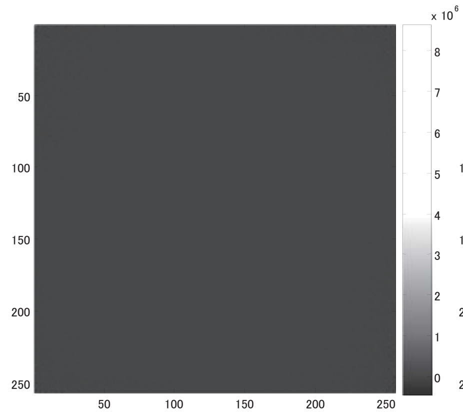
$g[m,n]$

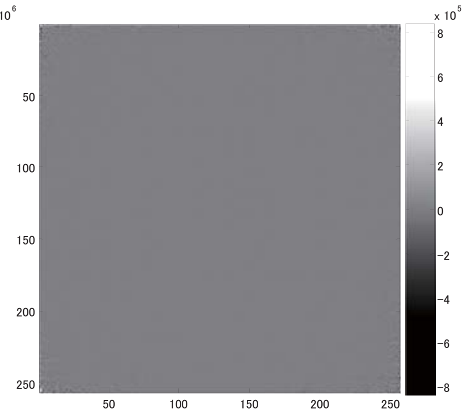
$(M+M_h-1)\times(N+N_h-1)$ pixels

```
cres = conv2( double(img), double(ci) );
imagesc( cres );
```

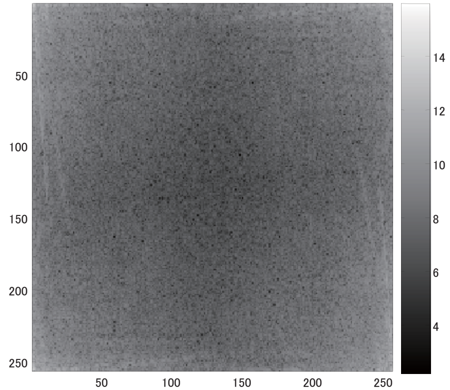


- fim = fft2(double(img));
- imagesc(real(fim));
- imagesc(imag(fim));

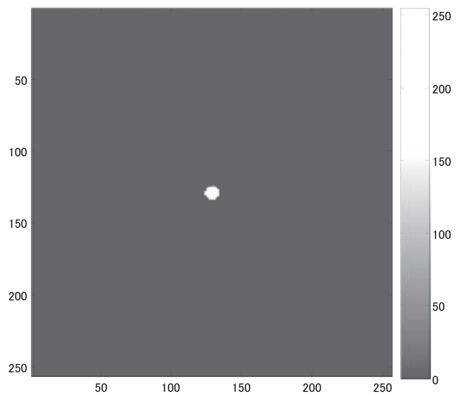


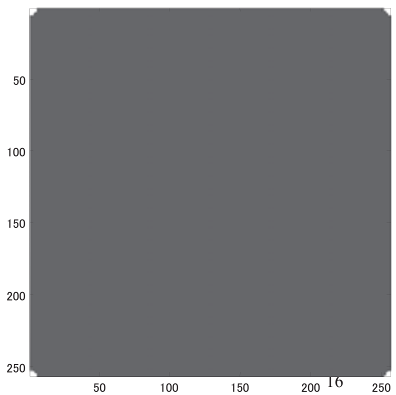


- imagesc(log(abs(fim)));

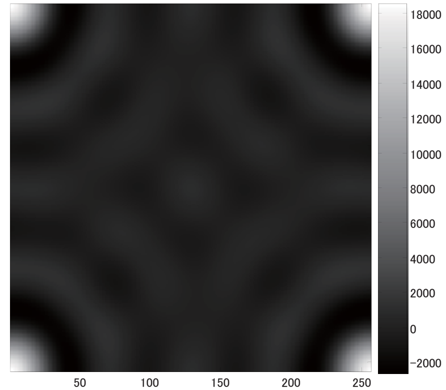


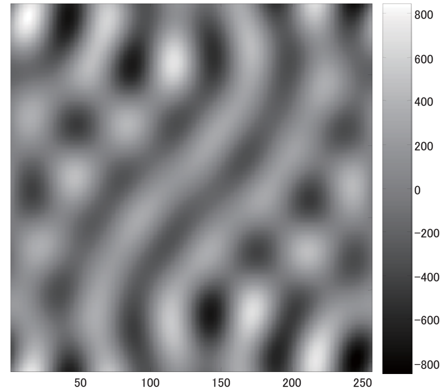
- cil = zeros(256,256);
- cil([97:160],[97:160])=ci;
- imagesc(cil);
- scil = fftshift(cil);
- imagesc(scil);



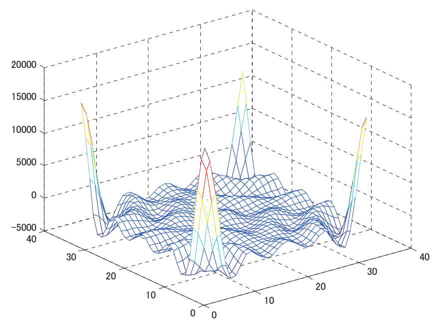


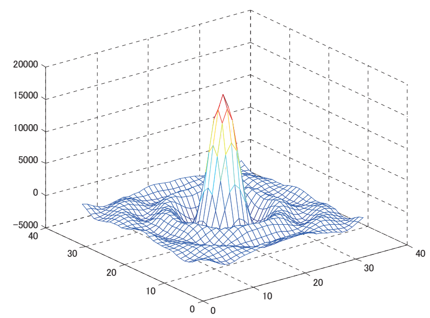
- fscil = fft2(scil);
- imagesc(real(fscil));
- imagesc(imag(fscil));



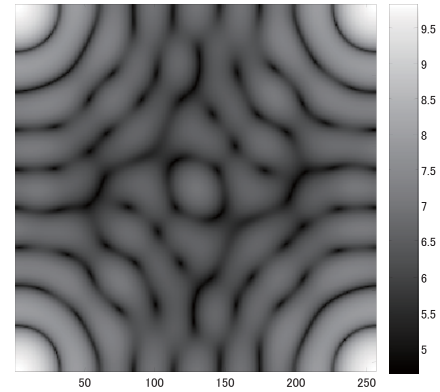


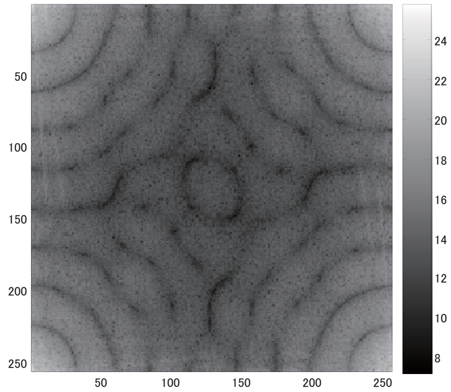
- mesh(real(fscil([1:8:256],[1:8:256])));
- mesh(fftshift(real(fscil([1:8:256],[1:8:256]))));



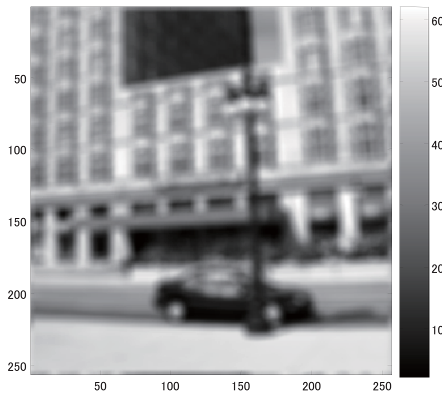


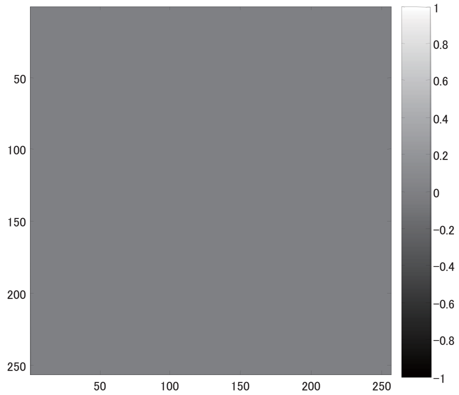
- imagesc(log(abs(fscil) + 100));
- fres = fim .* fscil;
- imagesc(log(abs(fres) + 1));





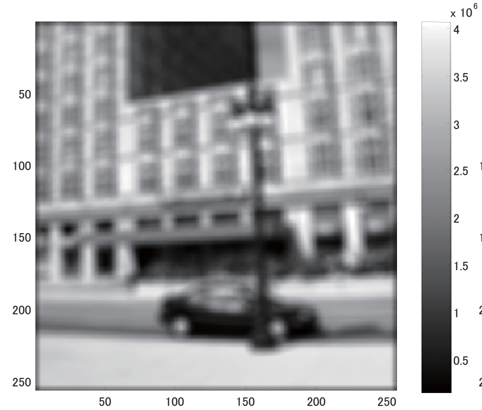
- res = ifft2(fres) / (256*256);
- imagesc(real(res));
- imagesc(imag(res));

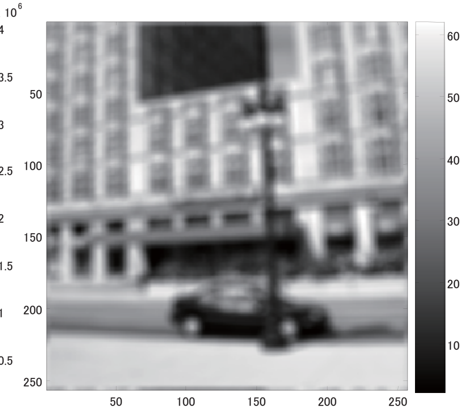


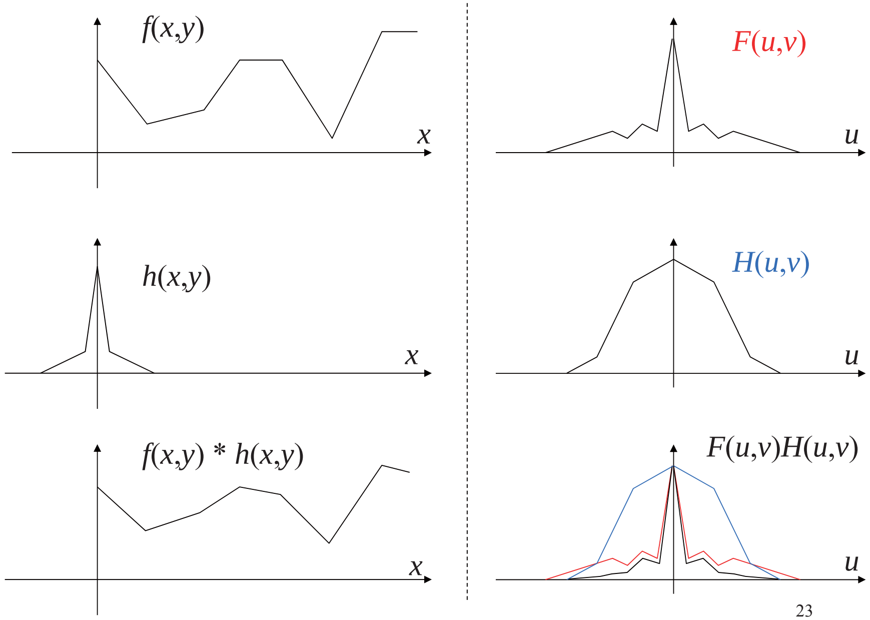


- cres = conv2(double(img), double(ci));
- imagesc(cres);

- cres = conv2(double(img), double(ci), 'same');
- imagesc(cres);
- imagesc(real(res));







Modulation transfer function

Transfer function : H
Modulation transfer function : | H |
Phase transfer function: arg{ H }

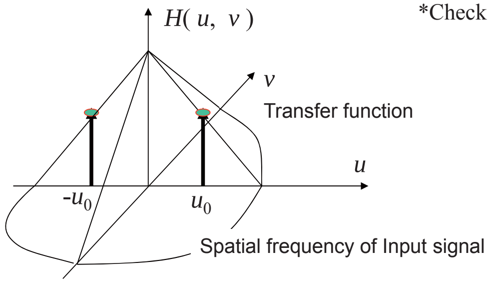
How to derive MTF (1)

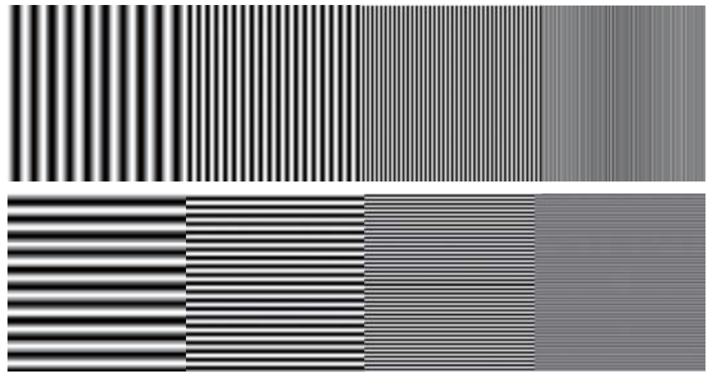
Input signal $A_I + A_I \cos 2\pi u_0 x$

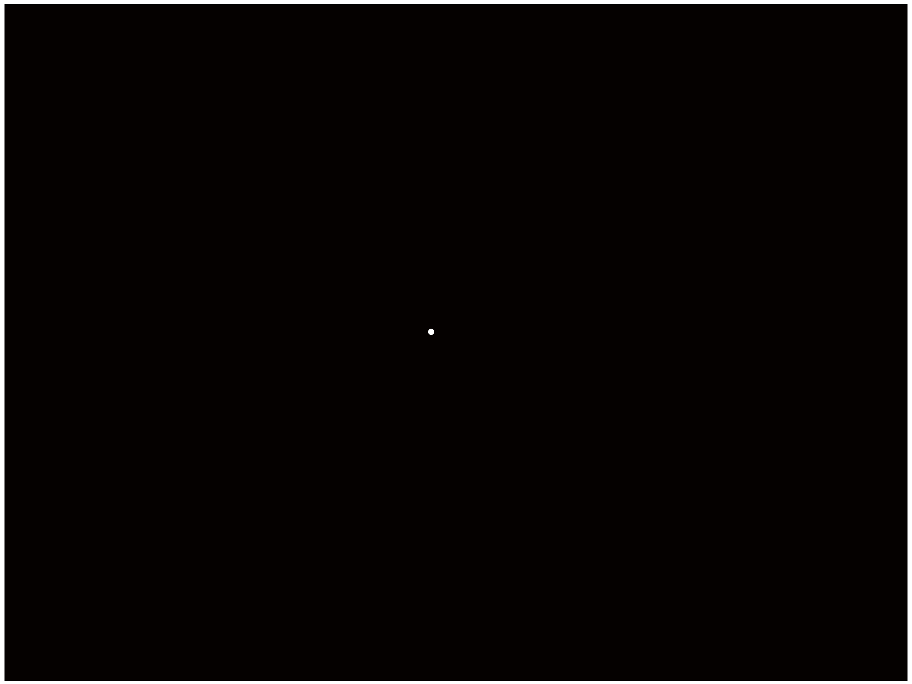
Output signal $A_I' + A_O \cos(2\pi u_0 x + \phi)$

$$| H(u_0,\ 0) | = \frac{A_O}{A_I}$$

*Check above equation!







MTF of Human visual system

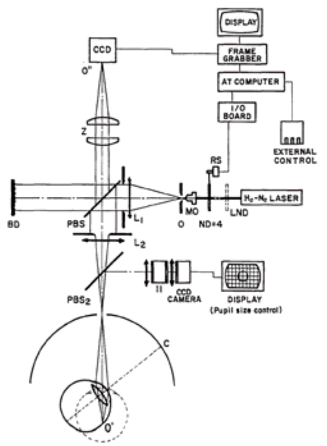
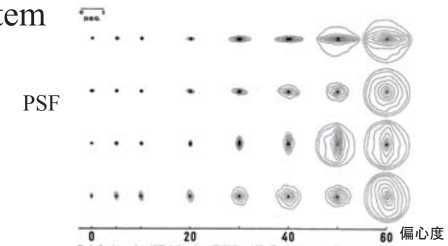


Fig. 1. Experimental setup for recording and digital processing of the double-pass aerial image of a point source (see text for a detailed description).



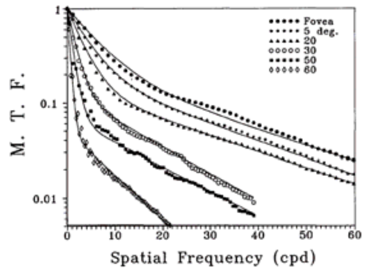
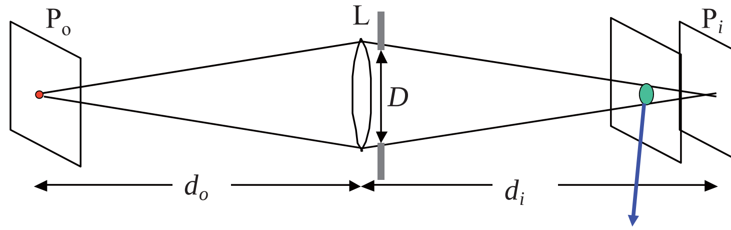


Fig. 9. Average radial profiles (symbols) and results of curve fitting (continuous curves) for six eccentricities. Results for 10° and 40° have been left out for the sake of clarity.

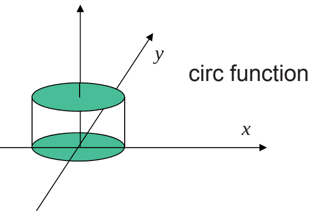
R. Navarro, P. Artal, D. R. Williams, "Modulation transfer of the human eye as a function of retinal eccentricity," JOSA A Vol. 10, No. 2, 201-212 (1993)

Impulse response of a defocused optical imaging system



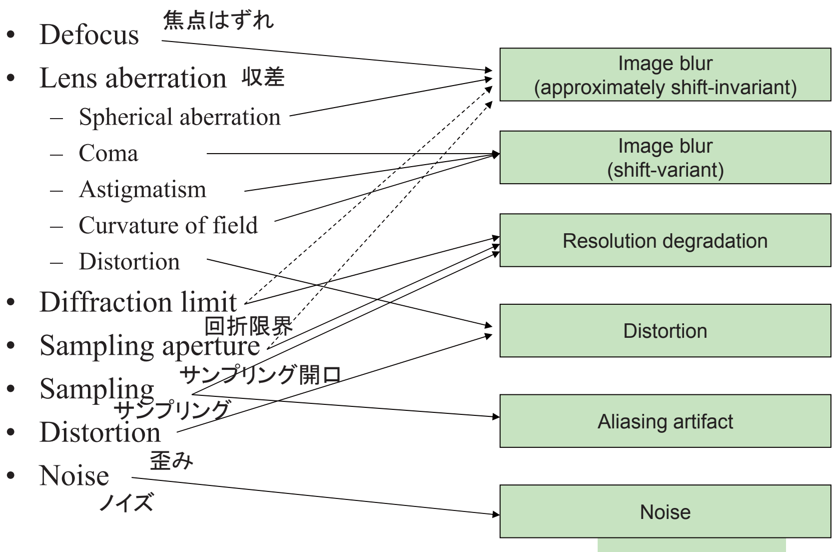
- Incoherent model (Geometrical model)

  2D object: (Approximately) shift-invariant
  3D object: Shift-variant



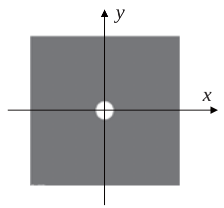
Coherent (wave optics model) case: see "diffraction and image formation."

1.8 Causes of image degradation
画像劣化とその原因

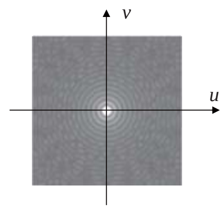


Transfer function of a defocused optical imaging system

- $\boldsymbol{F}\{\, \mathrm{circ}(\, r\,)\,\} = J_1(\, 2\,\pi\,\rho\,)\,/\,\rho$
  (Fourier-Bessel transform)
  - J_1 : Bessel function of the first kind, order 1.



Impulse response
Point spread function



Transfer function

Size	Large	Small
	Small	Large

IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 16, NO. 8, AUGUST 1994 824-831

Shape from Focus

Shree K. Nayar and Yasuo Nakagawa

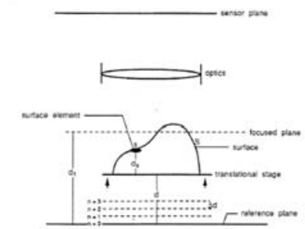


Fig. 3. Shape from focus.

optics [1]. Very often, a two-dimensional Gaussian function is used to approximate the physical model [13]. Then, the blurred or *defocused* image $I_d(x, y)$ formed on the sensor plane can be described as the result of convolving the focused image $I_f(x, y)$ with the blurring function $h(x, y)$:

$$I_d(x,y) = h(x,y) * I_f(x,y) \quad (3)$$

where

$$h(x,y) = \frac{1}{2\pi\sigma_h^2} e^{-\frac{x^2+y^2}{2\sigma_h^2}}. \quad (4)$$

The *spread parameter*, σ_h, is assumed to be proportional to the radius r [13].

One way to high-pass filter an image is to determine its second derivative. For two-dimensional images, the *Laplacian* may be used:

$$\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}. \quad (9)$$

In frequency domain, applying the Laplacian $L(u, v)$ to the defocused image $I_D(u, v)$ (5) gives

$$L(u,v) \cdot H(u,v) \cdot I_F(u,v) \quad (10)$$

where

$$L(u,v) \cdot H(u,v) = -(u^2+v^2) \cdot e^{-\frac{u^2+v^2}{2}\sigma_h^2}. \quad (11)$$

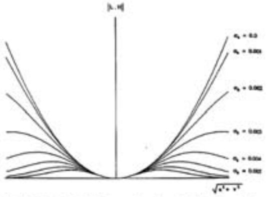


Fig. 4. The effect of defocusing and second-order differentiation in frequency domain.

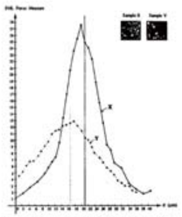


Fig. 5. SML focus measure function computed for two [illegible] samples.

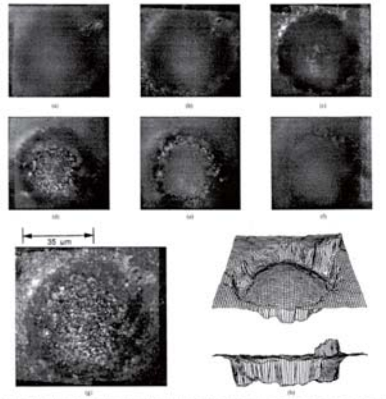


Fig. 6. Result obtained using the automated system: Via-hole filling on a ceramic substrate. The via-hole is approximately 70 μm and has insufficient filling. (a) i = 2; (b) i = 5; (c) i = 8; (d) i = 11; (e) i = 14; (f) i = 16; (g) Reconstructed image; (h) Depth maps.

M. Watanabe, S.K. Nayar and M. Noguchi, "Real-Time Computation of Depth from Defocus," Proc. SPIE, Vol.2599, pp.14-25, Jan, 1996

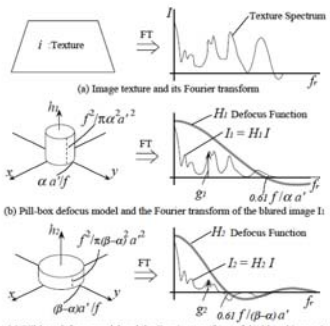


Figure 4: Image blaring and focus measure.

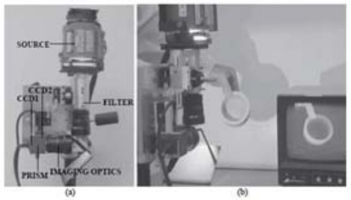


Figure 8: (a) The real-time focus range sensor and its key components. (b) The sensor can produce depth maps up to 512×480 in resolution at 30 Hz.

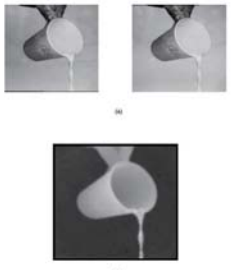


Figure 1: (a) Two images of a scene taken using different focus settings. (b) A depth map of the scene computed in 33 msec by the focus range sensor.

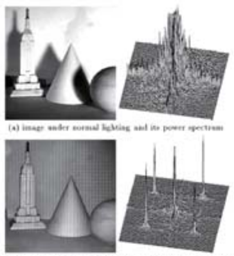


Figure 5: Effect of active pattern projection.

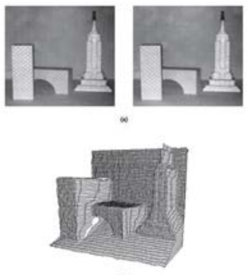


Figure 13: (a) Near and far focused images of a set of polyhedral objects. (b) Computed depth map.