# Advanced Macroeconomics (Tokyo Tech, 2014) The Search-theoretic Model of Labor Market

Ryoji Ohdoi

Department of Social Engineering

# Setup of a basic search model (partial equilibrium)

- Consider a continuous time, infinite horizon economy.
- In this basic model, we consider behaviors of workers only, each of whom is employed or unemployed.
- The wage rate, $w$, is exogenously given (that is, we consider a partial eq.).
- More specifically, we assume
  - $w \in [w_*, w^*]$.
  - The c.d.f of wage is given by $F(w)$

# Behavior of job seekers (i.e, unemployed workers)

- Each unemployed worker finds a job w.p. $\alpha\Delta t$ during a small time interval $\Delta t$.
  - In this model, $\alpha$ is exogenous, which is assumed to be endogenous below.
- Then, the worker draws the wage rate from the distribution $F$.
  - The realized wage rate does not change until an exogenous job separation occurs at the rate $\lambda$.

# Behavior of job seekers (cont'd)

- After observing the wage rate, the worker has the two options:
    1. rejecting the offer: in this case he receives the unemployed benefit and waits until he finds another job.
    2. accepting the offer: in this case he receives $w$ until an exogenous job separation occurs.
- Let
    - $V_U(t)$ denote the expected utility of unemployed; and
    - $V_E(t, w)$ denote the conditional expected utility of employed, given that the realized wage rate is $w$.

# Expected utility of employed with wage $w$

- Now focus on a steady state with $dV_E(t, w)/dt = 0$ and $\dot{V}_U(t) = 0$.
- $V_E$ is given by the following Bellman equation.

$$V_E(w) = w\Delta t + e^{-r\Delta t}\left[(1 - \lambda\Delta t)V_E(w) + \lambda\Delta t V_U\right],$$

which is reduced to

$$rV_E(w) = w - \lambda(V_E(w) - V_U). \tag{1}$$

# Expected utility of job seekers

- In a steady state with $\dot{V}_U(t) = 0$,

$$V_U = z\Delta t + e^{-r\Delta t}\left[(1-\alpha\Delta t)V_U + \alpha\Delta t\int_{w_*}^{w^*}\max\left\{V_E(w), V_U\right\}dF(w)\right],$$

where $z$ is the unemployment benefit minus search cost, assumed to be fixed.

- $z < w^*$ is assumed.

# Expected utility of job seekers (cont'd)

- Let $w_R$ denote the value of wage rate that solves $V_E(w) = V_U$.
- From (1), $w_R$ is implicitly given by

$$V_E(w_R) = \frac{w_R}{r} \tag{2}$$

  - Since the function $V_E$ is still unknown, so is $w_R$. For the moment, however, we assume that $w_R \in (w_*, w^*)$.

- Then,

$$rV_U = z + \alpha \int_{w_R}^{w^*} (V_E(w) - V_U) dF(w). \tag{3}$$

# Equilibrium

- In this model, equilibrium is given by $V_E(w)$, $V_U$ and $w_R$ that solve Eqs. (1)-(3).

- From these three equations, we have

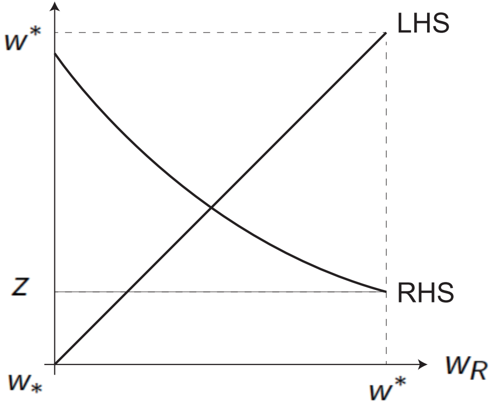
$$w_R = z + \frac{\alpha}{r+\lambda}\int_{w_R}^{w^*}(w - w_R)dF(w), \tag{4}$$

or equivalently

$$w_R = z + \frac{\alpha}{r+\lambda}\int_{w_R}^{w^*}(1 - F(w))dw. \tag{4'}$$

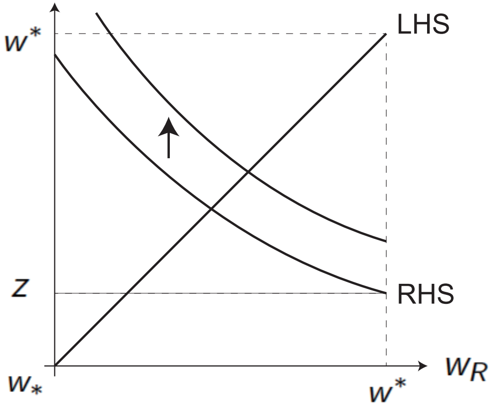
# Equilibrium (cont'd)

- The reservation wage in equilibrium:



# Comparative statics

- $z\uparrow$, $\alpha\uparrow$, $r\downarrow$ or $\lambda\downarrow$:



# Expected waiting times

- Consider a worker who became unemployed at time $T_0 > 0$.
- Define $\tilde{\alpha}(w_R) = \alpha(1 - F(w_R))$.
- Thus, the probability that the successful offer is encountered at time $T_1 > T_0$ is

$$\tilde{\alpha}(w_R)e^{-\tilde{\alpha}(w_R)(T_1 - T_0)}$$

  Duration of unemployment, denoted by $T \equiv T_1 - T_0$, follows a exponential distribution.

$\Downarrow$

- Expected duration of unemployment, $E[T]$, is

$$E[T] = \frac{1}{\tilde{\alpha}(w_R)}.$$

# Implications

- Expected unemployment duration extends according to

1. An increase in unemployment benefit ($z \uparrow$).
2. A rise in the arrival rate of job-finding ($\alpha \uparrow$)
3. A decline in the discount rate ($r \downarrow$)
4. A decline in the job separation rate ($\lambda \downarrow$)

# A random matching model

- So far only workers search their jobs. Now consider both workers and firms engage in search activities.
- Aims:
  - Point out the nature of unemployment in the steady state.
  - Show how wages and unemployment are jointly determined in equilibrium.
- Departures from the preceding section:
  - Matching: The labor market is decentralized.
  - Bargaining: Wage rate is negotiable.

# Setup of the model

- Each worker is either employed or unemployed.
  - A worker who has a job will never look for another job (no on-the-job search).
- Each firm has one job, which produces output. The job is either vacant or occupied.
  - Similarly, a firm whose job is occupied will not look for a new worker.
- Both workers and firms are risk-neutral.
- There is no coordination either among workers or firms.

# Setup of the model

- From now on, the number of notations becomes widespread.
- Therefore, for convenience, once reset the meanings of notations, and newly define
  - $L$ : total measure of workers.
  - $u(t)$ : the unemployment ratio at time $t$.

    $\Rightarrow u(t)L$ is the measure of unemployed workers.
  - $v(t)$ : the vacancy ratio.

    $\Rightarrow v(t)L$ is the measure of jobs still to be matched (vacancies)

$(*)$ "$(t)$" is omitted unless to do so would cause confusions.

# Matching function

- Let $M(t)\Delta t$ denote the measure of job matching during a small time interval $\Delta t$.
- It is assumed that the job matching takes place according to the following *matching function:*

$$M(t) = \Phi(u(t)L, v(t)L). \tag{5}$$

## Assumption

$\Phi : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ *satisfies*

$$\begin{aligned}
(i) &\quad \Phi_x(x,y) > 0,\ \Phi_y(x,y) > 0, \\
(ii) &\quad \Phi_{xx}(x,y) < 0, \Phi_{yy}(x,y) < 0, \\
(iii) &\quad \Phi(\lambda x, \lambda y) = \lambda \Phi(x,y) \forall \lambda \geq 0, \\
(iv) &\quad \Phi(x,0) = \Phi(0,y) = 0, \\
(v) &\quad \Phi(x,y) \leq \min\{x,y\}.
\end{aligned}$$

# Matching function (cont'd)

- Define $\theta(t) = v(t)/u(t)$, where $\theta(t)$ is the measure of vacant jobs per unemployed.
- During a small time interval $\Delta t$, one vacant job is matched to an unemployed with probability $q(\theta)\Delta t$, where

$$q(\theta) \equiv \frac{\Phi(uL, vL)}{vL} = \Phi\left(\frac{1}{\theta}, 1\right).$$

  - $\theta$ is also called *labor maker tightness* (LMT). LMT has a similar concept as the *jobs-to-applicants ratio*.
  - The mean duration is $1/q(\theta)$.
  - $q'(\theta) \leq 0$.
  - Let $\eta(\theta)$ denote the absolute value of the elasticity: $\eta(\theta) \equiv -\frac{\theta q'(\theta)}{q(\theta)}$. Then, $0 \leq \eta(\theta) \leq 1$.

# Matching function (cont'd)

- On the other hand, an unemployed worker is matched to one vacant job with probability $\theta q(\theta)\Delta t$, where

$$\theta q(\theta) \equiv \frac{\Phi(uL, vL)}{uL} = \Phi\left(1, \theta(t)\right).$$

  - $\dfrac{d[\theta q(\theta)]}{d\theta} \geq 0$. The elasticity is $1 - \eta(\theta) \geq 0$.
  - The mean duration of unemployment is $\dfrac{1}{\theta q(\theta)}$.
  - The more vacant jobs there are, the faster unemployed will find jobs.

# Note: “search externalities”

- Note that every job-seeker and every vacant job cause so-called *search externalities:*
  - An additional job-seeker ($u \uparrow$) cause a positive externality for the firm but a negative externality for the other job-seekers.
  - An additional vacancy will have an analogous effect.

# Labor market

- As in the preceding section, let $\lambda > 0$ denote the job destruction rate.
- Then, the evolution of the unemployment rate $u(t)$ is given by

$$\dot{u}(t) = \lambda(1 - u(t)) - \theta(t)q(\theta(t))u(t).$$

- From the above equation, the unemployment In steady state is

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}. \tag{6}$$

Thus, the equilibrium unemployment rate is dependent on both job creation– and destruction rate.

# Labor market (cont'd)

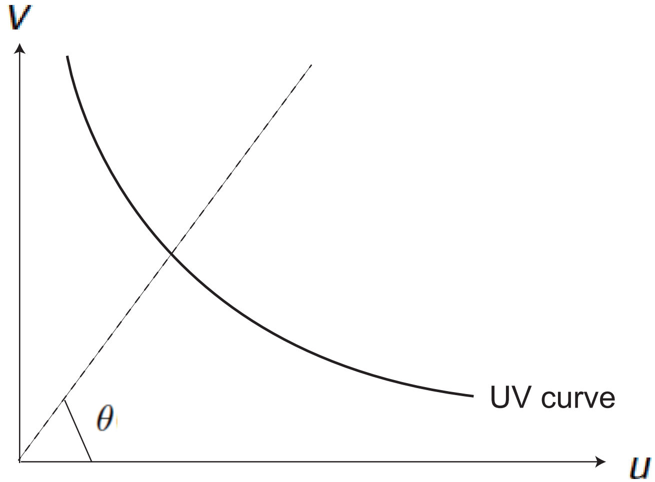
- (6):

$$u = \frac{\lambda}{\lambda + (v/u)q(v/u)}. \tag{6'}$$

  $\Rightarrow$ A negative correlation between the unemployment rate and the vacancy rate.

- A graphical representation of (6) is called the *beverage curve* or the *UV curve*.

# Beverage curve



# Jobs

- For simplicity, each firm has one job, and one job requires only one worker.
- A job produces one unit of output.
- If a firm and a worker are matched, and they agree to an employment contract, the detail of which is explained below, production occurs.
- Let $p > 0$ is the value of output, which is exogenous.

# Firms

- Let
  - $J$ : the expected value of the firm whose job is occupied.
  - $V$ : the expected value of the firm whose job is vacant.
- Then,

$$rJ = p - w - \lambda(J - V), \tag{7}$$

$$rV = -c + q(\theta)(J - V). \tag{8}$$

  - $p - w$ is the profit flow of the firm which is matched with a worker to engage in production.
  - $c$ is the cost which the firm whose job is vacant must pay in order to maintain the vacancy.

# Firms (cont'd)

- Free entry condition: $V = 0$.
- From the above condition and Eqs. (7) and (8),

$$J = \frac{p - w}{r + \lambda} = \frac{c}{q(\theta)}. \tag{9}$$

- Rearranging the second equation of (9):

$$w = p - \frac{(r + \lambda)c}{q(\theta)}. \tag{10}$$

- The wage rate $w$ must be lower if $\theta \uparrow$, that is $u \downarrow$.

# Workers

- Let
  - $W$ : the expected utility of the employed worker.
  - $U$ : the expected utility of the unemployed worker.
- Then,

$$rW = w - \lambda(W - U), \tag{11}$$

$$rU = z + \theta q(\theta)(W - U), \tag{12}$$

# Wage determination

- The wage rate $w$ is determined by the Nash bargaining.
- Therefore $w$ is given by solving

$$\max_{w} \, (W-U)^{\beta}(J-V)^{1-\beta}, \quad 0 \leq \beta \leq 1,$$

Since $V=0$, the first-order-condition is

$$\beta J \frac{dJ}{dw} + (1-\beta)(W-U)\frac{dW}{dw} = 0$$

- From (9) and (11), $\dfrac{dW}{dw} = -\dfrac{dJ}{dw} = \dfrac{1}{r+\lambda}$. Then,

$$\beta J = (1-\beta)(W-U). \tag{13}$$

# Wage determination (cont'd)

- Substituting (9) and (11) into (13),

$$w = \beta p + (1-\beta) rU.$$

- Substituting (12) into the above result,

$$w = (1-\beta)z + \beta(p + \theta c). \tag{14}$$

# Steady state equilibrium

- A steady state equilibrium is characterized by $u$, $w$ and $\theta$ which satisfy (6), (10) and (14):

$$u = \frac{\lambda}{\lambda + \theta q(\theta)}, \tag{6}$$

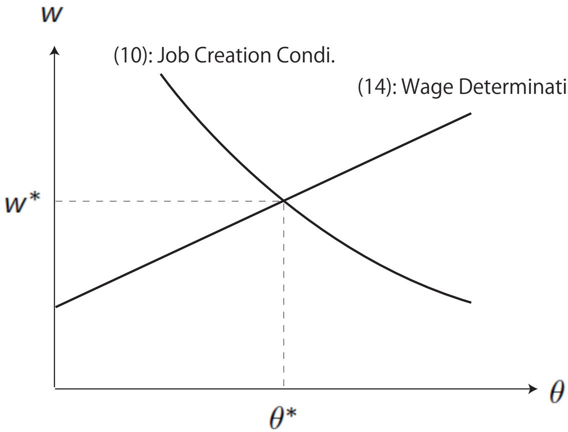
$$w = p - \frac{(r+\lambda)c}{q(\theta)}, \tag{10}$$

$$w = (1-\beta)z + \beta(p + \theta c). \tag{14}$$

- Let $u^*$, $w^*$, $\theta^*$ denote the steady-state values.

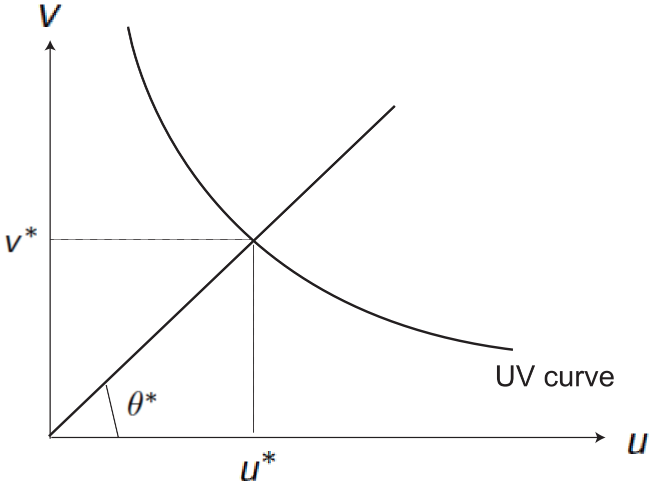
# Steady state equilibrium (cont'd)

- From (10) and (14), $w^*$ and $\theta^*$ are determined:



# Steady state equilibrium (cont'd)

- Using the above result, we can obtain $u^*$ and $v^*$:



# A question

- How do $w$, $u$, $v$ change according to an increase in $p$ (a positive output shock)?