

<b>Homework #3</b>
--------------------

Answer to some of the following questions. (You may solve at most two questions.) Due: by the next lecture.

**Questions:**

1. Prove Lemma 1.1 (5 points).
2. (1) Prove Fact 1. (4 points)  
(2) Consider a randomized algorithm for solving the Smallest Enclosing Disk problem by using the sampling technique discussed today. Determine the actual number of sample points sufficient for each iteration. (2 points).
3. Prove Theorem 3.1 (5 points).
4. Suppose we have, for some problem  $P$ , a randomized algorithm  $A$  that answers correctly with prob. 1 to any positive instance (i.e., an instance for which the answer is 'yes' w.r.t.  $P$ ) and answers correctly with prob.  $\geq 1/2$  to any negative instance (i.e., an instance for which the answer is 'no' w.r.t.  $P$ ). On the other hand, suppose that we also have an algorithm  $B$  that answers correctly with prob.  $\geq 1/2$  to any positive instance and answers correctly with prob. 1 to any negative instance. We assume that the time complexity of both algorithms is  $T(\ell)$  for instances of size  $\ell$ . By using these two algorithms, design a randomized algorithm  $C$  that does not make any mistake, and Explain the time complexity of  $C$  (5 points).