Advanced Macroeconomics

(Department of Social Engineering, Spring FY2015)

The Basic Real Business Cycle Model

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Course Guideline

- **Course Guideline**
  1. Dynamic Optimization (4 lectures)
  2. The Ramsey-Cass-Koopmans Model (4 lectures)
  3. Endogenous Growth Models (1 lecture)
  4. Time-inconsistent Preferences (Preference Reversals) (1 lecture)
  5. Some Macroeconomic Applications of Stochastic DP (3 lectures incl. today)

Plan of Lecutres in the Part of "Applications of Stochastic DP"

- July, 1 (Wed) (Today): A Basic model of real business cycles (RBC)
- July, 8 (Wed): An efficiency wage model

  (∗) A discrete-time version of the Shapiro and Stiglitz's (1984, AER) model.
- July, 15 (Wed): A search-theoretic model of the labor market (tentative)

Introduction

- The RBC models: Briefly speaking, the RBC models incorporate the following two features into the Ramsey-Cass-Koopmans model:

  1. The total factor productivity (全要素生産性, TFP) follows a stochastic process; and

  2. Labor supply is endogenously determined by a representative household's labor-leisure choice.

- Therefore, there is no
  1. market distortion; or
  2. market incompleteness

$\Rightarrow$ A competitive equilibrium allocation always corresponds to the allocation designed by the social planner.

Setup of the Model

- Time is discrete and indexed by $t = 0, 1, 2 \ldots$
- There is a continuum of homogenous households, whose measure is normalized to unity (i.e, $L_t = 1$).
- Notations:
  - $C_t \in \mathbb{R}_+$: amount of consumption in period t;
  - $K_t \in \mathbb{R}_+$: amount of physical capital stock in —–;
  - $H_t \in [0, 1]$: working time in —-.

$(*)$ Since the population size is normalized to 1, per-capita variables are also aggregate ones.

Production Technology

- Output, Y_t, is produced according to the following Cobb-Douglas production technology:

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}, \quad 0 < \alpha < 1. \tag{1}$$

where A_t is called the total factor productivity (TFP).

- It is assumed that A_t is specified as

$$A_t = \exp(z_t) \times \overline{A}, \quad \overline{A} > 0, \tag{2}$$

and z_t follows an AR(1) process:

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \quad 0 < \rho < 1, \tag{3}$$

where

$$E(\varepsilon_t) = 0,\ Var(\varepsilon_t) = \sigma_\varepsilon^2,\ E(\varepsilon_t \varepsilon_s) = 0\ \forall t \neq s.$$

Production Technology

- ε_{t+1} becomes observable after period $t+1$ comes.
- From (2) and (3),

$$\log A_{t+1} - \log \overline{A} = \rho(\log A_t - \log \overline{A}) + \varepsilon_{t+1}. \tag{4}$$

Thus, the rate of deviation of A_t from its mean value also follows an AR (1) process.

Households's Preferences

- A representative household's preferences are given by the following expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, H_t). \tag{5}$$

(∗) It is assumed that $\partial u/\partial H < 0$: i.e., disutility from labor supply.

- In the RBC models, u is often specified as

$$u(C_t, H_t) = \log C_t - \psi \frac{H_t^{1+\eta^{-1}}}{1+\eta^{-1}}, \quad \psi > 0, \ \eta > 0.$$

(∗) η is the Frisch elasticity of labor supply.

The Competitive Equilibrium

- Since there is no market distortion or incompleteness, the competitive equilibrium path is obtained by solving the social planner's problem.

- The social planner's problem in period t is given by

$$
\begin{aligned}
\max \quad & E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\log C_s - \psi \frac{H_s^{1+\eta^{-1}}}{1+\eta^{-1}} \right] \\
\text{s.t.} \quad & K_{s+1} = A_s K_s^{\alpha} H_s^{1-\alpha} + (1-\delta) K_s - C_s, \\
& \log A_{t+1} - \log \overline{A} = \rho (\log A_t - \log \overline{A}) + \varepsilon_{t+1}, \\
& K_t, \; A_t \text{ given.}
\end{aligned}
\tag{P}
$$

Definition

The sequence $\{(K_t, C_t, H_t)\}_{t=0}^{\infty}$ is the competitive equilibrium path if it solves the above problem (P).

The Bellman Equation

- Hereafter, let
  - $(H_t, K_t, A_t) = (H, K, A)$ and $(H_{t+1}, K_{t+1}, A_{t+1}) = (H', K', A')$
  - E_t, which is the expectation operator conditioned on the period t information, is denoted simply by E.

- The Bellman equation for the above problem can be set as

$$V(K, A) = \max_{K', H} \left\{ \log[AK^{\alpha}H^{1-\alpha} + (1-\delta)K - K'] - \psi \frac{H^{1+\eta^{-1}}}{1+\eta^{-1}} \right. \\ \left. + \beta E[V(K', A')] \right\} \quad (6)$$

The Competitive Equilibrium Path of a Simplified Model ($\delta = 1$)

If we assume $\delta = 1$, we can analytically obtain the competitive equilibrium path.

Proposition (Brock and Mirman (1972, JET))

Suppose that $\delta = 1$. Then, the competitive equilibrium path is characterized by

$$H_t = \overline{H} \equiv \left(\frac{1-\alpha}{\psi(1-\beta\alpha)} \right)^{\eta/(1+\eta)}, \tag{7}$$

$$K_{t+1} = \beta\alpha A_t K_t^{\alpha} \overline{H}^{1-\alpha}, \tag{8}$$

$$C_t = (1-\beta\alpha) A_t K_t^{\alpha} \overline{H}^{1-\alpha}, \tag{9}$$

Proof.

Exercise. □

The Competitive Equilibrium Path of a Simplified Model ($\delta = 1$)

- We can express the competitive equilibrium more simply:
  - Consider the hypothetical situation that there is no uncertainty: $\sigma_\varepsilon^2 = 0 \Rightarrow \varepsilon_t = 0 \Rightarrow A_t = \overline{A}$ for $t = 0, 1, \ldots$.
  - Let $\overline{K}$ denote the steady state of capital in such a deterministic economy:

$$\overline{K} = (\beta\alpha\overline{A})^{1/(1-\alpha)}\overline{H}. \tag{10}$$

- Using (10), the equilibrium dynamics of K_t, (8), is rewritten as

$$K_{t+1} = \overline{K}^{1-\alpha}(A_t/\overline{A})K_t^\alpha. \tag{11}$$

Effects of Shocks on Output

- Let $\hat{k}_t$ and $\hat{a}_t$ denote

$$\hat{k}_t = \log K_{t+1} - \log \overline{K}, \quad \hat{a}_t = \log A_{t+1} - \log \overline{A}.$$

- Then, (4) and (11) are respectively reduced to

$$\hat{a}_{t+1} = \rho \hat{a}_t + \varepsilon_{t+1}, \tag{12}$$

$$\hat{k}_{t+1} = \alpha \hat{k}_t + \hat{a}_t. \tag{13}$$

More General Case ($\delta \neq 1$)

- The assumption, $\delta = 1$, is problematic, because
  1. This is not realistic;
  2. More importantly, under $\delta = 1$, hours worked, H_t, does not respond to the productivity shocks, but fixed at $\overline{H}$.

Competitive Equilibrium Path when $\delta \neq 1$

- When $\delta \neq 1$, the equilibrium is characterized by the following system of stochastic difference equations:

$$\frac{1}{C_t} = \beta E_t \left[\frac{1 - \delta + r_{t+1}}{C_{t+1}} \right], \tag{14}$$

$$\psi C_t H_t^{1/\eta} = w_t, \tag{15}$$

$$K_{t+1} = \overline{A} \exp(z_t) K_t^{\alpha} H_t^{1-\alpha} + (1-\delta) K_t - C_t, \tag{16}$$

where

$$r_{t+1} = \overline{A} \exp(z_{t+1}) (K_{t+1}/H_{t+1})^{-(1-\alpha)}, \tag{17}$$

$$w_t = \overline{A} \exp(z_t) (K_t/H_t)^{\alpha}. \tag{18}$$

and z_t stochastically evolves according to

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}. \tag{19}$$

- It is known that if $\delta \neq 1$, the above equations are not analytically solved.

A Numerical Method

- This method consists of the following steps:

    1. Find the steady state of the model without shocks.
    (∗) Hereafter we call this steady state "deterministic steady state."
    2. Construct a log-linear approximation of the dynamic system (14)–(3) around the deterministic steady state.
    3. Solve the log-linearized system.
    4. Specify the parameter values involved in the model, say, α, β, δ and so on..., and calculate the coefficient values of the log-linearized system.
    (∗) This step is called "calibration."
    5. Conduct an impulse response analysis and/or simulation of the calibrated system.

Dynare

- To conduct this process, Matlab is often used.

  ⇒ In doing so, “Dynare” can greatly help writing this program.

- What is Dynare?
  - 一言でいえば，マクロ経済学の諸モデルを数値的に解くことに特化した Matlab のプログラム群
  - 注意：単体では動かない．Matlab，もしくはそのクローンである Octave にパスを通すことで使用可能.
  - 無償．ちなみに，Octave もフリーソフト．Matlab も今年度から東工大のライセンスを用いて学生の PC にインストール可能.

Dynare

- Matlab/Dynare 環境について:
  - Matlab を学内の自分にあてがわれた PC にダウンロードしたい場合：

    < http://tsubame.gsic.titech.ac.jp/MATLAB-TAH >

  - Dynare の入手法：

    < http://www.dynare.org/>

- 今回のモデルの Dynare Program: この講義の OCW-i でダウンロード可能

  (∗) ちなみに，多少モデルは違うが，以下の Dynare ホームページからも RBC の Dynare プログラムが入手可能.
  <http:
  //www.dynare.org/documentation-and-support/examples/rbc.zip>

Steps

- Step 1: ⇒ Exercise.

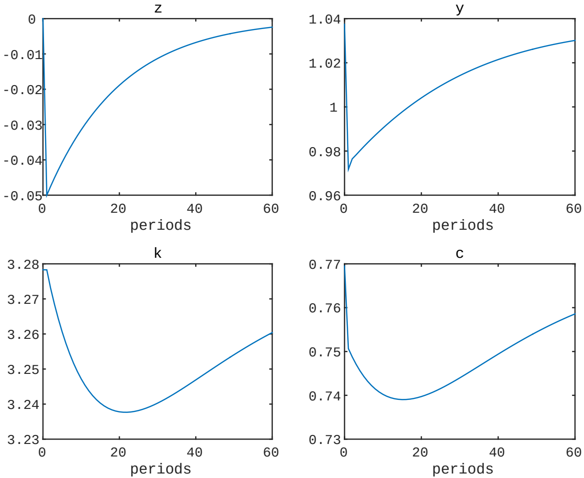
- Step 2 以降：モデル (14)–(19) をプログラムして，パラメータの値を決めれば，基本的に後はすべて Dynare が計算してくれる．
  - パラメータの値：ここでは,

$$
\begin{aligned}
\alpha &= 0.33 \\
\beta &= 0.99 \\
\delta &= 0.025 \\
\psi &= 1.0 \\
\eta &= 1.0 \\
\rho &= 0.95
\end{aligned}
$$

  - ショックの与え方：ここでは第 1 期にのみ $\epsilon_1 = -0.05$ となり，その後 0 に戻るというショック (インパルス).

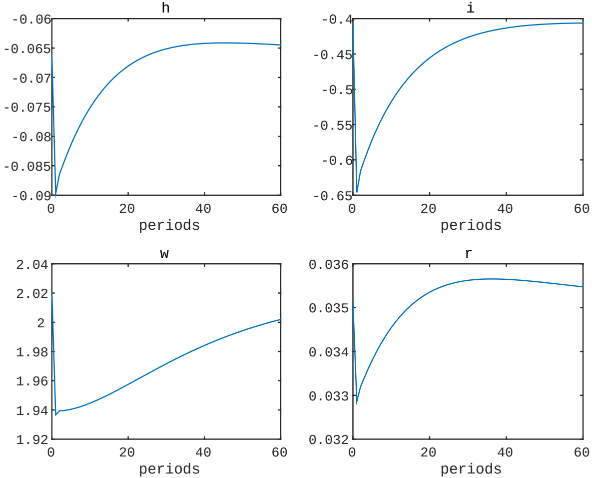
各変数の動き (1/2)

- 生産性，生産量，資本ストック，消費のインパルス応答
  (∗) z を除いて，各変数はすべて自然対数 (例えば，$k = \log K$)



各変数の動き (2/2)

- 労働時間，投資，賃金，利子率のインパルス応答
  (∗) r, w を除いて，各変数はすべて自然対数



まとめ

- この講義で扱うトピックの順番上，"確率的な動的計画法" の応用例の一つとして RBC モデルを挙げたが，歴史的経緯はむしろ逆.
    - Kydland and Prescott (1982, EMA), King, Plosser and Rebelo (1988, JME) などによる第一世代 RBC モデルの開発によって，動的計画法がマクロ経済分析 (特に景気循環の分析) に不可欠になった.
    - サーヴェイ論文としては，King and Rebelo (1999, Handbook) 等が参考になる.
- とはいえこの第一世代 RBC，その些かきつい仮定や，現実を説明する上でのパフォーマンスの悪さなどにより，当初の目的である現実の景気循環を説明するという目的が達成されたとは言いがたい.
  $\Downarrow$
  RBC を捨てるのではなく，この基本的なモデルに市場の不完全性や，貨幣的な側面を導入するなどの改造が進む (90 年代後半)
  $\Downarrow$
  (New Keynesian) Dynamic Stochastic General Equilibrium (DSGE) モデル

References

- Brock, W. and L. Mirman (1972) "Optimal Economic Growth and Uncertainty: The Discounted Case." *Journal of Economic Theory*, vol. 4, pp. 479–513.

- King R. G., C. I. Plosser and S. Rebero (1988) "Production, Growth and Business Cycles I. The Basic Neoclassical Model." *Journal of Monetary Economics*, vol. 21, pp. 195–232.

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- Kydland, F. E. and E. C. Prescott (1982) "Time to Build and and Aggregate Fluctuation." *Econometrica*, vol. 50, pp. 1345-1370.