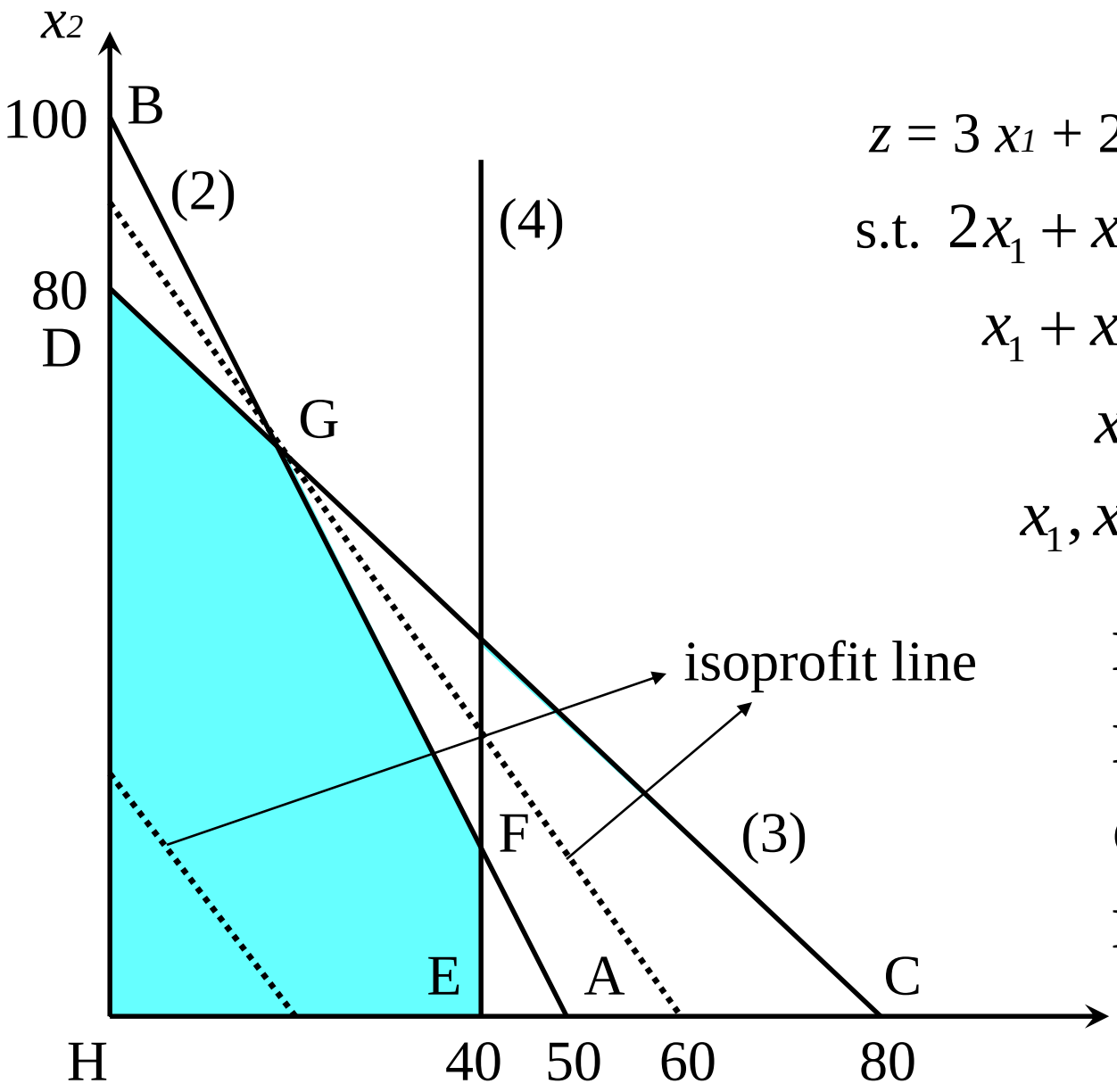
Finding the Feasible Solution



$$z = 3x_1 + 2x_2$$

$$\text{s.t. } 2x_1 + x_2 \le 100 \quad (2)$$

$$x_1 + x_2 \le 80 \quad (3)$$

$$x_1 \le 40 \quad (4)$$

$$x_1, x_2 \ge 0$$

Binding

Nonbinding

Convex Set

Extreme point

Initial Tableau

H

z	x_1	x_2	s_1	s_2	s_3	rhs	BV
1	-3	-2	0	0	0	0	z
0	2	1	1	0	0	100	s_1
0	1	1	0	1	0	80	s_2
0	1	0	0	0	1	40	s_3

First Tableau

E

z	x_1	x_2	s_1	s_2	s_3	rhs	BV
1	0	-2	0	0	3	120	z
0	0	1	1	0	-2	20	s_1
0	0	1	0	1	-1	40	s_2
0	1	0	0	0	1	40	x_1

Second Tableau

F

z	x_1	x_2	s_1	s_2	s_3	rhs	BV
1	0	0	2	0	-1	160	z
0	0	1	1	0	-2	20	x_2
0	0	0	-1	1	1	20	s_2
0	1	0	0	0	1	40	x_1

Optimal Tableau

G

z	x_1	x_2	s_1	s_2	s_3	rhs	BV
1	0	0	1	0	0	180	z
0	0	1	-1	2	0	60	x_2
0	0	0	-1	1	1	20	s_3
0	1	0	1	-1	0	20	x_1

4.4 Simplex Algorithm to Solve Minimization Problems

$$\min \; z = 2x_1 - 3x_2$$
$$\text{s.t.} \quad x_1 + x_2 \le 4$$
$$x_1 - x_2 \le 6$$
$$x_1, x_2 \ge 0$$

equivalent

Method 1

$$\max \; -z = -2x_1 + 3x_2$$
$$\text{s.t.} \quad x_1 + x_2 \le 4$$
$$x_1 - x_2 \le 6$$
$$x_1, x_2 \ge 0$$

Nonnegative (Max problem)

Initial Tableau

$-z$	x_1	x_2	s_1	s_2	rhs	BV
1	2	-3	0	0	0	$-z = 0$
0	1	1	1	0	4	$s_1 = 4$
0	1	-1	0	1	6	$s_2 = 6$

Optimal Tableau

$-z$	x_1	x_2	s_1	s_2	rhs	BV
1	5	0	3	0	12	$-z = 12$
0	1	1	1	0	4	$x_2 = 4$
0	2	0	1	1	10	$s_2 = 10$

z = -12

Method 2

$\min\ z = 2x_1 - 3x_2$

s.t. $x_1 + x_2 \le 4$

$x_1 - x_2 \le 6$

$x_1, x_2 \ge 0$

Initial Tableau

z	x_1	x_2	s_1	s_2	rhs	BV
1	-2	3	0	0	0	$z = 0$
0	1	1	1	0	4	$s_1 = 4$
0	1	-1	0	1	6	$s_2 = 6$

Optimal Tableau

Nonpositive

z	x_1	x_2	s_1	s_2	rhs	BV
1	-5	0	-3	0	-12	$z = -12$
0	1	1	1	0	4	$x_2 = 4$
0	2	0	1	1	10	$s_2 = 10$

Optimal (Min Problem)

Nonpositive coefficient of NBV in Row 0

$z - 5x_1 - 3s_1 = -12$

$z = -12 + 5x_1 + 3s_1$

4.5 Alternative Optimal Solution

$$\max \quad z = 60x_1 + 35x_2 + 20x_3$$

Optimal Tableau

	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	rhs	BV
	1	0	0	0	0	10	10	0	280	$z = 280$
	0	0	-2	0	1	2	-8	0	24	$s_1 = 24$
	0	0	-2	1	0	2	-4	0	8	$x_3 = 8$
	0	1	1.25	0	0	-0.5	1.5	0	2	$x_1 = 2^*$
	0	0	1	0	0	0	0	1	5	$s_4 = 5$

Another Optimal Tableau

	z	x_1	x_2	x_3	s_1	s_2	s_3	s_4	rhs	BV
	1	0	0	0	0	10	10	0	280	$z = 280$
	0	1.6	0	0	1	1.2	-5.6	0	27.2	$s_1 = 27.2$
	0	1.6	0	1	0	1.2	-1.6	0	11.2	$x_3 = 11.2$
	0	0.8	1	0	0	-0.4	1.2	0	1.6	$x_2 = 1.6$
	0	-0.8	0	0	0	0	-1.2	1	3.4	$s_4 = 3.4$

4.6 Unbounded LPs

$$\max \quad z = 36x_1 + 30x_2 - 3x_3 - 4x_4$$

$$\text{s.t.} \quad x_1 + x_2 - x_3 \le 5$$

$$6x_1 + 5x_2 - x_4 \le 10$$

$$x_1, x_2, x_3, x_4 \ge 0$$

Initial Tableau

z	x_1	x_2	x_3	x_4	s_1	s_2	rhs	BV
1	-36	-30	3	4	0	0	0	$z = 0$
0	1	1	-1	0	1	0	5	$s_1 = 5$
0	6	5	0	-1	0	1	10	$s_2 = 10$

First Tableau

z	x_1	x_2	x_3	x_4	s_1	s_2	rhs	BV
1	0	0	3	-2	0	6	60	$z = 60$
0	0	1/6	-1	1/6	1	-1/6	10/3	$s_1 = 10/3$
0	1	5/6	0	-1/6	0	1/6	5/3	$x_1 = 5/3$

Second Tableau

z	x_1	x_2	x_3	x_4	s_1	s_2	rhs	BV
1	0	2	-9	0	12	4	100	$z = 100$
0	0	1	-6	1	6	-1	20	$x_4 = 20$
0	1	1	-1	0	1	0	5	$x_1 = 5$

Impossible to do ratio test

⇒ Arbitrarily large z values