平成26年度前期東京工業大学学部講義

電子デバイス

Electron Devices

Handout

Supplement (2) [重要数式集（1）]

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A) Carrier density calculation

1. Fermi level of n and p-type semiconductors

B) pn junctions

1. Built-in potential
2. Depletion layer width
3. Current voltage characteristics

C) Tables

●宿題説明と新規宿題

Equations for carrier concentration calculation

handout

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Effective density of states at conduction band 伝導帯における電子の実効状態密度	N_c
Effective density of states at conduction band 価電子帯における正孔の実効状態密度	N_v
Intrinsic carrier density	n_i

for Si		
effective mass for density calculation	m_{de}/m_o	0.32
	m_{dh}/m_o	0.54
number of minima in conduction band	M_c	8

$$N_c = 2\left(\frac{2\pi m_{de} kT}{h^2}\right)^{3/2} M_c = \boldsymbol{N_o} M_c \left(\frac{m_{de}}{m_o}\right)^{3/2} T^{3/2}$$

$$N_v = 2\left(\frac{2\pi m_{dh} kT}{h^2}\right)^{3/2} = \boldsymbol{N_o}\left(\frac{m_{dh}}{m_o}\right)^{3/2} T^{3/2}$$

$$\boldsymbol{N_o} = 2\left(\frac{2\pi m_o k}{h^2}\right)^{3/2} = 2\left(\frac{2\,\pi \times\ 9.109 \times 10^{-31} \times 1.38 \times 10^{-23}}{(6.64 \times 10^{-34})^2}\right)^{3/2}$$

$$= 4.83 \times 10^{21}\ [\mathrm{cm}^{-3}]$$

$$n_i{}^2 = np = N_c N_v exp\left(-\frac{E_c - E_v}{kT}\right) = N_c N_v exp\left(-\frac{E_g}{kT}\right) \cdots (3)$$

$n_i =$ $[\mathrm{cm}^{-3}]$ for Si at 300K

N-type semiconductor

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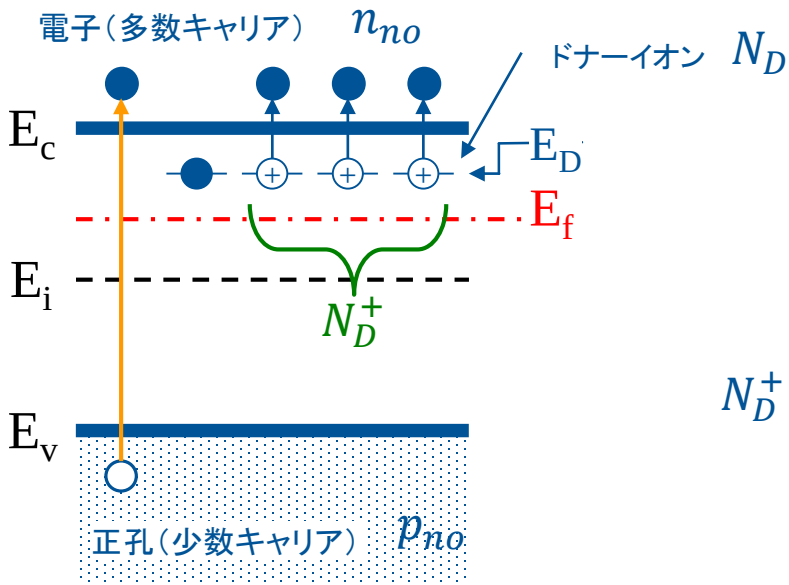
N-type semiconductor



$$n_{no} = N_c \cdot exp\left(-\frac{E_c - E_f}{kT}\right)$$

$$p_{no} = N_v \cdot exp\left(-\frac{E_f - E_v}{kT}\right)$$

$$N_D^+ = N_D\{1 - f_e(E_f)\}, f_e(E_f) = \frac{1}{1 + exp\left(\frac{E_d - E_f}{kT}\right)}$$

Doping density （ドーピング量）	N_D
Ionized Donor conc.（イオン化ドナー密度）	N_D^+
Majority carrier: electron density （多数キャリア）電子密度	p_{no}
Minority carrier: hole density （小数キャリア）正孔濃度	n_{no}
Charge neutrality condition（電荷中性条件） $N_D^+ + p_{no} = n_{no}$	

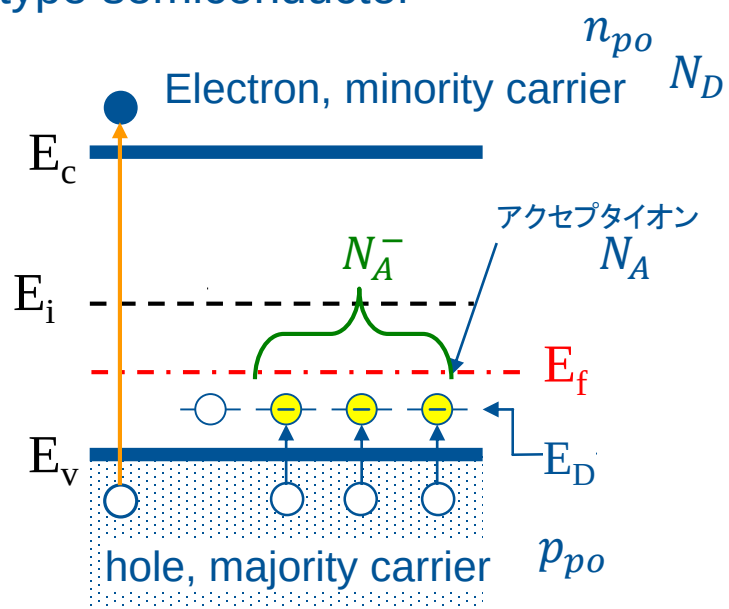
At room Temperature (300K): Donor atoms are almost ionized. 室温でドナー原子は殆どイオン化

$$n_{no} \approx N_D^+ \approx N_D, \quad p_{no} = \frac{n_i^2}{n_{no}} = \frac{n_i^2}{N_D}$$

$$E_f = E_i + \frac{kT}{q} log_e \frac{N_D}{n_i}$$

P-type semiconductor

P-type semiconductor



$$n_{po} = N_c \cdot exp\left(-\frac{E_c - E_f}{kT}\right)$$

$$p_{po} = N_v \cdot exp\left(-\frac{E_f - E_v}{kT}\right)$$

$$N_A^- = N_A f_e(E_f), f_e(E_f) = \frac{1}{1 + exp\left(\frac{E_d - E_f}{kT}\right)}$$

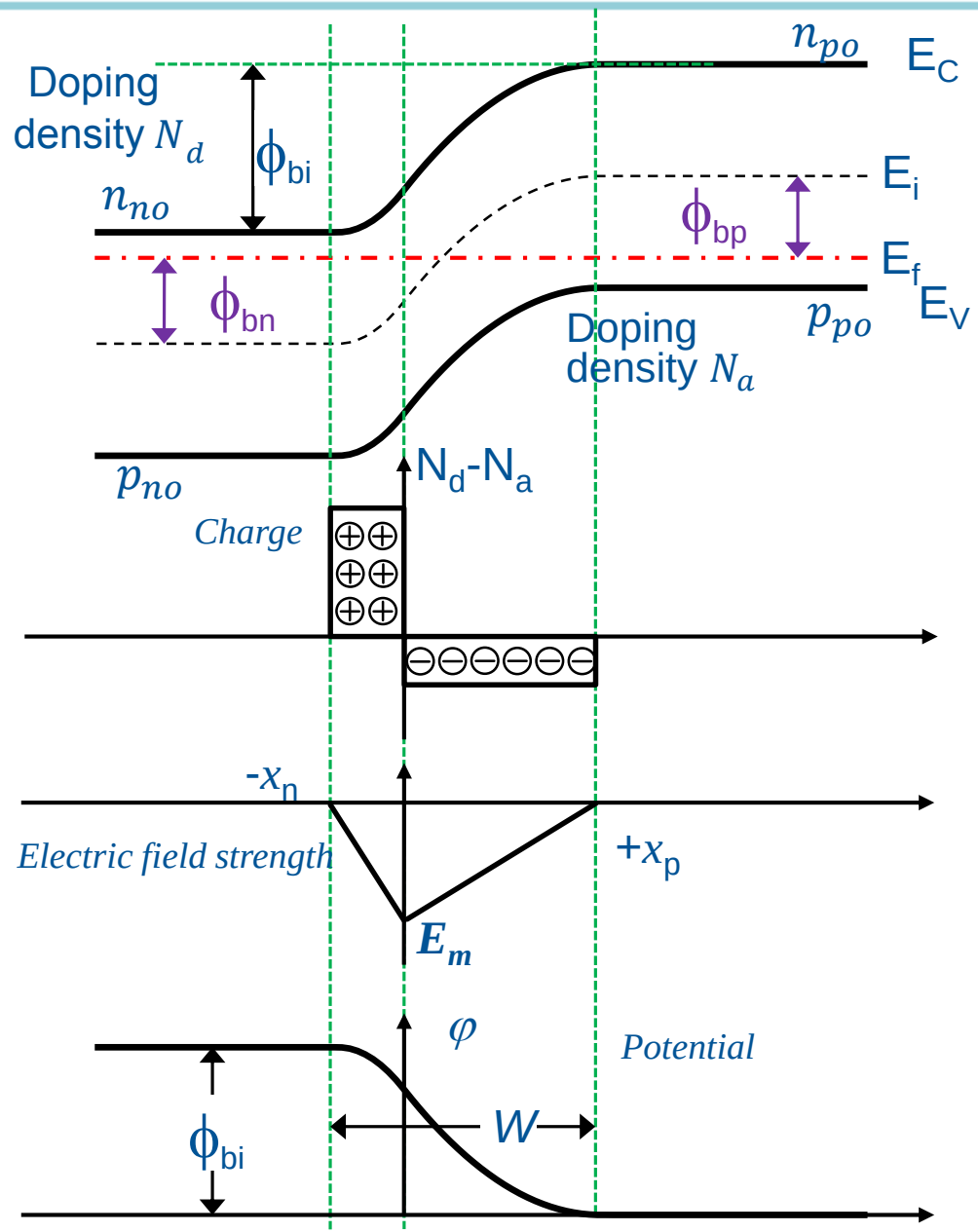
Doping density （ドーピング量）	N_A
Ionized Donor conc.(イオン化ドナー密度）	N_A^-
Majority carrier: hole density （多数キャリア）正孔密度	p_{po}
Minority carrier: electron density （小数キャリア）電子濃度	n_{po}
Charge neutrality condition（電荷中性条件） $N_A^- + n_{po} = p_{po}$	

At room Temperature (300K): Acceptor atoms are almost ionized. 室温でアクセプタ原子は殆どイオン化

$$p_{po} \approx N_A^- \approx N_A, \quad n_{po} = \frac{n_i^2}{p_{po}} = \frac{n_i^2}{N_A}$$

$$E_f = E_i - \frac{kT}{q} log_e \frac{N_A}{n_i}$$

Equations relating to n⁺p junction



Assumptions:

- Depletion layer approximation
- Room temperature
- Potential variation inside the depletion region is parabolic.

1. Built in potential

$$|\Phi_{bn}| = \frac{kT}{q} log_e \frac{N_d}{n_i},$$

$$|\Phi_{bp}| = \frac{kT}{q} log_e \frac{N_a}{n_i}, and$$

$$\Phi_{bi} = |\Phi_{bn}| + |\Phi_{bp}|$$

$$= \frac{kT}{q} log_e \frac{N_d N_a}{{n_i}^2}$$

Equations relating to n^+p junction

2. The relation among Φ_{bi}, W, x_n, x_p.

$$\Phi_{bi} = \frac{q\,N_a}{2\varepsilon}\,{x_p}^2 + \frac{q\,N_d}{2\varepsilon}\,{x_n}^2 \qquad N_d x_n = N_a x_p$$

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon(N_a + N_d)}{q\,N_a N_d}\,\Phi_{bi}}$$

$$x_n = \sqrt{\frac{2\varepsilon N_a}{q(N_a + N_d)N_d}\,\Phi_{bi}}\ ,\ x_p = \sqrt{\frac{2\varepsilon N_d}{q(N_a + N_d)N_a}\,\Phi_{bi}}$$

- $\varepsilon = \epsilon_r \epsilon_o$,
- ϵ_r:dielectric constant
- ϵ_r for Si: $\epsilon_{Si} = 11.9$

- When the forward bias voltage V is applied, Φ_{bi} is replaced to be $\Phi_{bi} - V$.

Equations relating to n^+p junction

1. Current thru the n^+p junction

$$J_{total} = q\left(\frac{n_{p0}D_e}{L_e} + \frac{p_{n0}D_h}{L_h}\right)\left\{exp\left(\frac{qV}{kT}\right) - 1\right\}$$

2. Current ratio

$$\gamma = \frac{electron\ current}{hole\ current} = \frac{n_{p0}}{p_{n0}}\frac{L_h}{L_e}\frac{D_e}{D_h} = \frac{N_d}{N_a}\frac{L_h}{L_e}\frac{D_e}{D_h}$$

3. For n^+p junction, $N_d \gg N_a$, $\gamma \gg 1$

N_d: n^+-region doping concentration
p_{no}: n^+-region electron density (minority carrier)
$N_d p_{no} = n_i{}^2$
L_e: electron diffusion length in n^+-region
D_e: electron diffusion coefficient in n^+-region

N_a: p-type doping concentration
n_{po}: p-type hole density (minority carrier)
$N_a n_{po} = n_i{}^2$
L_h: hole diffusion length in p-type region
D_h: hole diffusion coefficient in p-type region

SI単位系（MKSA単位系）

組立単位	読み	単位	他の表記	基本単位による組立
速度				m/s
加速度				m/s^2
力	Newton	N	$kg\cdot m/s^2$	F=ma
エネルギー	Joule	J	$kg\cdot m^2/s^2$	J = N·m
電流	Ampere	A		
電圧	Volt	V	J/C	$m^2\cdot kg\cdot s^{-3}\cdot A^{-1}$
電力 or 仕事率	Watt	W	$kg\cdot m^2/s^3$	W = J/s
電荷	Coulomb	C	A·s	s·A
磁束	Weber	Wb	V·s	$m^2\cdot kg\cdot s^{-2}\cdot A^{-1}$
磁束密度	Tesla	T	Wb/m^2	$kg\cdot s^{-2}\cdot A^{-1}$
抵抗	Ohm	Ω	V/A	$m^2\cdot kg\cdot s^{-3}\cdot A^{-2}$
インダクタンス	Henry	H	Wb/A	$m^2\cdot kg\cdot s^{-2}\cdot A^{-2}$
容量	Farad	F	C/V	$m^{-2}\cdot kg^{-1}\cdot s^4\cdot A^2$

$$\mu_o = 4\pi \times 10^{-7}[H/m] \quad \epsilon_o = (1/36\pi) \times 10^{-9}[F/m]$$

物理定数 Physical constant

Quantity	物理量	Symbol	Value 値	Unit 単位
Boltzmann constant	Boltzmann定数	k	$1.3806488 \times 10^{-23}$	J/K
Avogadro constant	アボガドロ数	N_A	$6.02214129 \times 10^{23}$	1/mol
Planck constant	Plank定数	h	$6.62606957 \times 10^{-34}$	Js
elementary charge	電気素量	q	$1.602176565 \times 10^{-19}$	C
electron charge	電子電荷	e		
electron mass	電子質量	m_o	$9.10938291 \times 10^{-31}$	kg
speed of light in vacuum	光速	c_o	2.99792458×10^{8}	m/sec
permittivity in vacuum	真空の誘電率	ϵ_o	$8.854187817 \times 10^{-12}$	F/m
permeability in vacuum	真空の透磁率	μ_o	$4\pi \times 10^{-7}$	H/m
electron volt		eV	$1.602176565 \times 10^{-19}$	J
thermal energy at 300K		kT	4.14195×10^{-21}	J
thermal voltage at 300K		kT/q	25.85199715	meV

The relation among c_o, μ_o, ϵ_o.

1. The relation among c_o, μ_o, ϵ_o are given by $c_o = 1/\sqrt{\mu_o \epsilon_o}$ from Maxwell equations. （Maxwellの方程式から導かれる））
2. At first, the speed of light is c_o measured experimentally. （光速は実測される）
3. The vacuum permeability μ_o is defined as $\mu_o = 4\pi \times 10^{-7}$ [H/m] （真空の透磁率は定義）
4. Then, ϵ_o is calculated by $\epsilon_o = c_o = 1/(c_o^2 \mu_o)$. （真空の誘電率は、光速と透磁率から求まる）

Properties of Ge, Si, and GaAs at 300K

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Properties	物理量		Ge	Si	GaAs
Atomic weight	原子量	g/mol	72.60	28.09	144.63
density	密度	g/cm^3	5.3267	2.328	5.32
crystal structure	結晶構造		Diamond	Diamond	Zincblende
lattice constant a	格子定数	Å	5.64613	5.43095	5.6533
dielectric constant ϵ_r	比誘電率		16	11.9	13.1
energy gap at 300K E_g	禁制帯幅	eV	0.66	1.12	1.424
electron affinity $q\chi$	電子親和力	eV	4	4.05	4.07
Intrinsic carrier conc. n_i	真性キャリア密度	cm^{-3}	2.4×10^{13}	1.5×10^{10}	1.79×10^{6}
effective density of states in conduction band N_c	実効状態密度（伝導帯）	cm^{-3}	1.04×10^{19}	2.8×10^{19}	4.7×10^{17}
effective density of states in valence band N_v	実効状態密度（価電子帯）	cm^{-3}	6.0×10^{18}	1.04×10^{19}	$7.0\mathrm{X}10^{18}$
electron effective mass for density of state calculation	実効状態密度有効質量（電子）	m_{de}/m_o	0.22	0.32	0.067
hole effective mass for density of state calculation	実効状態密度有効質量（正孔）	m_{dh}/m_o	0.29	0.54	0.47
electron effective mass for conductivity calculation	導電率有効質量（電子）	m_{ce}/m_o	0.12	0.26	0.067
hole effective mass for conductivity calculation	導電率有効質量（正孔）	m_{ch}/m_o	0.21	0.36	0.34

Effective masses of Ge, Si, and GaAs

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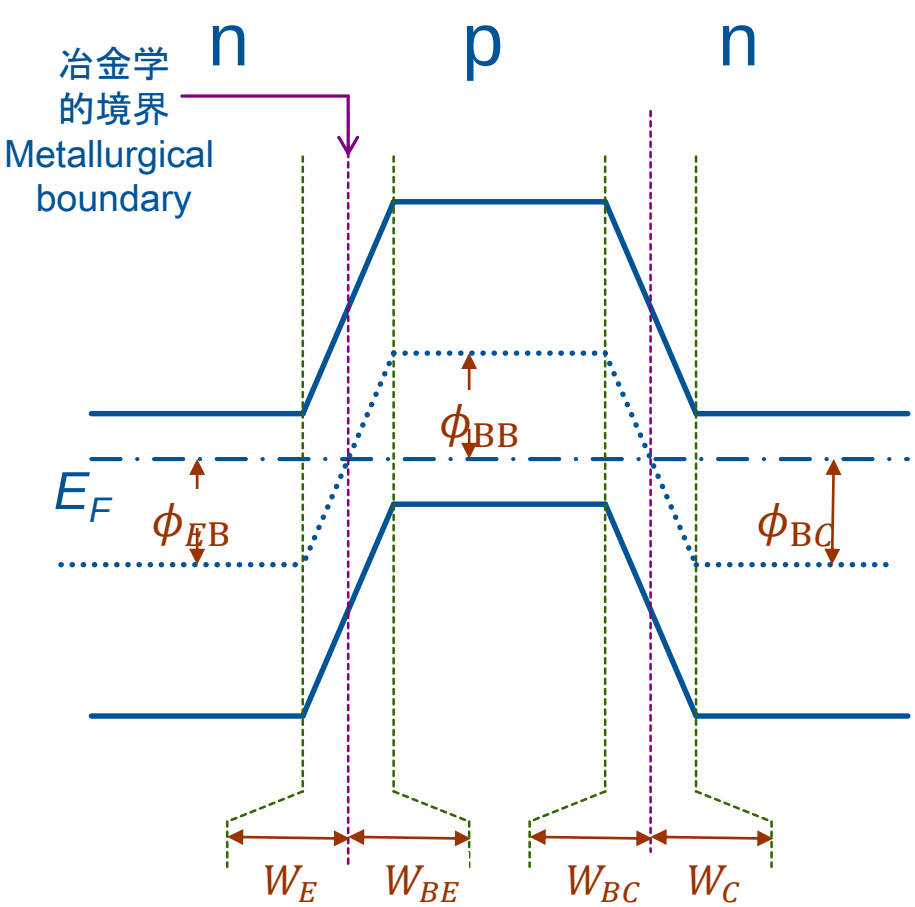
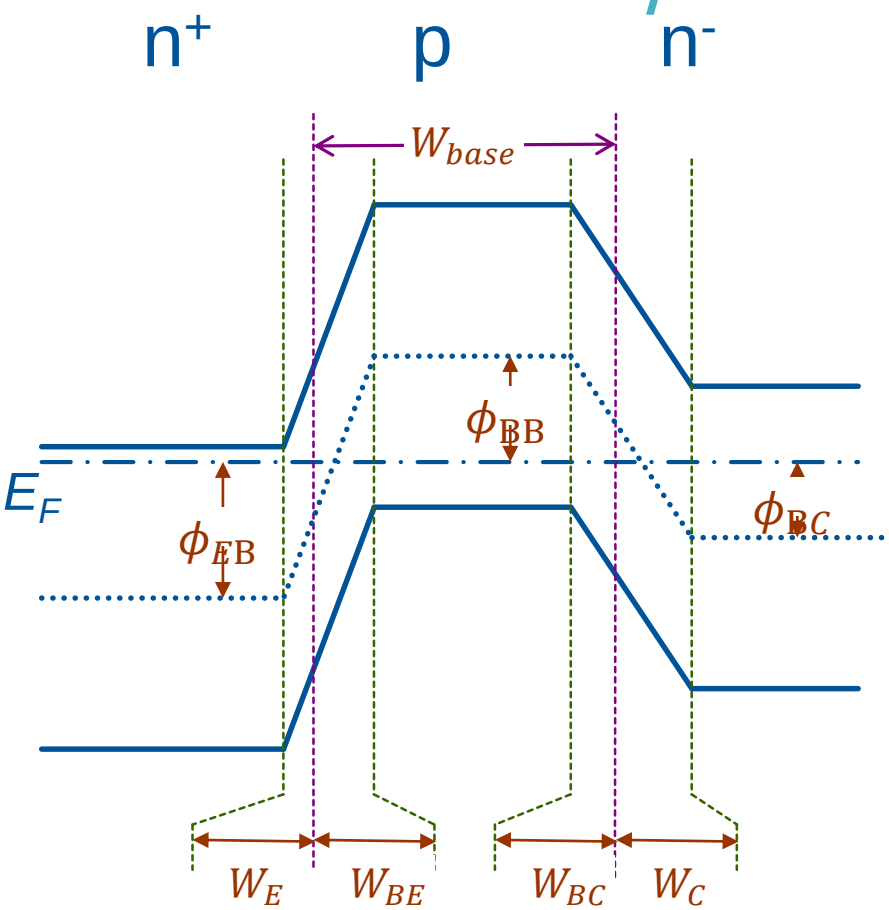
Properties			Ge	Si	GaAs
Band structure	遷移型		indirect 間接遷移	Indirect 間接遷移	Direct 間接遷移
Electron, longitudinal effective mass	電子「縦」有効質量	m_l^*/m_o	1.64	0.98	m_e^*/m_o =0.067
Electron transverse effective mass	電子「横」有効質量	m_t^*/m_o	0.082	0.19	
Longitudinal direction			<111>	<100>	---
Number of minima in the conduction band	伝導帯の最低エネルギー点の数	M_c	8	6	1
heavy hole effective mass	m_{hh}^*/m_o	m_{hh}^*/m_o	0.28	0.49	0.45
light hole effective mass	m_{lh}^*/m_o	m_{lh}^*/m_o	0.044	0.16	0.082

effective mass for density-of-sate electron calculation	$m_{de} = \left(m_l^* \, m_t^{*2}\right)^{1/3}$
effective mass for density-of-sate electron calculation	$m_{dh} = \left(m_{lh}^{*\,3/2} + m_{hh}^{*\,3/2}\right)^{2/3}$
Conductivity electron effective mass	$\frac{1}{m_{ce}} = \frac{1}{3}\left(\frac{1}{m_l^*} + \frac{2}{m_t^*}\right)$

宿題説明 : npn and n^+pn^- junctions





- npn structure: doping concentrations of each regions are the same.
- npn構造（全てのドーピング量が等しいとき）

- n^+pn^- structure

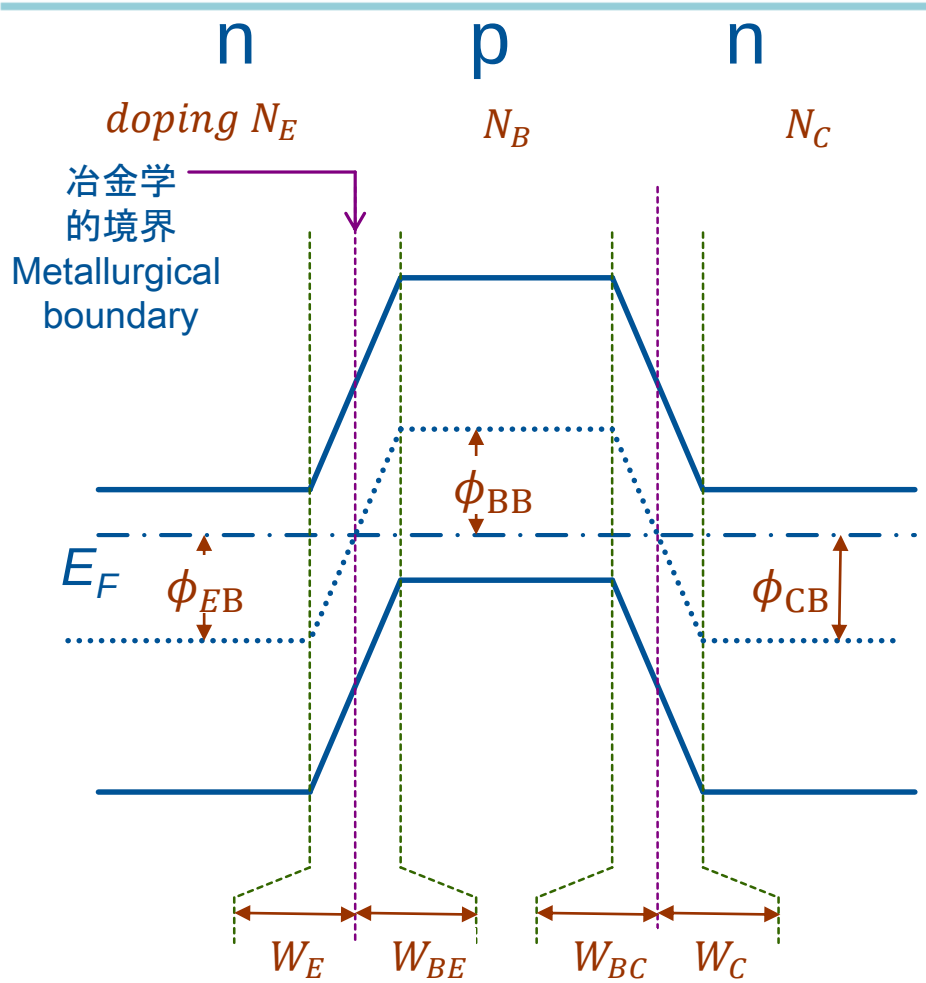
数値をいれて考える

- npnとn^+pn^-接合について、具体的に下記の数値を入れたときのバンド図を描け。
- ベース幅W_{base}は、0.6μmと仮定せよ。
- 描くときは、横軸1cm=0.1μm,（3～5）cm = 1eVというように適当に数値を仮定して描け。
- $W_E, W_{BE}, W_{CE}, W_C,$ $\phi_{E\mathrm{B}}, \phi_{B\mathrm{B}}, \phi_{C\mathrm{B}}$ をきちっと明示せよ。

- npn接合
  - $N_E(emitter) = 2 \times 10^{16}(cm^{-3})$
  - $N_B(base) = 2 \times 10^{16}(cm^{-3})$
  - $N_C(collector) = 2 \times 10^{16}(cm^{-3})$
- n^+pn^-接合
  - $N_E(emitter) = 1 \times 10^{19}(cm^{-3})$
  - $N_B(base) = 2 \times 10^{16}(cm^{-3})$
  - $N_C(collector) = 1 \times 10^{15}(cm^{-3})$

npn junctions

handout



$$\phi_{EB} = \frac{kT}{q} log_e \frac{N_E}{n_i}$$

$$\phi_{BB} = \frac{kT}{q} log_e \frac{N_B}{n_i}$$

Because $N_E = N_B = N_C$,

$$\phi_{EB} = \phi_{BB} = \phi_{CB}$$

$$\phi_{bi} = \phi_{EB} + \phi_{BB}$$

npn junctions

handout

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