2nd Lecture

12 Robust Performance

12.1 Modeling Uncertainty (pp.347 to 352)
(9.2 The Nyquist Criterion) (pp.270 to 278)
(9.3 Stability Margins) (pp.278 to 282)
12.2 Stability in the Presence of Uncertainty (pp.352 to 358)

Keyword : Modeling Uncertainty
Robust Stability

12.2 Stability in the Presence of Uncertainty
(12.3 Performance in the Presence of Uncertainty) (pp.358 to 361)
(11.5 Fundamental Limitation) (pp.331 to 340)

Keyword : Complementary Sensitivity Function
Small Gain Theorem

12.1 Modeling Uncertainty

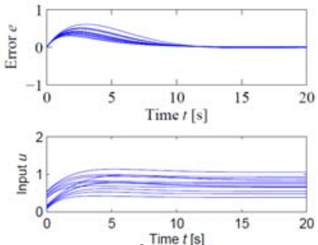
- **Parametric uncertainty**

parameters describing the system are unknown

Mass of a car changes with the number of passengers

mass $1600 < m < 2000$

gear ratio $\alpha = 10, 12, 14$ (3rd) (4th) (5th)   speed $10 \le v_e \le 40$



Disturbance response

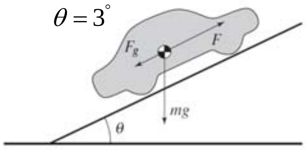


Fig. 3.3

Modeling Uncertainty

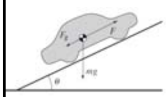
- **Parametric uncertainty**

parameters describing the system are unknown

Ex. 12.1 ⟶ The design based on a simple nominal model will give satisfactory control.

- **Unmodeled dynamics**

neglected mechanisms such that the simple model does not include.



- detailed model of the engine dynamics
- slight delays that can occur in electronically controlled engines

Unmodeled Dynamics

Vibration mode

$$\tilde{P} = \frac{0.5}{s} + \Delta \qquad \Delta = \sum_{i=1}^{4} \frac{0.2s}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}$$

P vibration mode

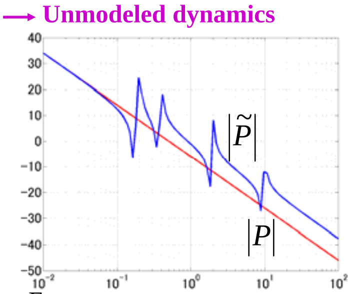
$\omega_1 = 0.2, \omega_2 = 0.5, \omega_3 = 2$ ⟶ **Unmodeled dynamics**

$\omega_4 = 10, \zeta_i = 0.02$

Nominal System P
(Rigid body mode)

$$P = \frac{0.5}{s}$$

$$\tilde{P} = P + \Delta$$

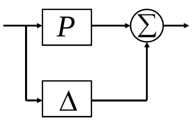


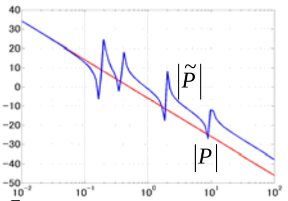
Frequency response

Unmodeled Dynamics

Additive perturbations

$$\tilde{P} = P + \Delta$$

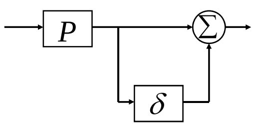




Frequency response

Multiplicative perturbations

$$\tilde{P} = (1 + \delta) P$$



$$\delta := \Delta / P$$

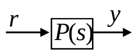
P : nominal model
$\tilde{P}$: actual model

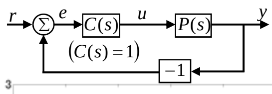
Δ, δ : unmodeled dynamics

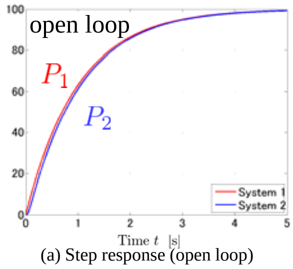
[When Are Two Systems Similar ?]

[Ex. 12.2] Similar in Open Loop but Large Differences in Closed Loop

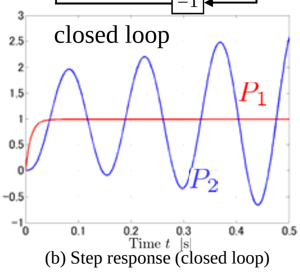
$$P_1(s) = \frac{k}{s+1} \qquad P_2(s) = \frac{k}{(s+1)(sT+1)^2} \qquad T = 0.025, \; k = 100 \qquad (12.1)$$







(a) Step response (open loop)



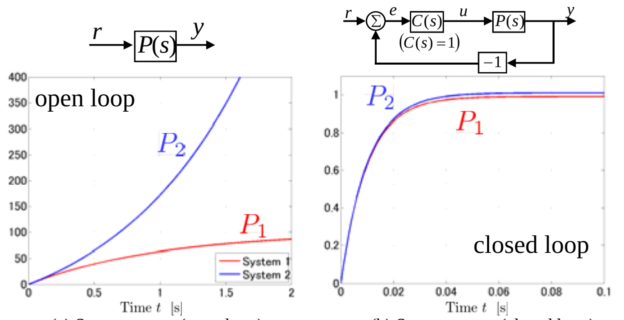
(b) Step response (closed loop)

Fig. 12.3(a)

[When Are Two Systems Similar ?]

[Ex. 12.3] Different in Open Loop but Similar in Closed Loop

$$P_1(s) = \frac{k}{s+1} \qquad P_2(s) = \frac{k}{(s-1)} \qquad k = 100 \qquad (12.2)$$



(a) Step response (open loop) (b) Step response (closed loop)

Fig. 12.3(b)

Nyquist Criterion (**§** 9.2)

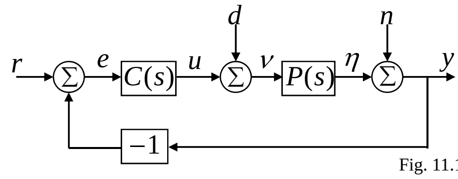


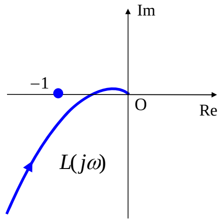
Fig. 11.1

Loop Transfer Function

$L = PC$

Nyquist's Stability Theorem

Theorem 9.1 and 9.2



Stability Margin (**§** 9.3)

Gain Margin

$g_m = 1/|L(i\omega_{pc})|$ (9.5) $g_m = 2-5$ (6-14 dB)

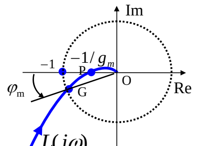


Fig. 9.9 (a)

Phase Margin

$\varphi_m = \pi + \arg L(i\omega_{gc})$ (9.6) $\varphi_m = 30^\circ - 60^\circ$

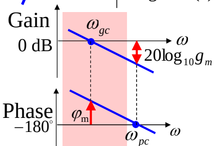


Fig. 9.9 (b)

Stability Margin s_m

$s_m = 1/M_s \qquad 0.5 < s_m < 0.8$

($M_s = 1/s_m$: maximum sensitivity)

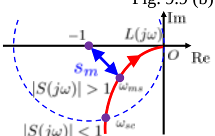
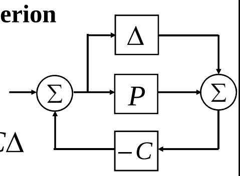


Fig. 11.6(b)

Robust Stability Using Nyquist's Criterion

Additive Uncertainty

$P \longrightarrow P + \Delta$ Δ : stable perturbations



Loop Transfer Function

$$L = PC \longrightarrow (P+\Delta)C = PC + C\Delta$$
$$= L + C\Delta$$

Perturbed Nyquist curve doesn't reach -1 (i.e. perturbed closed loop is stable) when

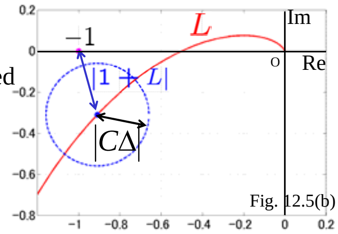


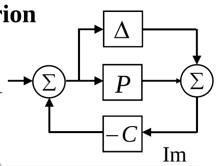
Fig. 12.5(b)

Nyquist plot (additive uncertainty)

Robust Stability Using Nyquist's Criterion

Additive Uncertainty

Perturbed Nyquist curve doesn't reach -1 when



$$|C\Delta| < |1+L| \qquad |\Delta| < \left|\frac{1+PC}{C}\right|$$

$$|\Delta| < \frac{1}{|CS|} \qquad \left(\because S = \frac{1}{1+PC}\right)$$

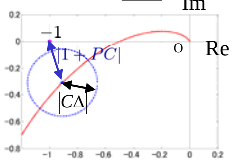
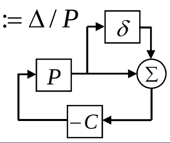


Fig. 12.5(b)

Multiplicative Uncertainty

$\delta := \Delta / P$

$$|\delta| = \left|\frac{\Delta}{P}\right| < \left|\frac{1+PC}{PC}\right| = \frac{1}{|T|} \qquad \left(\because T = \frac{PC}{1+PC}\right)$$



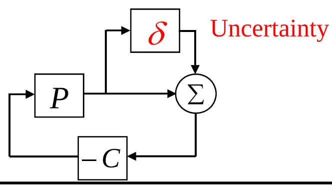
Robust Stability Using Nyquist's Criterion

Sufficient condition for robust stability

$$|\delta(j\omega)| < \frac{1}{|T(j\omega)|} \qquad \forall \omega \geq 0 \qquad (12.6)$$

small $|T(j\omega)| \longrightarrow$ big $|\delta(j\omega)|$ is allowed

big $|T(j\omega)| \longrightarrow$ small $|\delta(j\omega)|$ is allowed



Performance in the Presence of Uncertainty (§ 12.3)

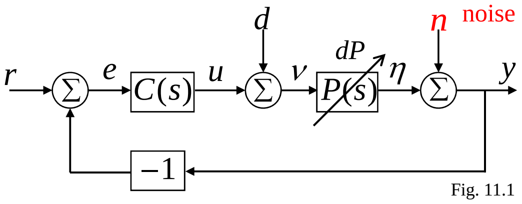


Fig. 11.1

$n \to u$

$$G_{un} = -\frac{C}{1+PC} = -\frac{T}{P} \;\Rightarrow\; \frac{dG_{un}}{G_{un}} = -T\frac{dP}{P} \quad (12.13)$$

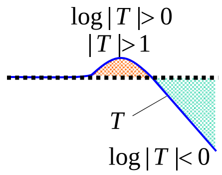
Exercise

Measurement noise typically has high frequencies

Bode's Integral Formula (§ 11.5)

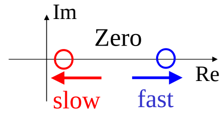
Complementary sensitivity function:

$$\int_0^\infty \frac{\log|T(j\omega)|}{\omega^2}d\omega = \pi\sum\frac{1}{z_i} \quad (11.20)$$



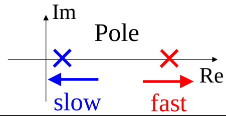
where the summation is over all right half-plane zeros.

RHP zeros fast (big): better
slow (small): worse



Sensitivity function: (1st lecture)

$$\int_0^\infty \log|S(j\omega)|d\omega = \pi\sum p_k \quad (11.19)$$



Complementary Sensitivity Function

$$T(s) = \frac{P(s)C(s)}{1+P(s)C(s)} \qquad \left(S+T = \frac{1}{1+PC} + \frac{PC}{1+PC} = 1 \right)$$

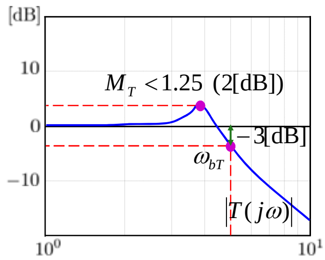
ω_{bT} : Complementary Sensitivity Bandwidth Frequency

$$|T(j\omega)| = \frac{1}{\sqrt{2}} \quad (-3[\text{dB}])$$

M_T : Maximum Peak Magnitude of $T(j\omega)$

$$M_T = \max_\omega |T(j\omega)|$$

$$M_T < 1.25 \;\; (2[\text{dB}])$$



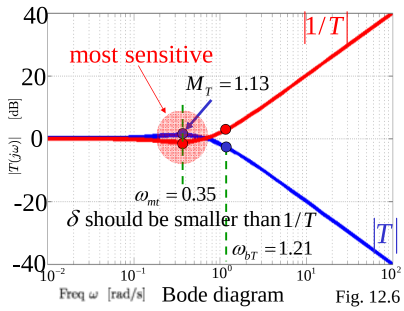
[Ex. 12.5] Cruise Control

Robust Stability (sufficient condition)

$$|\delta(j\omega)| < \frac{1}{|T(j\omega)|}$$

$$P(s) = \frac{1.38}{s+0.0142}$$

$$C(s) = 0.72 + \frac{0.18}{s} \quad \text{: PI controller}$$



Bode diagram Fig. 12.6

[Ex. 12.5] Cruise Control

Robust Stability (sufficient condition)

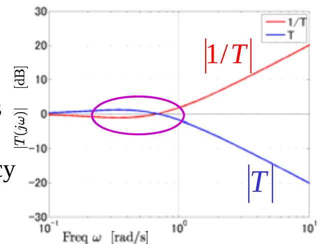
$$|\delta| < \frac{1}{|T|}$$

around the gain crossover frequencies

small $|\delta(j\omega)|$ is required

⬇

A simple model that describes the process dynamics well around the crossover frequency is often sufficient for design



Bode diagram Fig. 12.6 '

Robust Stability Using Small Gain Theorem

sufficient condition for robust stability

$$|\delta(j\omega)| < \frac{1}{|T(j\omega)|} \quad \forall\, \omega \geq 0 \quad (12.6)$$

another interpretation by using small gain theorem

Theorem 9.4 **Small Gain Theorem** (§ 9.5)

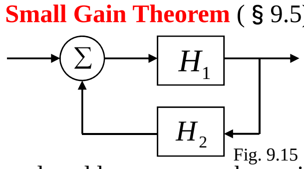


Fig. 9.15

Consider the closed loop system shown in Fig. 9.15, where H_1 and H_2 are stable systems and the signal spaces are properly defined. Let the gains of the systems H_1 and H_2 be γ_1 and γ_2 . Then the closed loop system is input/output stable if $\gamma_1\gamma_2 < 1$.

Robust Stability Using Small Gain Theorem

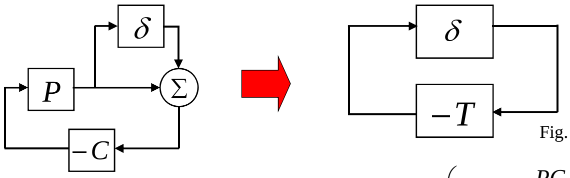


Fig. 12.7

$\left(\because T = \frac{PC}{1+PC} \right)$

sufficient condition for robust stability

$$|\delta| < \frac{1}{|T|}$$

$$|\delta| \cdot |T| < 1$$

↓ Small Gain Theorem

Closed loop system is input/output stable

Load Sensitivity Function* (§ 11.3)

$$G_{yd} = \frac{P}{1+PC} = PS = \frac{T}{C}$$

$C(s)$: with integral action $\frac{k_i}{s}$

(11.8, 11.9)

$$G_{yd} = \frac{T}{C} \approx \frac{1}{C} \approx \frac{s}{k_i} \quad \text{for small } \omega \quad (T \approx 1)$$

$$G_{yd} = PS \approx P \quad \text{for large } \omega \quad (S \approx 1)$$

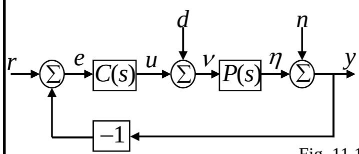
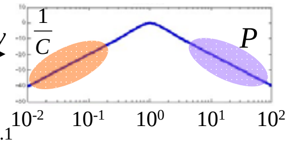


Fig. 11.1



$$P = \frac{1}{s+1}$$

Noise Sensitivity Function* (§ 11.3)

$$-G_{un} = \frac{C}{1+PC} = CS = \frac{T}{P} \quad (11.10)$$

$$-G_{un} = \frac{T}{P} \approx \frac{1}{P} \quad \text{for small } \omega \quad (T \approx 1)$$

$$-G_{un} = CS \approx C \quad \text{for large } \omega \quad (S \approx 1)$$

$$P = \frac{1}{(s+1)^3}$$

$$C = \left(k_p + \frac{k_i}{s} + k_d s \right) \frac{1}{T_f^2 / 2s^2 + T_f s + 1}$$

PID + 2nd-order noise filter

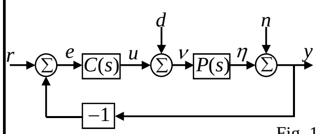
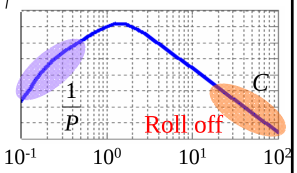


Fig. 11.1



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(9.2 The Nyquist Criterion) (pp.270 to 278)

(9.3 Stability Margins) (pp.278 to 282)

12.2 Stability in the Presence of Uncertainty (pp.352 to 358)

Keyword : Modeling Uncertainty
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12.2 Stability in the Presence of Uncertainty

(12.3 Performance in the Presence of Uncertainty) (pp.358 to 361)

(11.5 Fundamental Limitation) (pp.331 to 340)

Keyword : Complementary Sensitivity Function
Small Gain Theorem

Next: 3rd Lecture

11 Frequency Domain Design

11.4 Feedback Design via Loop Shaping (pp.326 to 331)

(9.4 Bode's Relations and Minimum Phase Systems) (pp.283 to 285)

Keyword : Loop Shaping
Bode's Relations

11.5 Fundamental Limitations (pp.331 to 340)

Keyword : Right Half-Plane Poles and Zeros
Gain Crossover Frequency Inequality