Nuclear Reactor Physics Lecture Note (1)
-Nuclear Reactions and Nuclear Cross Sections-

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1. Nuclear Reactions and Nuclear Cross Sections

1.1 Reactions between Neutrons and nuclei

- Nuclear fission (n, fission):

$$ {}^{1}_{0}\mathrm{n} + {}^{A_1}_{Z_1}\mathrm{X} \rightarrow {}^{A_2}_{Z_2}\mathrm{X} + {}^{A_3}_{Z_3}\mathrm{X} + \text{neutrons} + 200\mathrm{MeV} $$

- Radioactive capture (n, γ): excited state

$$ {}^{1}_{0}\mathrm{n} + {}^{A}_{Z}\mathrm{X} \rightarrow \left({}^{A+1}_{Z}\mathrm{X}\right)^{*} \rightarrow {}^{A+1}_{Z}\mathrm{X} + \gamma $$

- Scattering (n,n) or (n,n'):

$$ {}^{1}_{0}\mathrm{n} + {}^{A}_{Z}\mathrm{X} \rightarrow {}^{1}_{0}\mathrm{n} + {}^{A}_{Z}\mathrm{X} \qquad \text{elastic scattering (n, n)} $$

$$ {}^{1}_{0}\mathrm{n} + {}^{A}_{Z}\mathrm{X} \rightarrow {}^{1}_{0}\mathrm{n} + \left({}^{A}_{Z}\mathrm{X}\right)^{*} $$

$$ \rightarrow {}^{1}_{0}\mathrm{n} + {}^{A}_{Z}\mathrm{X} + \gamma \qquad \text{inelastic scattering (n, n}'\text{)} $$

1.2 Microscopic Cross Sections

The probability that a neutron-nuclear reaction will occur is characterized by a quantity called a nuclear cross section.

(1) Definition

Consider a beam of neutrons travelling with the same speed and direction and a sufficiently thin target (one atomic layer thick).

The rate R at which reactions occur per unit area on the target:

$$ \mathrm{R} = \sigma \cdot \mathrm{I} \cdot \mathrm{N_A} \ \left[\mathrm{cm^{-2}} \cdot \mathrm{S^{-1}}\right] $$

The microscopic cross section σ is defined as

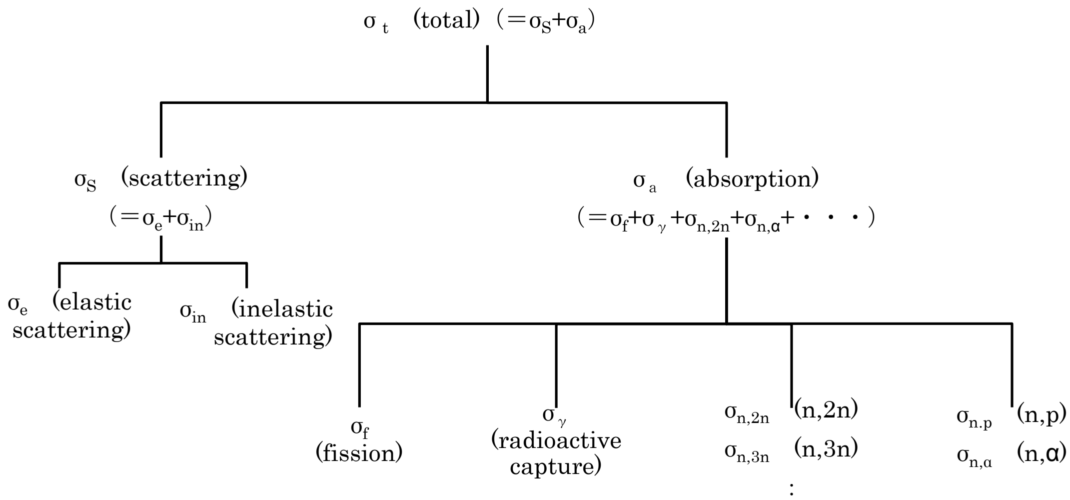
$$ \sigma = \frac{\left(\mathrm{R}/\mathrm{N_A}\right)}{\mathrm{I}} \left[\mathrm{cm^2}\right] $$

$$ = \frac{(\text{Number of reactions/nucleus/sec})}{(\text{Number of incident neutrons/cm}^2\text{/sec})} $$

The unit：b（barn）

$1b = 10^{-24} cm^2$

(2) Type of neutron cross sections



Microscopic cross sections depend on the nucleus, reactions and incident neutron speed (energy)

1.3 Macroscopic Cross Sections

(1) Definition

Consider a monoenergetic neutron beam incident on the surface of a target of arbitrary thickness.

The total reaction rate per unit area in dx:

$$dR = \sigma_t I N dx$$

The dR can be equal to the decrease in beam intensity between x and x+dx (with the prescription that any type of interactions will remove an incident neutron)

$$-dI(x) = \sigma_t I N dx$$

$$\therefore \frac{dI}{dx} = -N\sigma_t I(x)$$

$$\therefore I(x) = I_0 \exp(-\sigma_t N x)$$

The definition of total macroscopic cross section

$\Sigma_t \equiv \sigma_t N[\mathrm{cm}^{-1}]$

(2) Neutron mean free path and collision frequency

(Probability that a neutron moves a distance x without any interaction)

$= \frac{I_0 \exp(-\Sigma_t x)}{I_0} = \exp(-\Sigma_t x)$

So

(probability that a neutron has its first interaction in dx)

$= \exp(-\Sigma_t x) \cdot \Sigma_t dx \equiv p(x)dx$

We can calculate the average distance a neutron travels before interacting with a nucleus in the sample.

$$\bar{x} \equiv \int_0^\infty x p(x) dx = \Sigma_t \int_0^\infty x \exp(-\Sigma_t x) dx$$

$$= \frac{1}{\Sigma_t} \qquad \text{(neutron mean free path)}$$

Σ_t is the probability per unit path length that a neutron will undergo a reaction, so

$$\text{(The collision frequency)} = v\Sigma_t [\mathrm{s}^{-1}]$$

where v is the neutron speed.

(3) Macroscopic cross sections for specific reaction

ex. macroscopic fission cross section

$$\Sigma_f = \sigma_f N$$

macroscopic absorption cross section

$$\Sigma_a = \sigma_a N$$

(4) Macroscopic cross sections for mixture

ex. The total cross section of homogenized mixture of three different species of nuclide X, Y, and Z:

$$\Sigma_t = N_X \sigma_t^X + N_Y \sigma_t^Y + N_Z \sigma_t^Z$$