部材の弾塑性解析

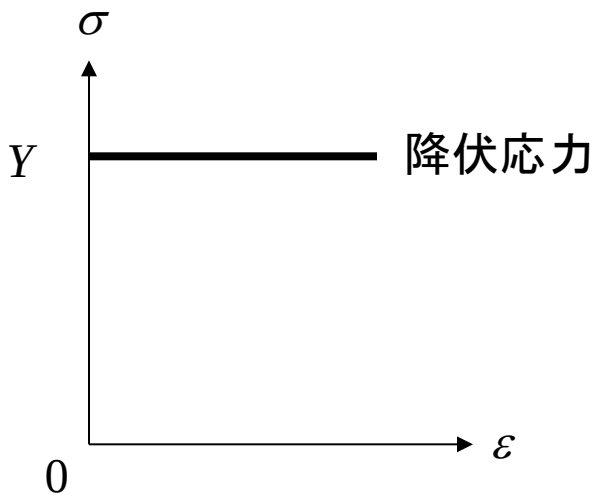
弾塑性力学

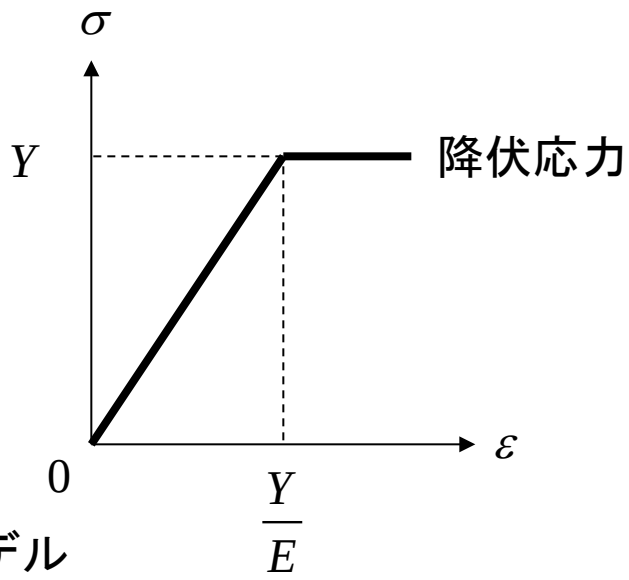
P.ティラポン

第9回 : 2014/7/3

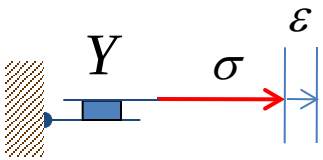
応力･ひずみ曲線の数式化

- 弾完全塑性体（ヤング率E，降伏強度Yが必要）
- 微小変形（形状の変化は無視できるものとする）

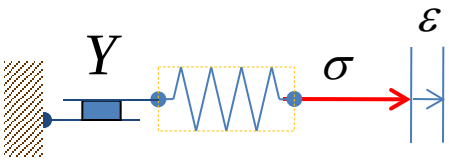




スライダー･バネのモデル



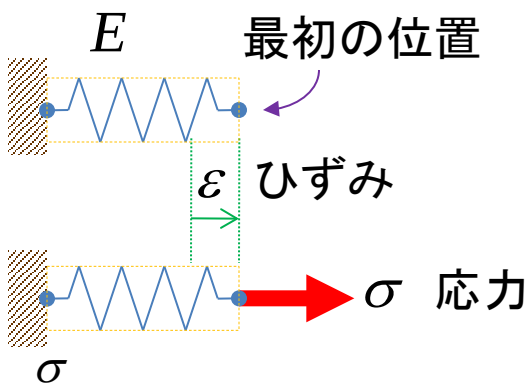
剛完全塑性(rigid-perfect plastic)

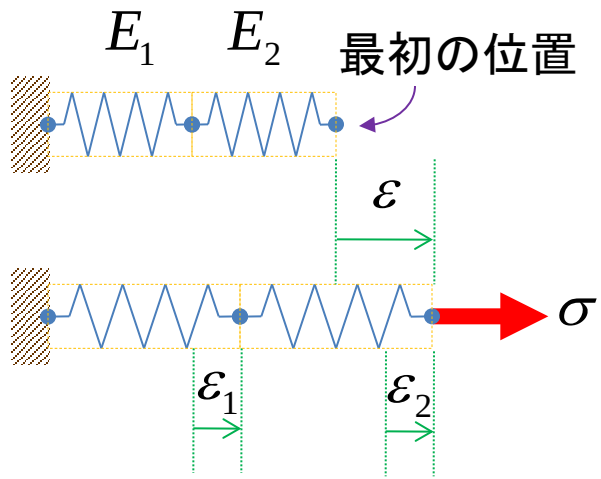


弾完全塑性(elastic-perfect plastic)

フック法則による
一次元問題

一次元の弾性問題



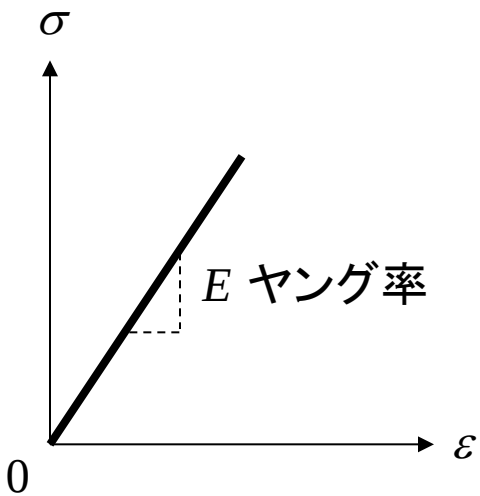


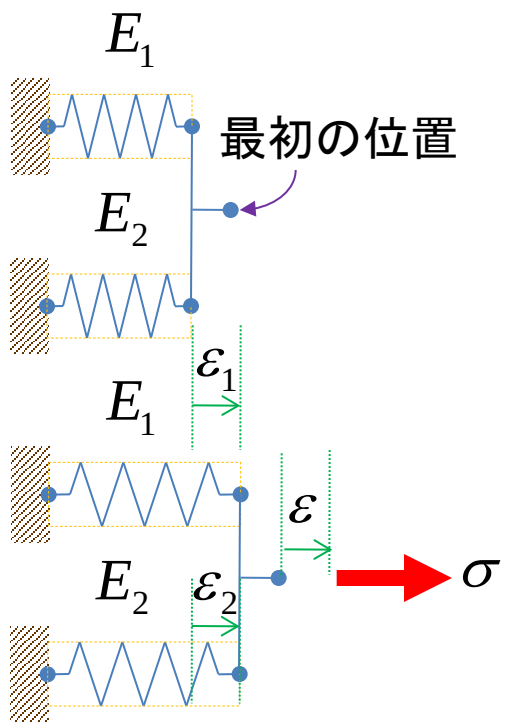
$$\sigma = \sigma_1 = \sigma_2$$
$$\varepsilon = \varepsilon_1 + \varepsilon_2$$

問題の条件

$$\frac{\sigma}{E} = \frac{\sigma_1}{E_1} + \frac{\sigma_2}{E_2}$$

$$E = \frac{E_1 E_2}{E_1 + E_2}$$





$$\sigma = \sigma_1 + \sigma_2$$
$$\varepsilon = \varepsilon_1 = \varepsilon_2$$

問題の条件

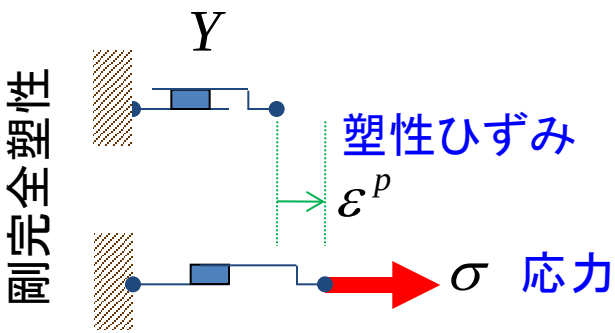
$$E\varepsilon = E_1\varepsilon_1 + E_2\varepsilon_2$$

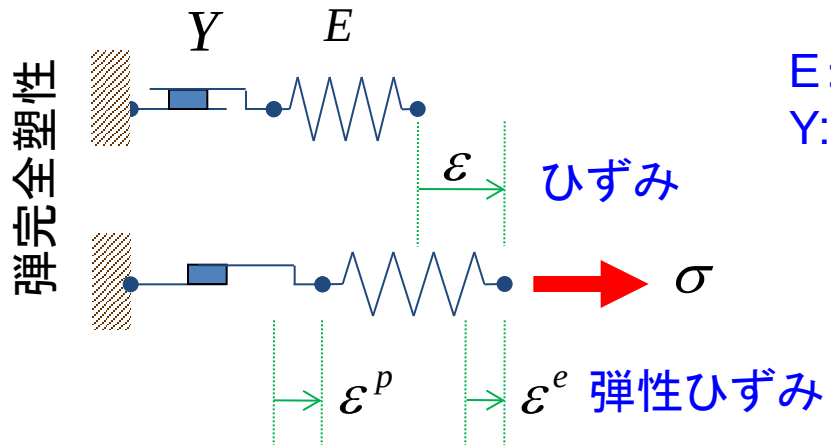
$$E = E_1 + E_2$$

ひずみエネルギ

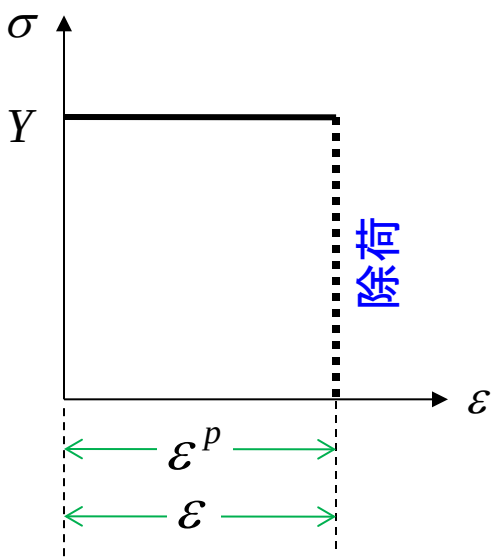
$$U = \frac{1}{2}\sigma\varepsilon = \frac{\varepsilon^2}{2E}$$

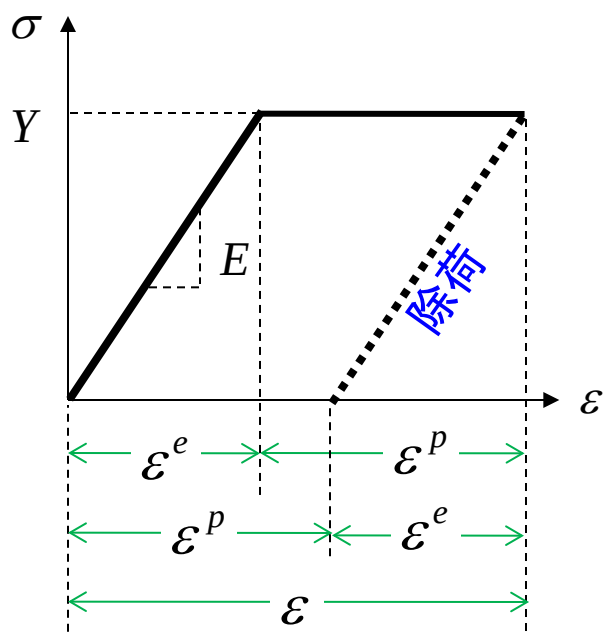
一次元の完全塑性問題





E:ヤング率
Y: 降伏応力





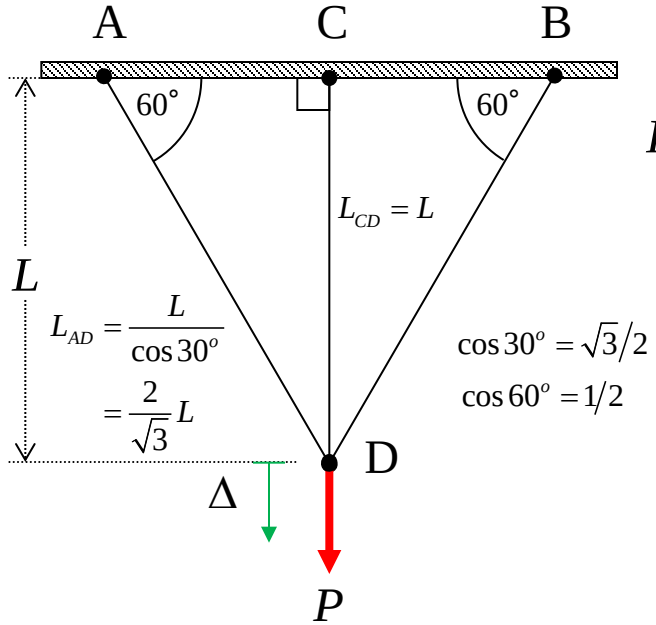
$f = |\sigma| - Y$ 降伏関数

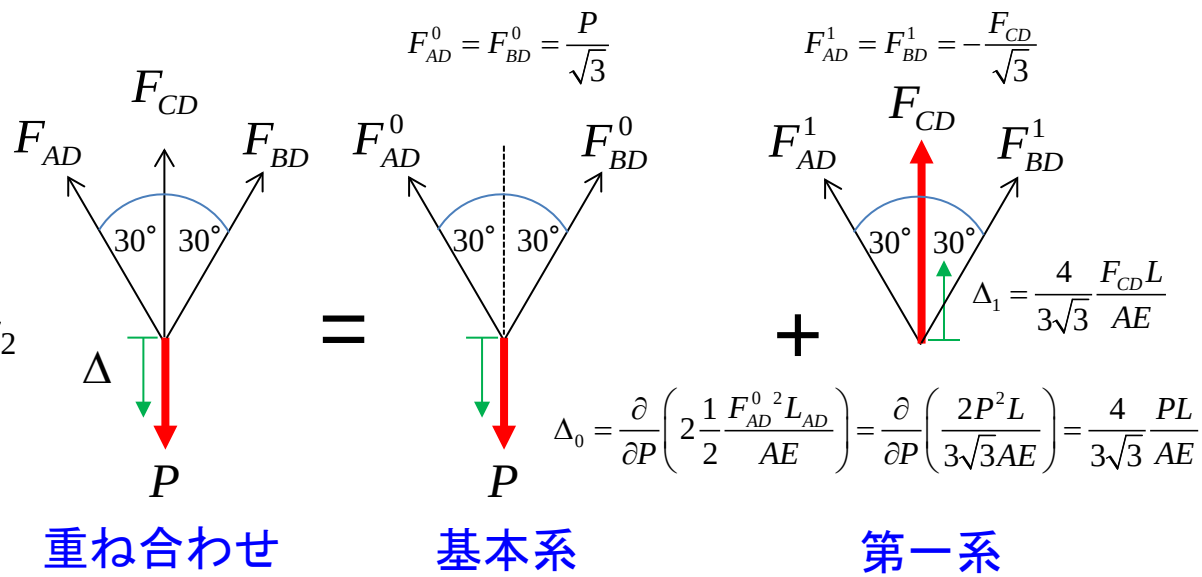
$f < 0 \rightarrow \varepsilon = \varepsilon^e$

$f = 0 \rightarrow \varepsilon = \varepsilon^e + \varepsilon^p$

3本ワイヤ(弾性)

応力法による解法





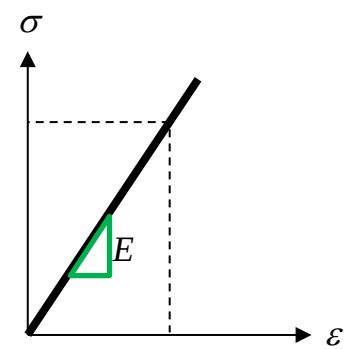
ワイヤの縦弾性係数Eを，断面積を等しくAとし，ワイヤの質量は無視できるものとする

$$\because F_{CD} = AE\frac{\Delta}{L} \Rightarrow \Delta = \frac{F_{CD}L}{AE}$$

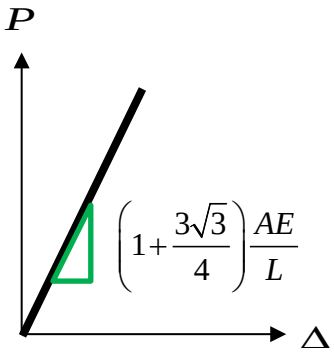
重ね合わせ

$$\Delta = \Delta_0 - \Delta_1 \Rightarrow \frac{F_{CD}L}{AE} = \frac{4}{3\sqrt{3}}\frac{PL}{AE} - \frac{4}{3\sqrt{3}}\frac{F_{CD}L}{AE}$$

$$\therefore P = \left(1 + \frac{3\sqrt{3}}{4}\right)F_{CD} = \left(1 + \frac{3\sqrt{3}}{4}\right)AE\frac{\Delta}{L}$$

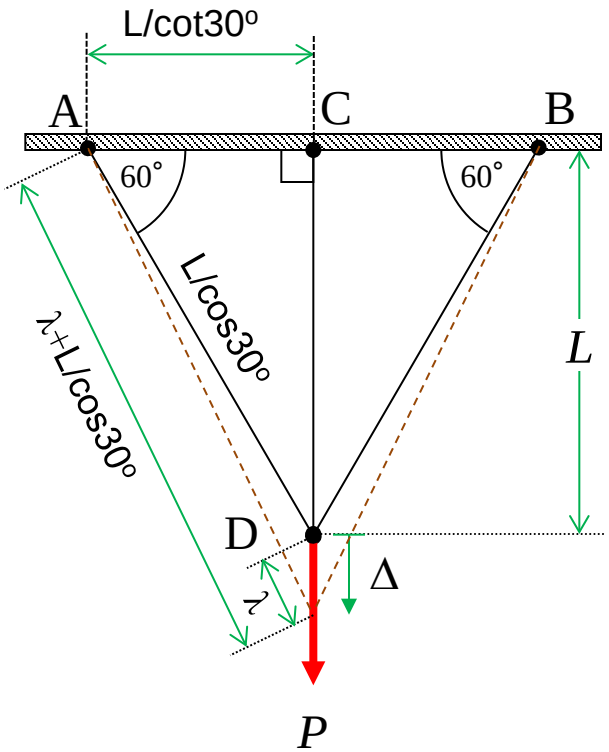


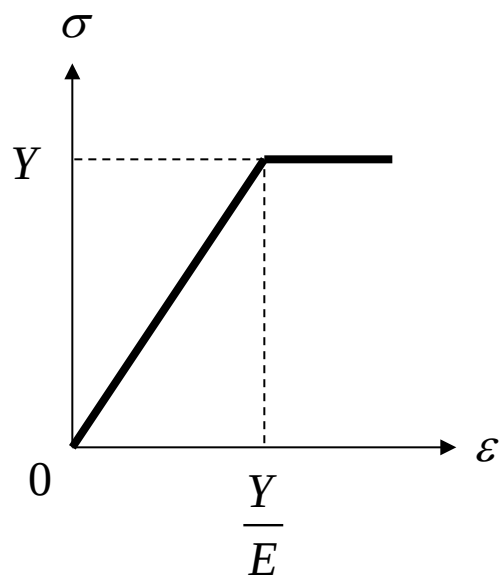
応力・ひずみ関係

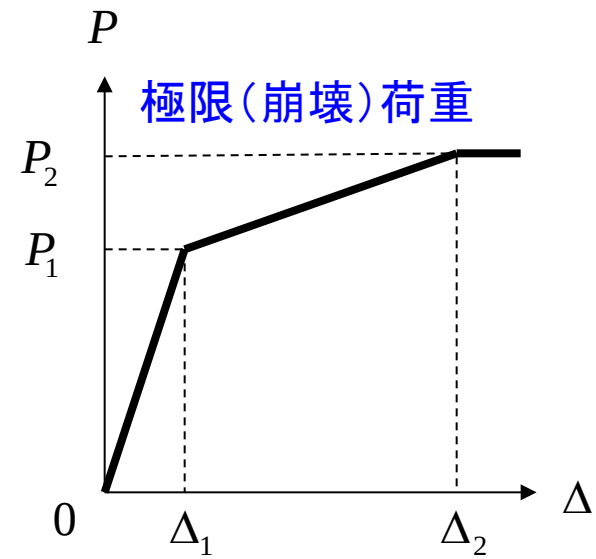


力・変位関係

3本ワイヤ(弾塑性)







変位法による解

$$\left(\frac{L}{\cos 30^o}+\lambda\right)^2=\left(\frac{L}{\cot 30^o}\right)^2+(L+\Delta)^2$$

$$\left(\frac{L}{\cos 30^o}\right)^2+2\frac{\lambda L}{\cos 30^o}+\lambda^2=\left(L\frac{\sin 30^o}{\cos 30^o}\right)^2+L^2+2\Delta L+\Delta^2$$

微小変形 $\lambda^2=\Delta^2\approx 0$

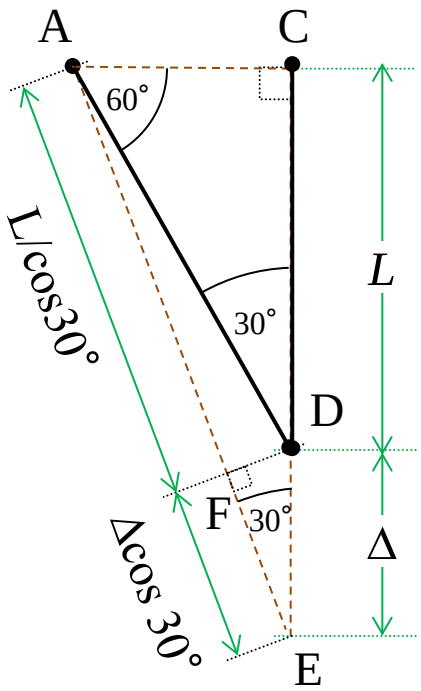
$$\left(\frac{L}{\cos 30^o}\right)^2+2\frac{\lambda L}{\cos 30^o}=L^2\left(1+\left(\frac{\sin 30^o}{\cos 30^o}\right)^2\right)+2\Delta L$$

$$\lambda=\Delta\cos 30^o$$

$$\Delta_1=\frac{Y}{E}L \qquad P_1=\left(1+\frac{3\sqrt{3}}{4}\right)AY\approx 2.299A\sigma_y$$

$$\Delta_2=\frac{4}{3}\frac{Y}{E}L=1.333\frac{Y}{E}L \qquad P_2=\left(1+\sqrt{3}\right)AY\approx 2.732A\sigma_y$$

変位法による解



ワイヤADに生じる伸びとワイヤCD に生じる伸びΔとの間の幾何学的関係から，次式が得られる $\Delta L_{CD} = \Delta \quad \Delta L_{AD} = \Delta\cos(30^o) = \frac{\sqrt{3}}{2}\Delta$

ワイヤの長さ $L_{CD} = L \quad L_{AD} = \frac{L}{\cos(30^o)} = \frac{2}{\sqrt{3}}L$

ワイヤAD, CDに生じるひずみ $\varepsilon_{CD} = \frac{\Delta}{L} \quad \varepsilon_{AD} = \frac{\Delta L_{AD}}{L_{AD}} = \frac{\frac{\sqrt{3}}{2}\Delta}{\frac{2}{\sqrt{3}}L} = \frac{3}{4}\frac{\Delta}{L}$

ワイヤAD, CDに生じる応力 $\sigma_{CD} = E\varepsilon_{CD} = \frac{E}{L}\Delta \quad \sigma_{AD} = E\varepsilon_{AD} = \frac{3}{4}\frac{E}{L}\Delta$

ワイヤCD, ADに生じる内力F_{CD}, F_{AD} $F_{CD} = A\sigma_{CD} = AE\frac{\Delta}{L} \quad F_{AD} = A\sigma_{AD} = \frac{3}{4}AE\frac{\Delta}{L}$

左右対称を考えて、点Dにおける鉛直方向の力の釣り合い $P = F_{CD} + 2\cos(30^o)F_{AD} = F_{CD} + \sqrt{3}F_{AD}$

$\sigma_{CD} > \sigma_{AD}$ であるため、ワイヤCD最初に降伏するので，$\Delta = \Delta_1$のとき $\sigma_{CD} = Y = \frac{E}{L}\Delta_1$ とすると $\Delta_1 = \frac{L}{E}Y$

力の釣り合い式に代入すれば $P_1 = F_{CD} + \sqrt{3}F_{AD} = AY + \sqrt{3}\left(\frac{3}{4}AE\frac{\Delta_1}{L}\right) = AY + \frac{3\sqrt{3}}{4}AE\frac{\left(\frac{L}{E}Y\right)}{L} = \left(1 + \frac{3\sqrt{3}}{4}\right)AY$

続いて$\Delta = \Delta_2$のとき，ワイヤADは降伏する $\sigma_{AD} = Y = \frac{3}{4}\frac{E}{L}\Delta_2$ とすると $\Delta_2 = \frac{4}{3}\frac{L}{E}Y$

力の釣り合い式に代入すれば $P_2 = AY + \sqrt{3}AY = \left(1 + \sqrt{3}\right)AY$

宿題（2013年試験問題）

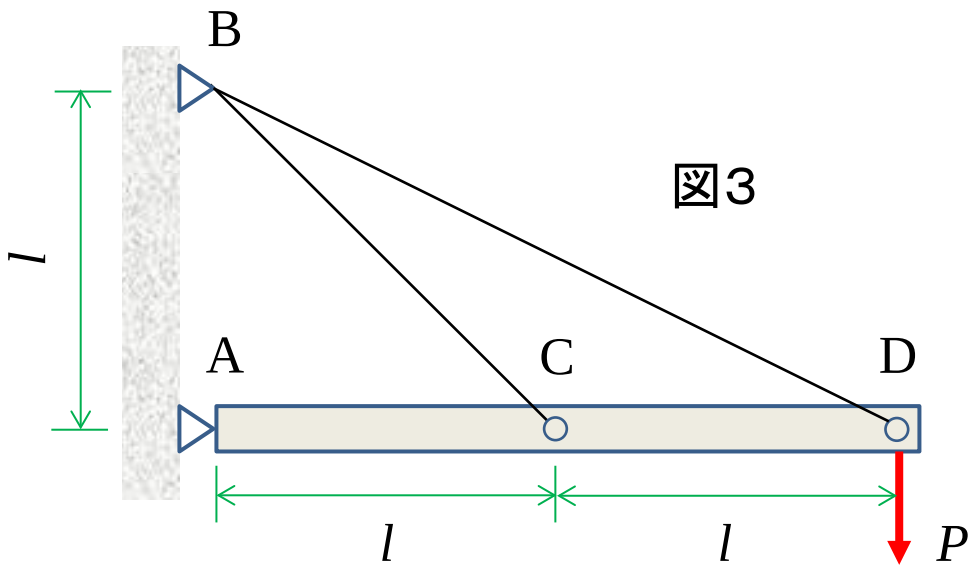


図3のように，剛体棒ADは壁と2本のワイヤによって水平に支持されている。各ワイヤの降伏応力は Y, 面積は a である。先端に垂直荷重が作用したときの崩壊荷重 P を求めよ。