Chapter 3 Force Acting on a Dislocation

3.1 Peach-Koehler equation

$$\begin{cases} \tau : \text{applied shear stress} \\ l : \text{the length of the dislocation} \\ x : \text{distance that the dislocation moved} \\ b : \text{magnitude of the Burgers vector } \mathbf{b} \end{cases}$$

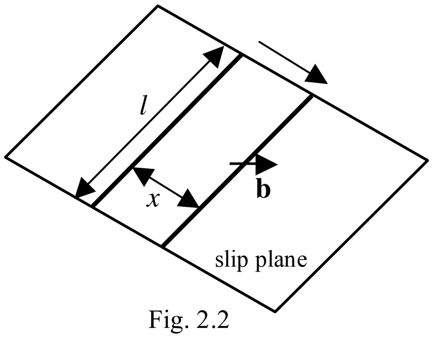

Fig. 2.2

the work done W by the applied stress

$$W = (\text{force}) \times (\text{distance}) = (\tau l x) \times b$$

Rearranging variables, we have

$$W = (\tau b l) \times x = (\text{force } \tau b l) \times (\text{distance } x) \tag{3.1}$$

$$\boxed{\text{Force acting on a unit length of a dislocation} = \tau b}$$

More strictly,

$$\begin{cases} \sigma_{ij} : \text{applied shear stress} \\ \mathbf{t} : \text{dislocation line (unit) vector} \\ \mathbf{f} : \text{force acting on a unit length of a dislocation} \\ A_j \equiv \sum_{i=1}^{3} \sigma_{ij} b_i \ : \text{a vector} \end{cases}$$

$$\boxed{\mathbf{f} = \mathbf{A} \times \mathbf{t}} \quad \text{(Peach-Koehler equation)} \tag{3.2}$$

The force is always acting perpendicularly to the dislocation.

$$\begin{cases} A_1 \equiv \sigma_{11} b_1 + \sigma_{21} b_2 + \sigma_{31} b_3 \\ A_2 \equiv \sigma_{12} b_1 + \sigma_{22} b_2 + \sigma_{32} b_3 \\ A_3 \equiv \sigma_{13} b_1 + \sigma_{23} b_2 + \sigma_{33} b_3 \end{cases} \qquad \begin{cases} f_1 = A_2 t_3 - A_3 t_2 \\ f_2 = A_3 t_1 - A_1 t_3 \\ f_3 = A_1 t_2 - A_2 t_1 \end{cases} \tag{3.3}$$

3.2 Interaction between two dislocations

3.2.1 parallel screw dislocations

dislocation I at $[0, 0, x_3]$: $\mathbf{b} = [0, 0, b]$, $\mathbf{t} = [0, 0, 1]$
dislocation II at $[x, d, x_3]$: $\mathbf{b} = [0, 0, b]$, $\mathbf{t} = [0, 0, 1]$

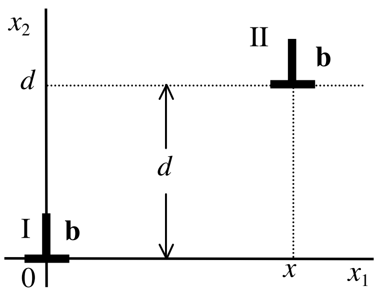

Fig. 2.3 two parallel screw dislocations

Force acting on dislocation II from dislocation I (from Eqs. (3.3))

$$f_1 = A_2 t_3 - A_3 t_2 = A_2 = \sigma_{32} b$$
$$f_2 = A_3 t_1 - A_1 t_3 = -A_1 = -\sigma_{31} b$$

$$f_3 = A_1 t_2 - A_2 t_1 = 0$$

From the above and Eq. (2.2)

$$\mathbf{f} = [f_1, f_2, f_3] = \left[\frac{\mu b^2}{2\pi} \frac{x}{x^2 + d^2}, \quad \frac{\mu b^2}{2\pi} \frac{d}{x^2 + d^2}, \quad 0 \right] \tag{3.4}$$

$$|\mathbf{f}| = \frac{\mu b^2}{2\pi r}, \qquad (r = \sqrt{x^2 + d^2}\,) \tag{3.5}$$

$$\mathbf{f} \begin{cases} \text{: repulsive (if the two screw dislocations have the same sign)} \\ \text{: attractive (if the two screw dislocations have opposite sign)} \end{cases}$$

3.2.2 parallel edge dislocations

dislocation I at $[0, 0, x_3]$: $\mathbf{b} = [b, 0, 0]$, $\mathbf{t} = [0, 0, 1]$
dislocation II at $[0, 0, x_3]$: $\mathbf{b} = [b, 0, 0]$, $\mathbf{t} = [0, 0, 1]$

$$\mathbf{f} = [f_1, f_2, f_3] = \left[\frac{Dx(x^2 - d^2)}{(x^2 + d^2)^2}, \quad \frac{Dd(3x^2 + d^2)}{(x^2 + d^2)^2}, \quad 0 \right], \quad D \equiv \mu b^2 / \{2\pi(1-\nu)\}$$

where

$D \equiv \pm \mu b^2 / \{2\pi(1-\nu)\}$, (+: same sign, -: opposite sign)

If the slip plane of each dislocation is unchanged, the force component f_1 determines the interaction.

$$f_1 = \frac{Dx(x^2 - d^2)}{(x^2 + d^2)^2} \tag{3.6}$$

same sign : stable when I and II line up vertically
opposite sign : stable when I and II are at the 45° position

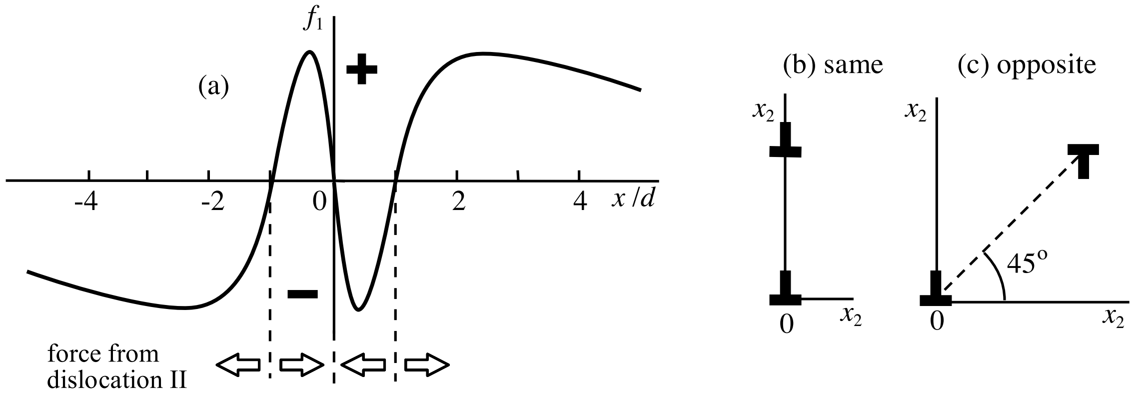

Fig. 2.4 Force acting on dislocation I from dislocation II