7th Lecture

11 Frequency Domain Design

11.4 Feedback Design via Loop Shaping: Example (pp.326 to 331)

Keyword : Lead and Lag Compensation

11.2 Feedforward Design (pp.319 to 322)

Keyword : Feedforward
2 Degree of Freedom

11.3 Performance Specifications (pp.322 to 326)

Keyword : Time Domain Analysis
Step Response

[Ex. 11.6] Roll control for a vectored thrust aircraft*

(§ 2.4, Exe. 8.10)

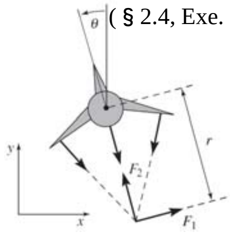


Fig. 2.17 (a) Harrier “jump jet” (b) Simplified model

$$m\ddot{x} = -mg\sin\theta - c\dot{x} + u_1\cos\theta - u_2\sin\theta$$
$$m\ddot{y} = mg(\cos\theta - 1) - c\dot{y} + u_1\sin\theta + u_2\cos\theta \qquad (2.27)$$
$$J\ddot{\theta} = ru_1$$

$u_1 = F_1$ m : mass J : inertia
$u_2 = F_2 - mg$ c : damping r : force moment arm

[Ex. 11.6] Roll control for a vectored thrust aircraft*

from u_1 to θ

$$P(s) = G_{\theta u_1} = \frac{r}{Js^2}$$

$C(s) = k \qquad k = 200$

Specification
Error in stationary state:
Less than 1%
Tracking error up to around
10 [rad/s]: Less than 10%

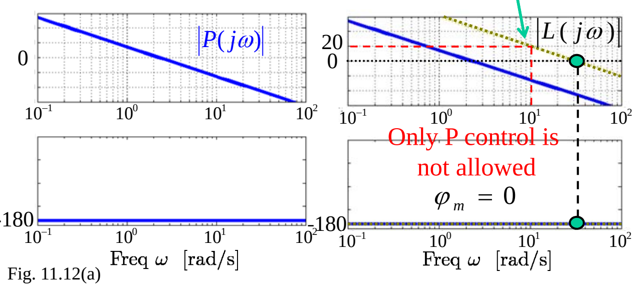
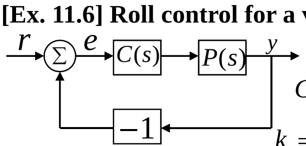


Fig. 11.12(a)

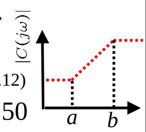
[Ex. 11.6] Roll control for a vectored thrust aircraft*



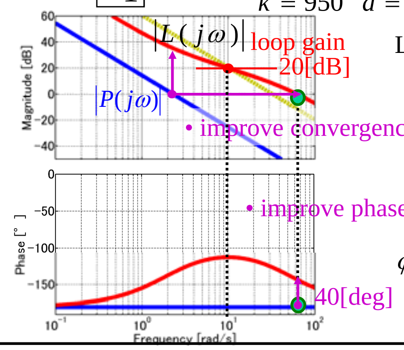
Lead compensator

$$C(s) = k\frac{s+a}{s+b} \quad (11.12)$$

$k = 950 \quad a = 2 \quad b = 50$



Loop transfer function
$L(s) = P(s)C(s)$



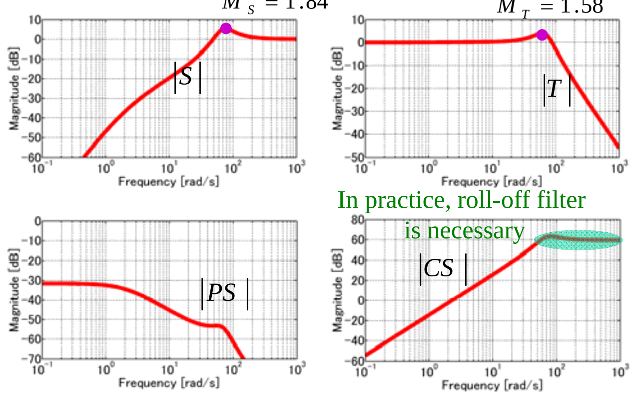
• improve convergence speed (by k)

• improve phase margin (by $\frac{s+2}{s+50}$)

$\varphi_m = 40^\circ$

[Ex. 11.6] Roll control for a vectored thrust aircraft*

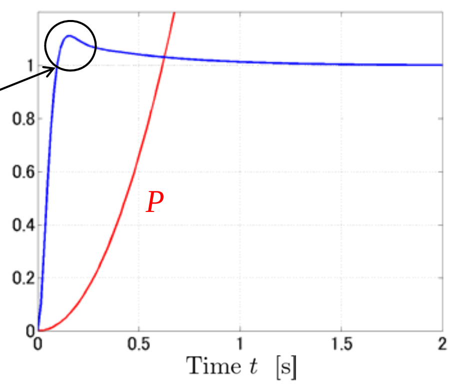
Gang of Four



Frequency response

[Ex. 11.6] Roll control for a vectored thrust aircraft*

Step response



11.6 Design Example

[Ex. 11.12] Lateral control of a vectored thrust aircraft

(§ 2.4, Exe. 8.10)

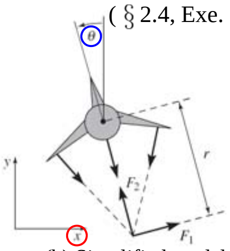


Fig. 2.17(a) Harrier "jump jet" (b) Simplified model

Ex. 11.6: controller for the roll dynamics

Ex. 11.12: controller for the position of the aircraft

(stabilization of both the attitude and the position)

↓

inner / outer loop design methodology

[Ex. 11.12] Lateral control of a vectored thrust aircraft

2. Design C_o for the lateral position under the approximation that we can directly control the roll angle .

1. Design C_i so that H_i provides fast and accurate control of the roll angle.

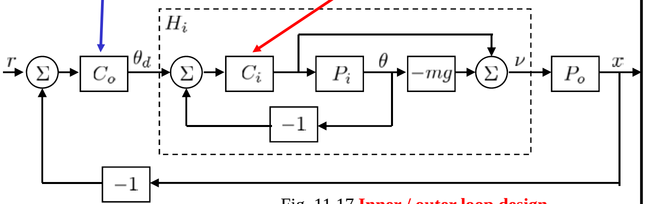


Fig. 11.17 **Inner / outer loop design**

inner loop (H_i): the roll dynamics and control

outer loop: the lateral position dynamics and controller

[Ex. 11.12] Lateral control of a vectored thrust aircraft

Performance specification (entire system)

- zero steady-state error in the lateral position
- a bandwidth of 1 rad/s
- a phase margin of 45°

Performance specification (inner loop)

- the low-frequency error to be no more than 5 %
- a bandwidth of 10 rad/s (10 times that of the outer loop)

[Ex. 11.12]

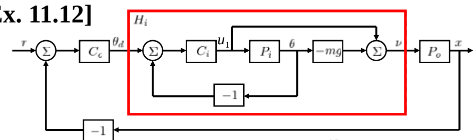


Fig. 11.17

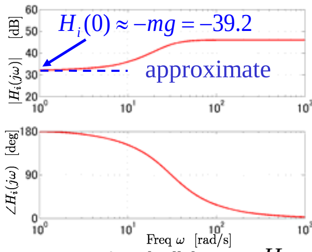
transfer function from u_1 to θ

$$P_i(s) = \frac{r}{Js^2}$$

lead compensator ([Ex. 11.6])

$$C_i(s) = \frac{200(s+2)}{s+50}$$

Performance specification (inner loop) O



Actual roll dynamics H_i

Fig. 11.18 (a)

inner loop

$$H_i(s) = \frac{C_i(1-mgP_i)}{1+C_iP_i}$$

[Ex. 11.12]

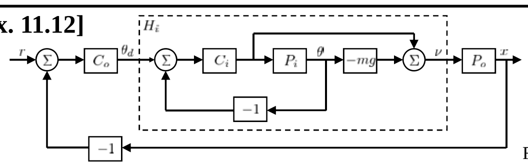


Fig. 11.17

Assume that the inner loop will eventually track our commanded input.

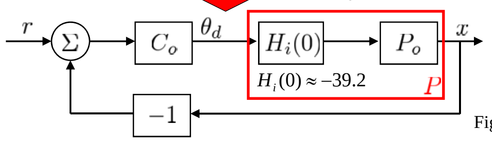


Fig. 11.18 (a)

Lateral position dynamics

$$P(s) = H_i(0)P_o(s) = \frac{-mg}{ms^2+cs}$$

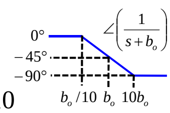
Lead compensator

$$C_o(s) = -k_o\frac{s+a_o}{s+b_o}$$

[Ex. 11.12] Lateral control of a vectored thrust aircraft

Lead compensator

$$C_o(s) = -k_o\frac{s+a_o}{s+b_o}$$

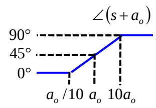


Phase lead flattens out at approximately $b_o/10$

⟶ Desired crossover $\omega_{gc} = 1$[rad/s]

⟶ $b_o = 10$

Ensure adequate phase lead



⟶ $b_o/10 < 10a_o < b_o$

⟶ $a_o = 0.3$ $(\varphi_m \geq 45°)$

At $\omega_{gc} = 1$[rad/s], magnitude 1

$$P(s) = H_i(0)P_0(s) = \frac{H_i(0)}{ms^2+cs} \quad \Rightarrow \quad C_o(s) = -0.98\frac{s+0.3}{s+10}$$

⟶ $k_o = 0.98$

[Ex. 11.12] Lateral control of a vectored thrust aircraft

Combine the inner and outer loop controllers and verify that the system has the desired closed loop performance.

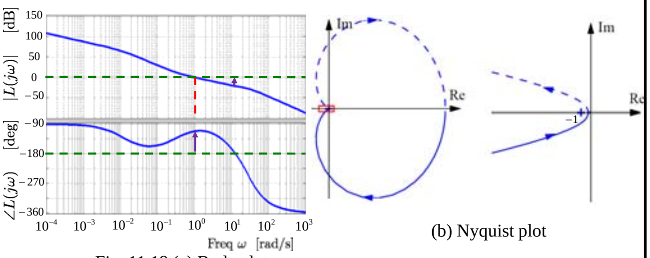


Fig. 11.19 (a) Bode plot (b) Nyquist plot

- ω_{gc} : 1 [rad/s]
- phase margin φ_m : 68°
- gain margin g_m : 10

➡ Performance specification (entire system) ○

[Ex. 11.12] Lateral control of a vectored thrust aircraft

Gang of Four

Not have integral action

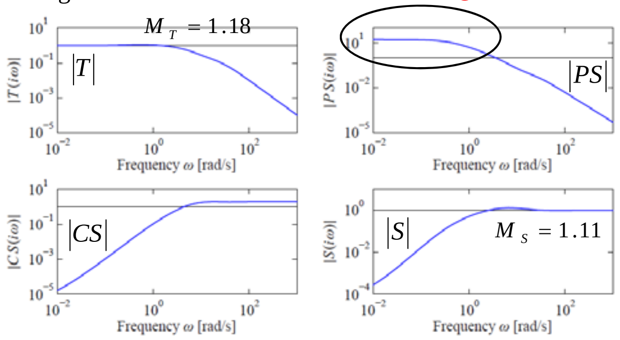


Fig. 11.20 Gang of Four for vectored thrust aircraft system

Feedforward Design

Controller with two degrees of freedom (2DOF)

- A combination of feedforward and feedback controllers.
- Response to reference signals can be designed **independently** of the design for disturbance attenuation and robustness.

Feedforward
- Improve the response to reference signals

Feedback
- Give good robustness
- Disturbance attenuation

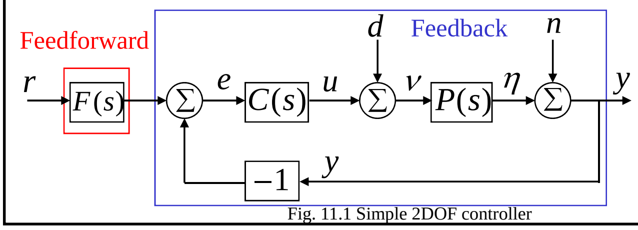
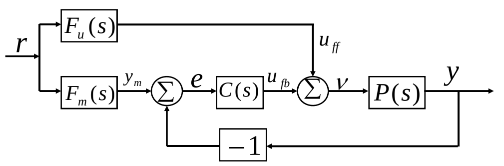


Fig. 11.1 Simple 2DOF controller



F_m: ideal response of the system to reference signals

F_u : feedforward reference controller

From reference signal r to process output y

$$G_{yr}(s) = \frac{P(CF_m + F_u)}{1+PC} = F_m + \frac{PF_u - F_m}{1+PC} \quad (11.4)$$

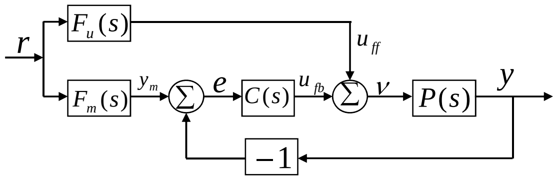
desired response

$$= F_m + (PF_u - F_m)S$$

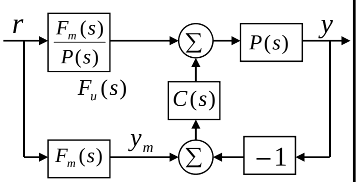
➡ Ideal Feedforward

Feedback (11.5)

small $S = \dfrac{1}{1+PC}$



$$G_{yr}(s) = F_m + \frac{PF_u - F_m}{1+PC} \quad (11.4)$$



Ideal Feedforward

$$F_u = \frac{F_m}{P} \quad (11.5)$$

Feedback

small $S = \dfrac{1}{1+PC}$ for all uncertainties δ

➡ **Robust Performance**

Generalized controller with two degrees of freedom (2DOF)

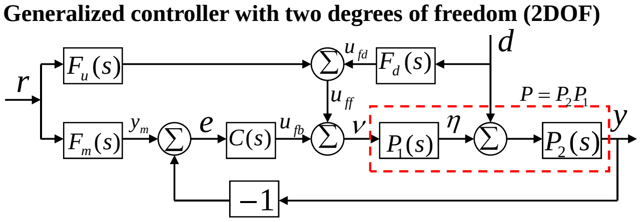


Fig. 11.3 2DOF controller for improved response to reference signals and measured disturbances

F_m: ideal response of the system to reference signals

F_u : feedforward reference controller

F_d : feedforward disturbance controller

C : feedback controller

P_1, P_2 : process $P = P_2P_1$

y_m: desired output

u_{ff}: signal which gives the desired output when applied as input to the process

u_{fb}: feedback control input

d : load disturbance

r : reference signal

Generalized controller with two degrees of freedom (2DOF)

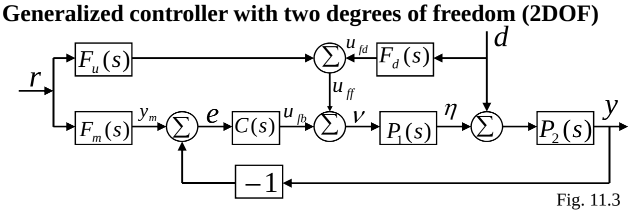


Fig. 11.3

from load disturbance d
to the process output y

$$G_{yd}(s) = \frac{P_2(1+F_dP_1)}{1+PC} = P_2(1+F_dP_1)S \quad (11.6)$$

$\Rightarrow$ ideal feedforward (11.7)

feedback
small $S = \dfrac{1}{1+PC}$

* $P = P_2P_1$

[Ex. 11.2] Vehicle steering

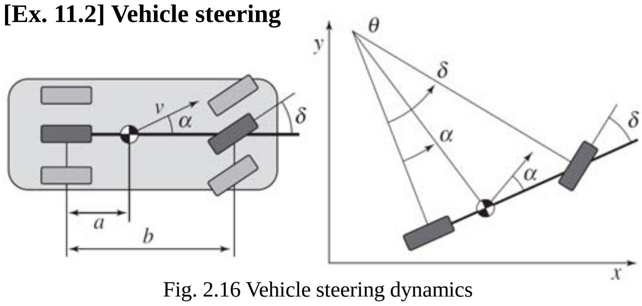


Fig. 2.16 Vehicle steering dynamics

from steering angle δ to lateral deviation y

$$P(s) = \frac{(\gamma s+1)}{s^2} \qquad * \ \gamma = a/b$$

[Ex. 11.2] Vehicle steering

$P(s) = \dfrac{(\gamma s+1)}{s^2}$

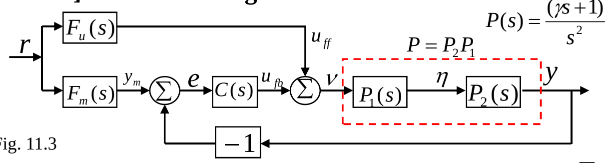
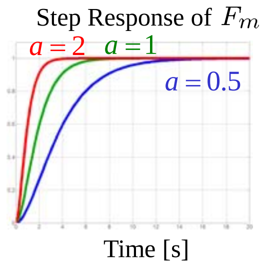


Fig. 11.3

desired response

$$F_m(s) = \frac{a^2}{(s+a)^2}$$

$\longrightarrow$ a nice response without overshoot



from (11.5)

$$F_u = \boxed{\frac{F_m}{P}} = \frac{a^2s^2}{(\gamma s+1)(s+a)^2}$$

must be proper (stable as long as $\gamma > 0$)

[Ex. 11.2] Vehicle steering

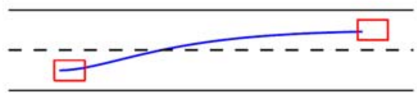


Fig. 11.4(a) Overhead view

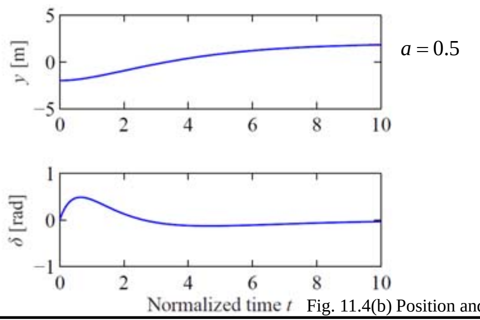
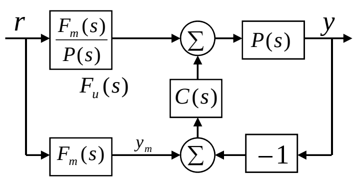
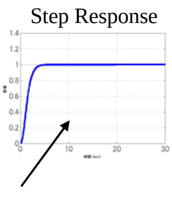


Fig. 11.4(b) Position and steering

[Ex. 11.3] Third-order system





process

$$P(s) = \frac{1}{(s+1)^3}$$

desired response

$$F_m(s) = \frac{1}{(0.5s+1)^3}$$

PI feedback controller

$$C(s) = \frac{0.6s+0.5}{s}$$

feedforward controller

$$F_u = \boxed{\frac{F_m}{P}} = \frac{(s+1)^3}{(0.5s+1)^3}$$

must be proper

[Ex. 11.3] Third-order system

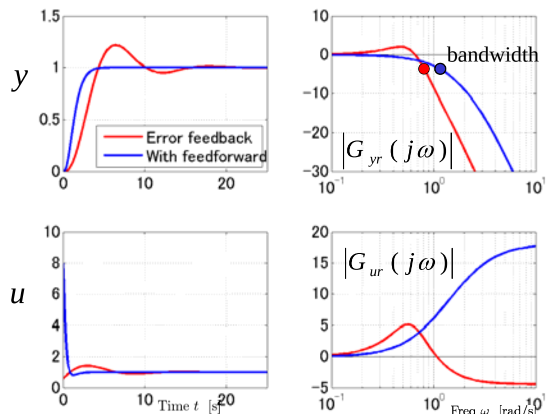


Fig. 11.5 (a) Step response   Fig. 11.5 (b) Frequency response

11.3 Performance Specifications*

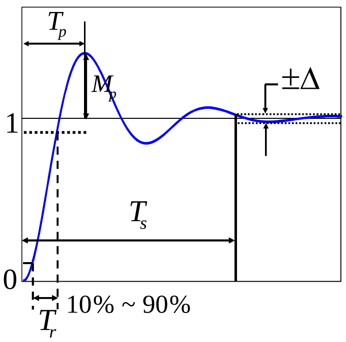
Time Responses (§ 5.3)

Performance criteria

- Rise time T_r
- Settling time T_s
- Peak time T_p
- Overshoot M_p
- Error tolerance Δ

$\Delta = 2\%$ and $\Delta = 5\%$ are the most widely used

Step response

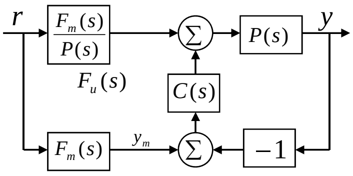


Feedforward Design*

Ideal feedforward

$$F_u = \frac{F_m}{P} \Rightarrow y = F_m r$$

How to build F_m?



First-order System

$$F_m(s) = \frac{K}{Ts+1}$$

Rise time

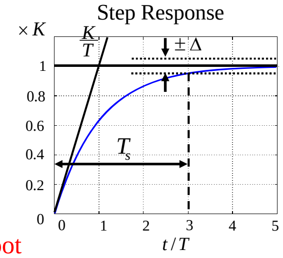
$$T_r = (\ln 9)T \approx 2.2T$$

Settling time

$$T_s \approx \begin{cases} 3T & \text{if } \Delta = 5\% \\ 4T & \text{if } \Delta = 2\% \end{cases}$$

The step response has no overshoot

Step Response



Second-order System*

$$F_m(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n > 0$: natural frequency

$\zeta \geq 0$: damping ratio

$\left(\omega_d = \omega_n\sqrt{1-\zeta^2}\right)$

Rise time

$$T_r \approx \frac{\pi/2 + \arcsin\zeta}{\omega_d}$$

Settling time

$$T_s = \begin{cases} \dfrac{4}{\zeta\omega_n} & \text{if } \Delta = 2\% \\ \dfrac{3}{\zeta\omega_n} & \text{if } \Delta = 5\% \end{cases}$$

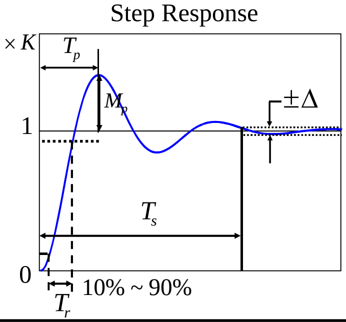
Peak time

$$T_p = \frac{\pi}{\omega_d}$$

Overshoot

$$M_p = Ke^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

Step Response



Butterworth Filter*

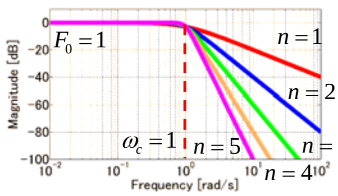
Low-pass filter

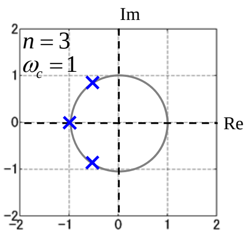
$$|F_m(j\omega)| = \frac{F_0}{\sqrt{1 + \left(\dfrac{\omega}{\omega_c}\right)^{2n}}}$$

n : order of filter

ω_c : cutoff frequency

F_0 : DC gain





slope: $-20n$ [dB/dec]

Butterworth Filter*

Denominator polynomial

$n=1$	$s + \omega_c$
$n=2$	$s^2 + 1.4\omega_c s + \omega_c^2$
$n=3$	$s^3 + 2.0\omega_c s^2 + 2.0\omega_c^2 s + \omega_c^3$
$n=4$	$s^4 + 2.6\omega_c s^3 + 3.4\omega_c^2 s^2 + 2.6\omega_c^3 s + \omega_c^4$
$n=5$	$s^5 + 3.24\omega_c s^4 + 5.24\omega_c^2 s^3 + 5.24\omega_c^3 s^2 + 3.24\omega_c^4 s + \omega_c^5$

Low-pass filter High-pass filter

$$s \Rightarrow \frac{1}{s}$$

7th Lecture

11 Frequency Domain Design

11.4 Feedback Design via Loop Shaping: Example (pp.326 to 331)

Keyword : Lead and Lag Compensation

11.2 Feedforward Design (pp.319 to 322)

Keyword : Feedforward, 2 Degree of Freedom

11.3 Performance Specifications (pp.322 to 326)

Keyword : Time Domain Analysis, Step Response