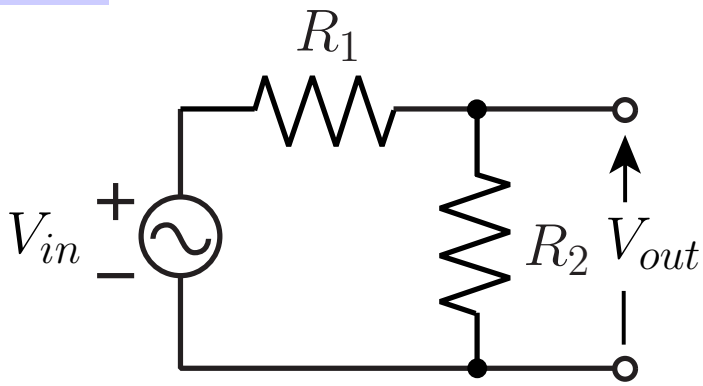
LCシミュレーション

電力伝送



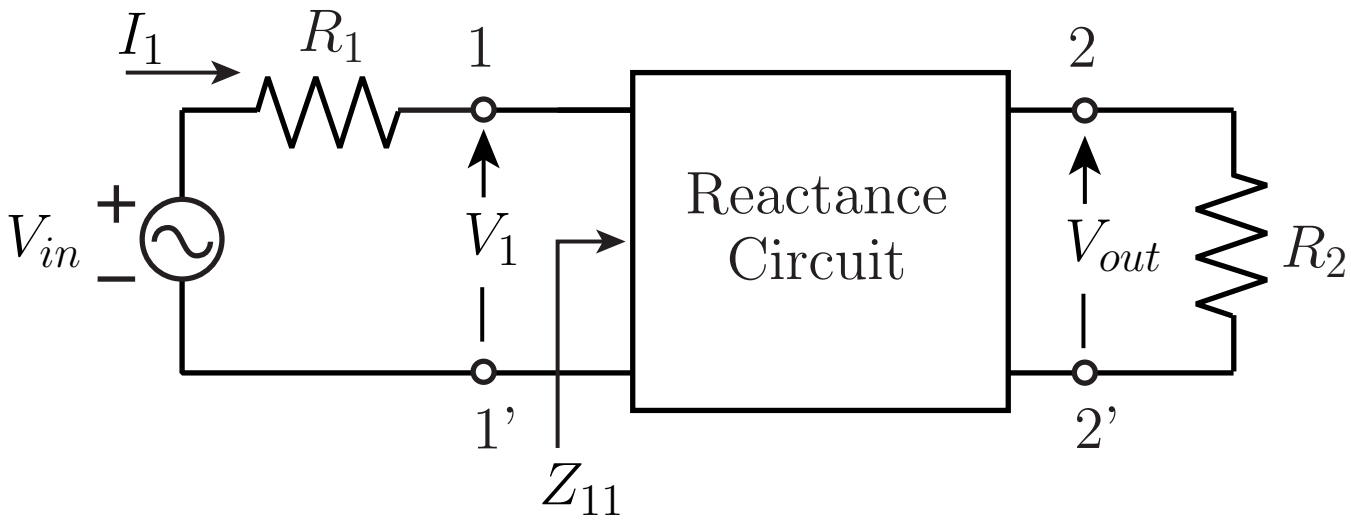
$$P_2=\frac{|V_{out}|^2}{R_2}=\frac{1}{R_2}\left|\frac{R_2}{R_1+R_2}V_{in}\right|^2=\frac{R_2}{(R_1+R_2)^2}|V_{in}|^2$$

$$=\frac{1}{\frac{R_1^2}{R_2}+2R_1+R_2}|V_{in}|^2\leq\frac{1}{4R_1}|V_{in}|^2$$

相加相乗平均の定理: $a+b\geq 2\sqrt{ab}$

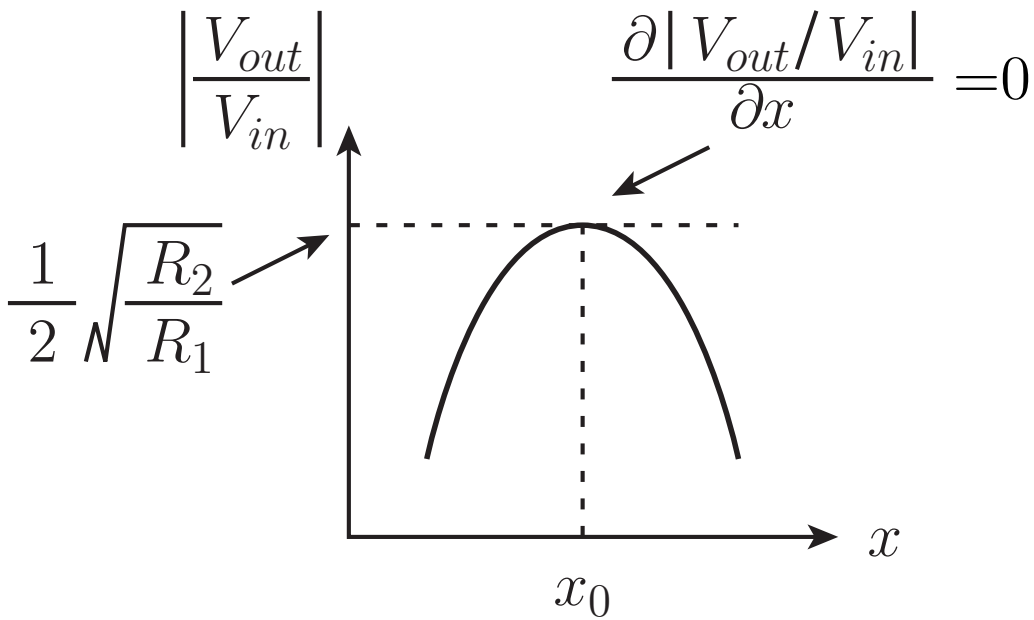
(等号は$a=b$のとき成立)

抵抗両終端型LCフィルタの性質



$$P_{max}=\frac{|V_{in}|^2}{4R_1} \quad \geq \quad P_2=\frac{|V_{out}|^2}{R_2}$$

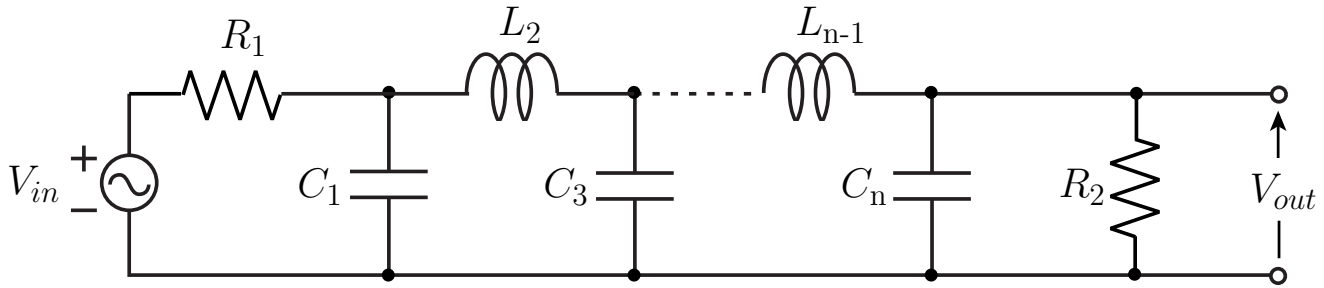
$$\left|\frac{V_{out}}{V_{in}}\right| \leq \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$$



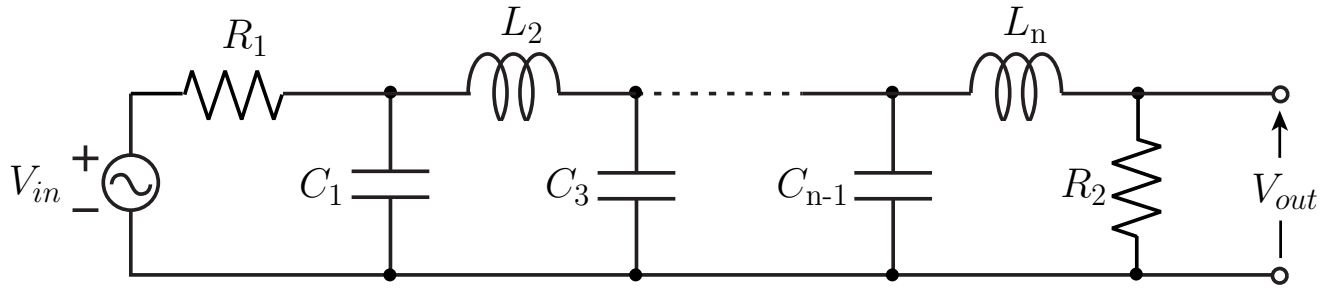
$$\frac{\partial|T(s)|}{\partial x}=\frac{\partial|V_{out}/V_{in}|}{\partial x}=0 \quad x=x_0, \quad \omega=\omega_0$$

$$\boxed{\Delta|T(s)|\approx\frac{\partial|T(s)|}{\partial x}\Delta x}$$

振幅平坦特性の設計公式



奇数次



偶数次

$R_1=R_2$　n：フィルタの次数　　$\beta_p=\sqrt{\alpha_p{}^2-1}$

遮断角周波数:1rad/s

$$C_1=\frac{2\beta_p{}^{1/n}\sin\frac{\pi}{2n}}{R_1}$$

$$L_iC_{i+1}\text{または}C_iL_{i+1}=4\beta_p{}^{2/n}\sin\frac{(2i-1)\pi}{2n}\sin\frac{(2i+1)\pi}{2n}$$

振幅等リプル特性の設計公式

n=奇数：$R_2=R_1$

n=偶数：$R_2=(\alpha_p-\sqrt{\alpha_p{}^2-1})^2 R_1$

$$\xi=\left(\frac{\alpha_p+1}{\alpha_p-1}\right)^{1/2n}-\left(\frac{\alpha_p-1}{\alpha_p+1}\right)^{1/2n}$$

$$b_i=\xi^2+4\sin^2\frac{i}{n}\pi$$

$$C_1=\frac{4\sin\frac{\pi}{2n}}{\xi R_1}$$

$$L_iC_{i+1}\text{または}C_iL_{i+1}=\frac{4\sin\frac{(2i-1)\pi}{2n}\sin\frac{(2i+1)\pi}{2n}}{b_i}$$

フィルタ特性の変換

$T_0(s)$ ：基準低域通過型関数

低域-低域通過変換

$$s \rightarrow \frac{s}{\omega_C}$$

$$T_L(s) = T_0\left(\frac{s}{\omega_C}\right)$$

低域-高域通過変換

$$s \rightarrow \frac{1}{s}$$

$$T_{H0}(s) = T_0\left(\frac{1}{s}\right)$$

$$T_H(s) = T_{H0}\left(\frac{s}{\omega_C}\right)$$

低域-帯域通過変換

$$s \to \frac{\omega_0}{\omega_b}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)$$

$$\boxed{T_B(s)=T_0\left(\frac{\omega_0}{\omega_b}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)\right)}$$

$$-\infty \to 0$$

$$\omega \to \frac{\omega_0}{\omega_b}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$

$$0 \to \omega_0$$

$$\infty \to \infty$$

$$\frac{\omega_0}{\omega_b}\left(\frac{\omega_{C1}}{\omega_0}-\frac{\omega_0}{\omega_{C1}}\right)=-1$$

$$\frac{\omega_0}{\omega_b}\left(\frac{\omega_{C2}}{\omega_0}-\frac{\omega_0}{\omega_{C2}}\right)=1$$

$$\omega_{C1}=\frac{-\omega_b+\sqrt{\omega_b{}^2+4\omega_0{}^2}}{2}$$

$$\omega_{C2}=\frac{\omega_b+\sqrt{\omega_b{}^2+4\omega_0{}^2}}{2}$$

$$\boxed{\omega_{C2}-\omega_{C1}=\omega_b}$$

帯域幅

$$\frac{\omega_0}{\omega_b}\left(\frac{\omega_{C1}}{\omega_0}-\frac{\omega_0}{\omega_{C1}}\right)=-1 \qquad \frac{\omega_0}{\omega_b}\left(\frac{\omega_{C2}}{\omega_0}-\frac{\omega_0}{\omega_{C2}}\right)=1$$

$$\omega_{C1}-\frac{\omega_0{}^2}{\omega_{C1}}=-\omega_b \qquad \omega_{C2}-\frac{\omega_0{}^2}{\omega_{C2}}=\omega_b$$

$$\omega_{C1}+\omega_{C2}=\frac{\omega_0{}^2}{\omega_{C1}}+\frac{\omega_0{}^2}{\omega_{C2}}=\frac{\omega_0{}^2\left(\omega_{C1}+\omega_{C2}\right)}{\omega_{C1}\omega_{C2}}$$

$$\boxed{\omega_{C1}\omega_{C2}=\omega_0{}^2}$$

幾何学対称

例題1

$$T_0(s)=\frac{1}{y_{min}(s^2+\sqrt{2}s+1)}$$: 遮断角周波数1rad/s

遮断周波数4kHzに変換

$$s \rightarrow \frac{s}{\omega_C}=\frac{s}{2\pi\times 4000}$$

$$\boxed{T(s)=\frac{(2\pi\times 4000)^2}{y_{min}\left\{s^2+2\sqrt{2}\pi\times 4000s+(2\pi\times 4000)^2\right\}}}$$

例題2 $T_0(s)=\dfrac{1}{y_{min}(s^2+\sqrt{2}s+1)}$: 遮断角周波数1rad/s

中心周波数1kHz，帯域幅450Hz

$$s\rightarrow\frac{\omega_0}{\omega_b}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)=\frac{1000}{450}\left(\frac{s}{2\pi\times1000}+\frac{2\pi\times1000}{s}\right)$$

$$T(s)=\frac{1/y_{min}}{\left\{\frac{\omega_0}{\omega_b}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)\right\}^2+\sqrt{2}\frac{\omega_0}{\omega_b}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)+1}$$

$$=\frac{(1/y_{min})\omega_b{}^2s^2}{s^4+\sqrt{2}\omega_b s^3+\left\{2\left(\frac{\omega_0}{\omega_b}\right)+1\right\}^2\omega_b{}^2s^2+\sqrt{2}\omega_b\omega_0{}^2s+\omega_0{}^4}$$

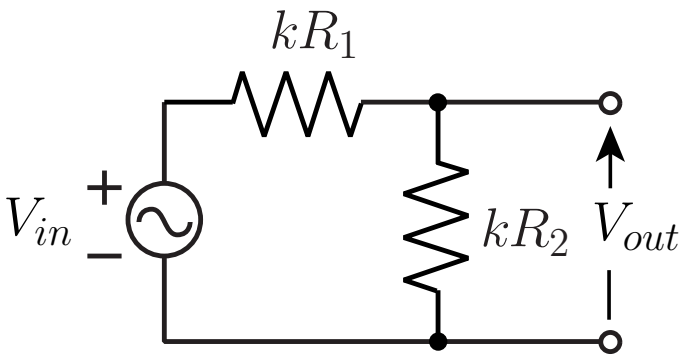
$$T(s)=\frac{(1/y_{\min})\omega_b{}^2 s^2}{s^4+\sqrt{2}\omega_b s^3+\left\{2\left(\frac{\omega_0}{\omega_b}\right)+1\right\}^2\omega_b{}^2 s^2+\sqrt{2}\omega_b\omega_0{}^2 s+\omega_0{}^4}$$

$$=\frac{(1/y_{min})(2\pi\times 450)^2 s^2}{s^4+2\pi\times 450\times\sqrt{2}s^3+\left\{2\left(\frac{20}{9}\right)^2+1\right\}(2\pi\times 450)^2 s^2+\sqrt{2}(2\pi\times 1000)^2(2\pi\times 450)s+(2\pi\times 1000)^4}$$

$\omega_{C1}=2\pi\times 800,\quad \omega_{C2}=2\pi\times 1250$

$$\boxed{\omega_{C1}-\omega_{C2}=2\pi\times 450}$$

インピーダンス・スケーリング



$$\frac{V_{out}}{V_{in}} = \frac{kR_2}{kR_1 + kR_2} = \frac{R_2}{R_1 + R_2}$$

例題　　現実的な素子値への変更

「インピーダンススケーリング」と「周波数スケーリング」

(1) 抵抗値を1kΩに（インピーダンススケーリング）

$$R_1=R_2=1\Omega\rightarrow 1\mathrm{k}\Omega$$

$$C=1\mathrm{F}\rightarrow\frac{1}{1000}\mathrm{F}$$

$$L=2\mathrm{H}\rightarrow 2\times 1000\mathrm{H}$$

(2) 遮断周波数を1kHzに（周波数スケーリング）

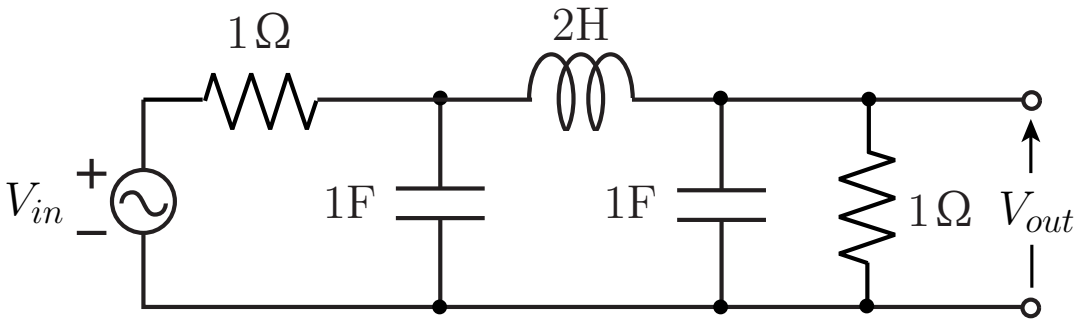
$$j\omega\rightarrow j\frac{\omega}{\omega_C}$$

$$C=\frac{1}{1000}\mathrm{F}\rightarrow\frac{1}{2\pi\times 1000000}\mathrm{F}=0.159\mu\mathrm{F}$$

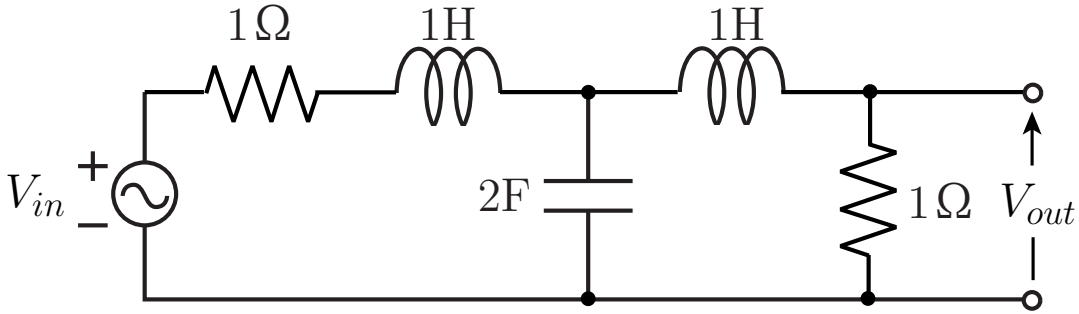
$$L=2\times 1000\mathrm{H}\rightarrow\frac{2\times 1000}{2\pi\times 1000}\mathrm{H}=0.318\mathrm{H}$$

3次抵抗両終端型LCフィルタ

遮断周波数が1rad/s，Qが$1/\sqrt{2}$の低域通過フィルタ



(a)

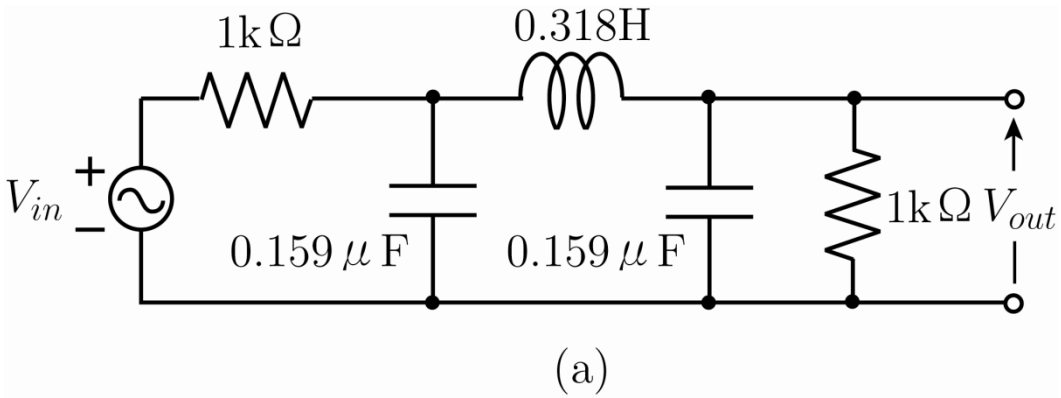


(b)

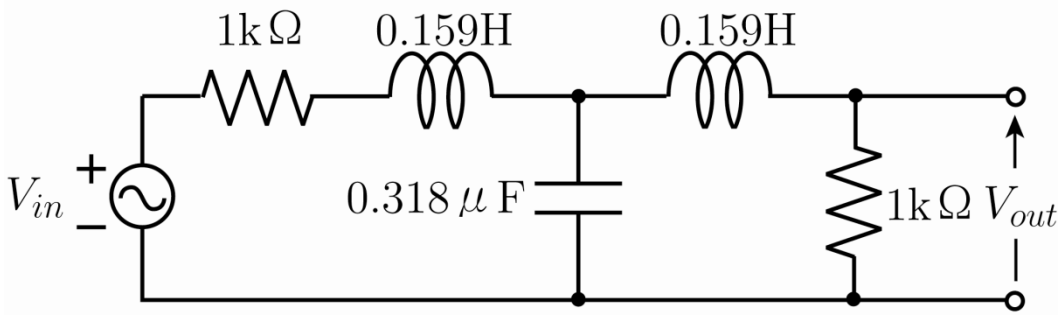
遮断周波数が1.00kHz，Qが$1/\sqrt{2}$の低域通過フィルタ

インピーダンスレベル：1000倍

インダクタンス，容量：1/(2000π)倍



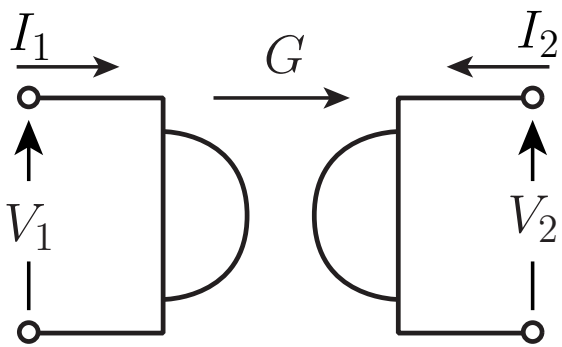
(a)



(b)

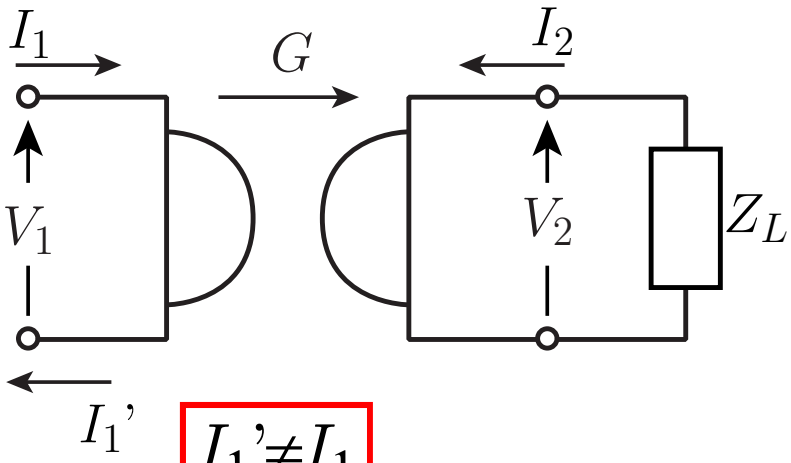
インピーダンス・シミュレーション

ジャイレータ



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & \pm G \\ \mp G & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

複号同順



$$\boxed{I_1' \neq I_1}$$

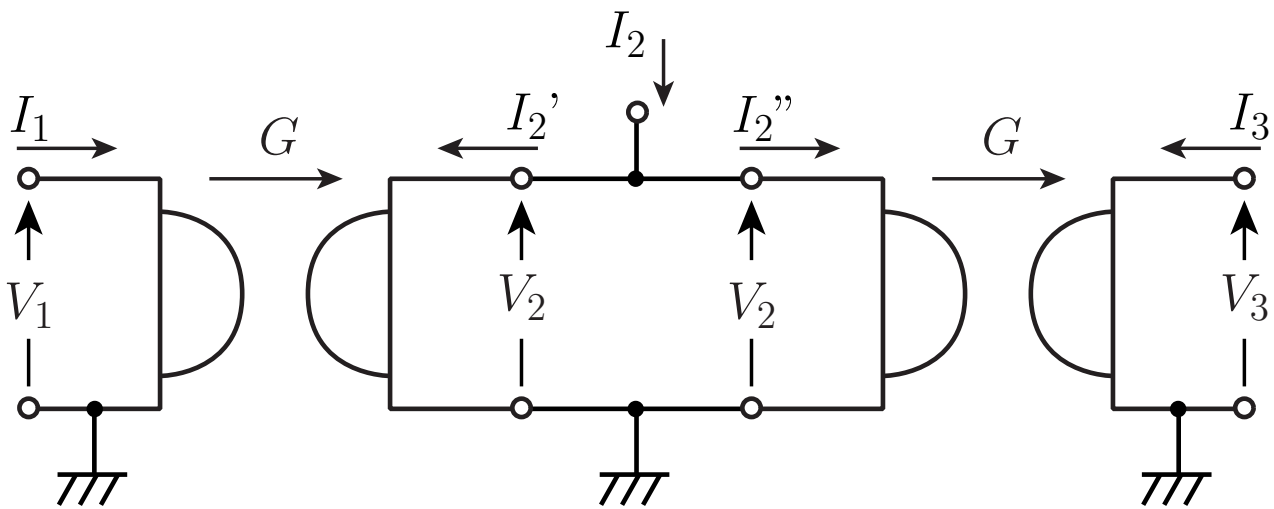
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & \pm G \\ \mp G & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = \pm G V_2 = \pm G(-Z_L I_2) = \pm G(\pm Z_L G V_1) = Z_L G^2 V_1$$

$$\boxed{Z_{in} = \frac{V_1}{I_1} = \frac{1}{G^2 Z_L}}$$

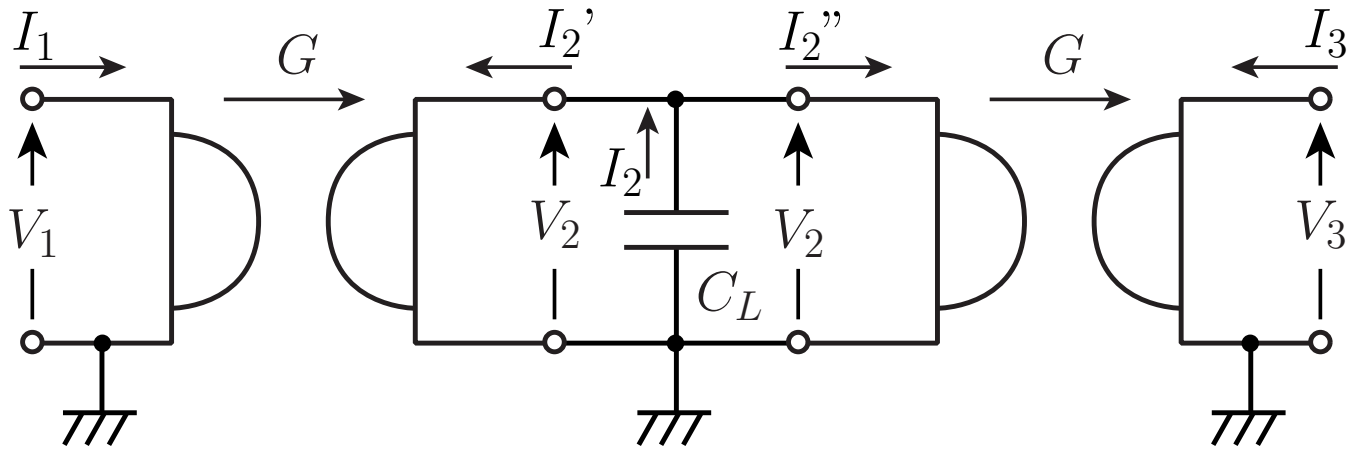
$$\boxed{Z_L = \frac{1}{sC_L} \rightarrow Z_{in} = s\frac{C_L}{G^2} \quad \left(L = \frac{C_L}{G^2}\right)}$$

3端子対ジャイレータによる非接地インダクタの構成



$$\begin{bmatrix} I_1 \\ I_2' \end{bmatrix} = \begin{bmatrix} 0 & \pm G \\ \mp G & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \begin{bmatrix} I_2'' \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 & \pm G \\ \mp G & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2'+I_2'' \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 & \pm G & 0 \\ \mp G & 0 & \pm G \\ 0 & \mp G & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$



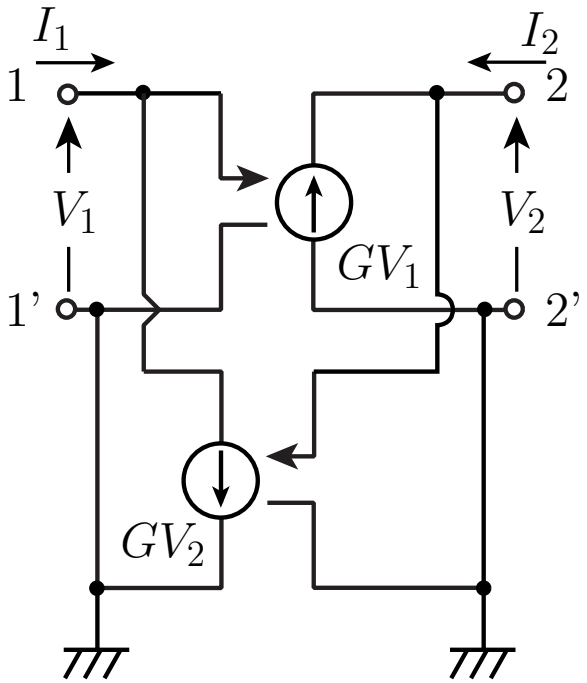
$$I_2 = -sC_L V_2$$

$$I_2 = \mp G(V_1 - V_3)$$

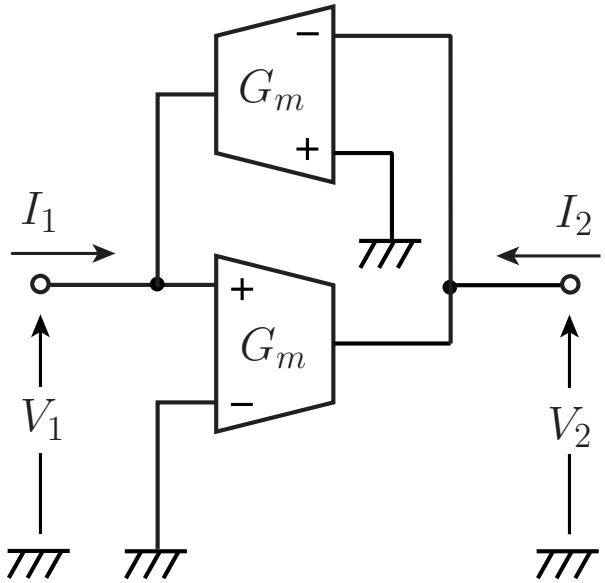
$$\longrightarrow \quad V_2 = \pm \frac{V_1 - V_3}{sC_L}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 & \pm G & 0 \\ \mp G & 0 & \pm G \\ 0 & \mp G & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \longrightarrow \boxed{\begin{bmatrix} I_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} \frac{G^2}{sC_L} & \frac{G^2}{sC_L} \\ \frac{G^2}{sC_L} & \frac{G^2}{sC_L} \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \end{bmatrix}}$$

ジャイレータの構成例



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & G \\ -G & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



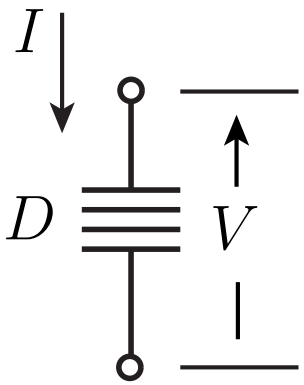
インピーダンス・スケーリング・シミュレーション

$$sL \xrightarrow{\times\frac{1}{s}} L \quad \text{抵抗}$$

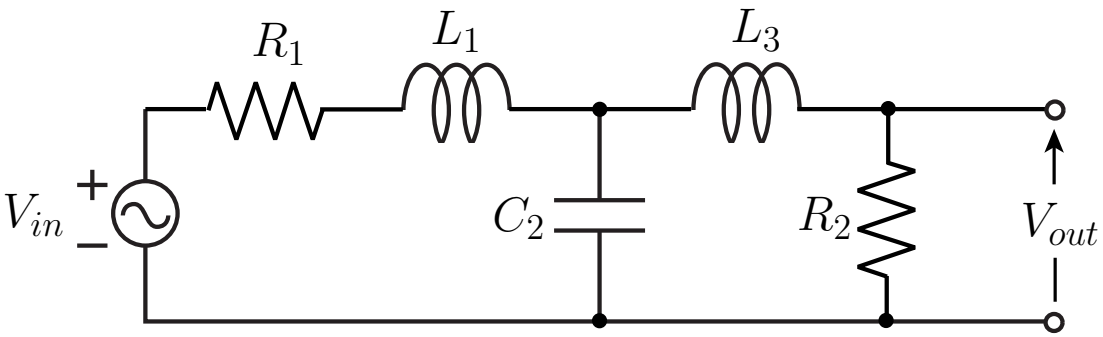
$$R \longrightarrow \frac{R}{s} \quad \text{容量}$$

$$\frac{1}{sC} \longrightarrow \frac{1}{s^2C} \quad \boxed{\text{FDNR}}$$

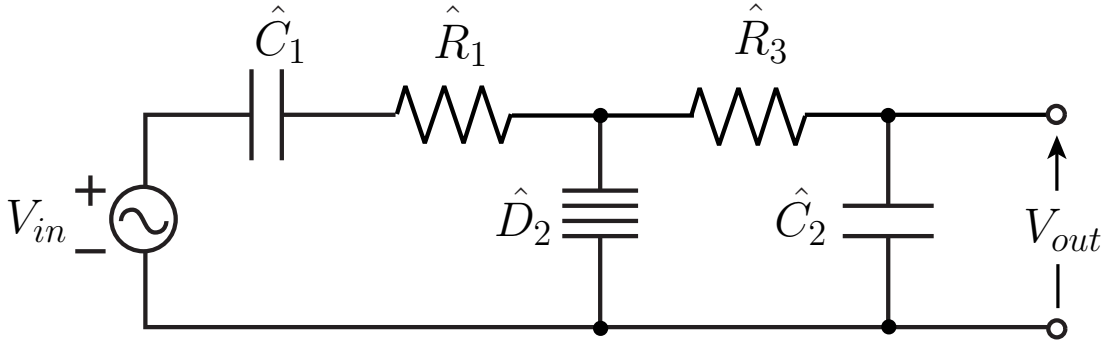
Frequency-Dependent Negative Resistance
(周波数依存性負性抵抗)



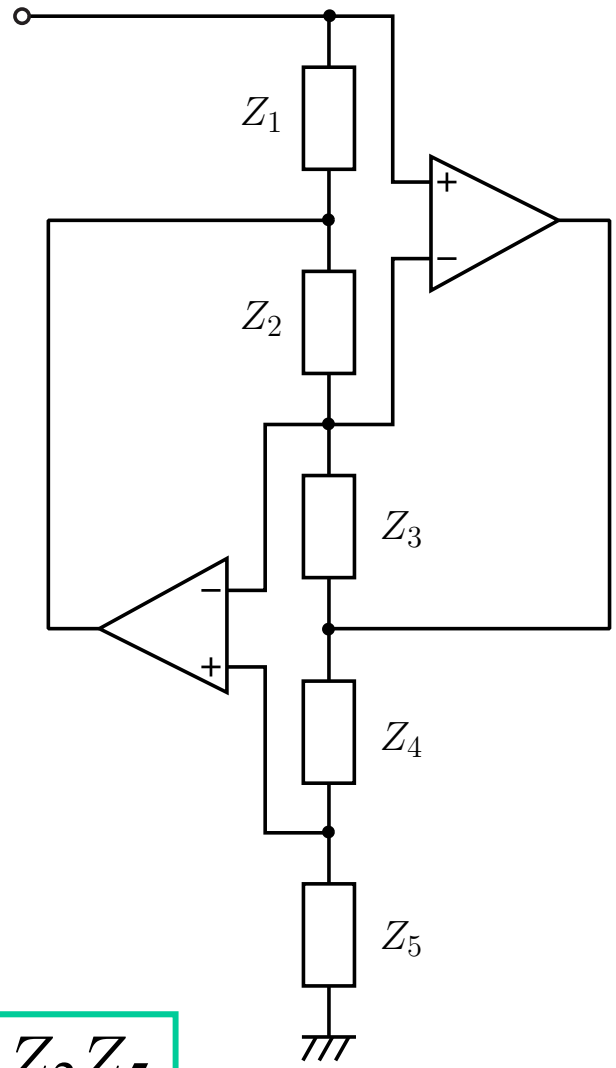
$$\frac{V}{I} = \frac{1}{s^2 D}$$



(a)



(b)



GIC

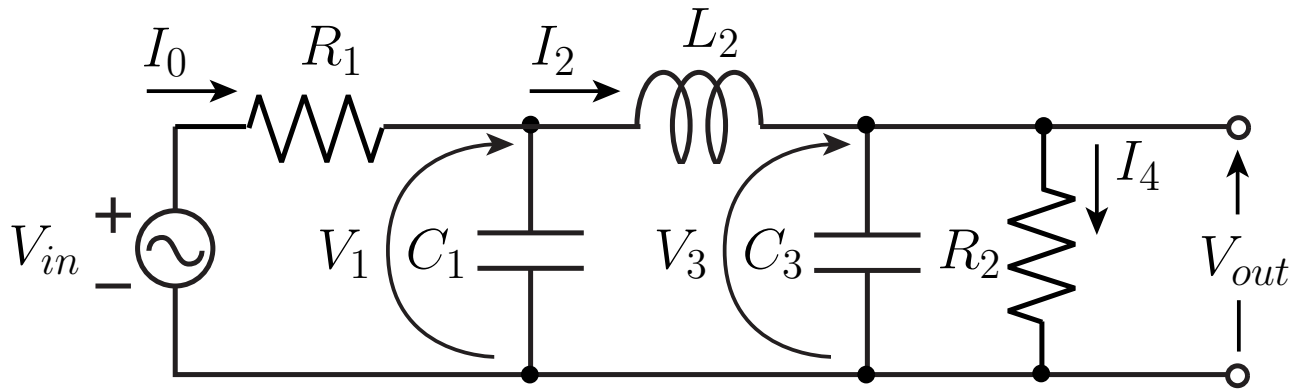
$$Z_{in} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

リープフログ・シミュレーション

リープフログ=Leapfrog

カエル跳び？

馬跳び!



$$I_0 = \frac{V_{in} - V_1}{R_1}$$

$$V_1 = \frac{I_0 - I_2}{sC_1}$$

$$I_2 = \frac{V_1 - V_3}{sL_2}$$

$$V_3 = \frac{I_2 - I_4}{sC_3}$$

$$I_4 = \frac{V_3}{R_2}$$

シグナルフローグラフ

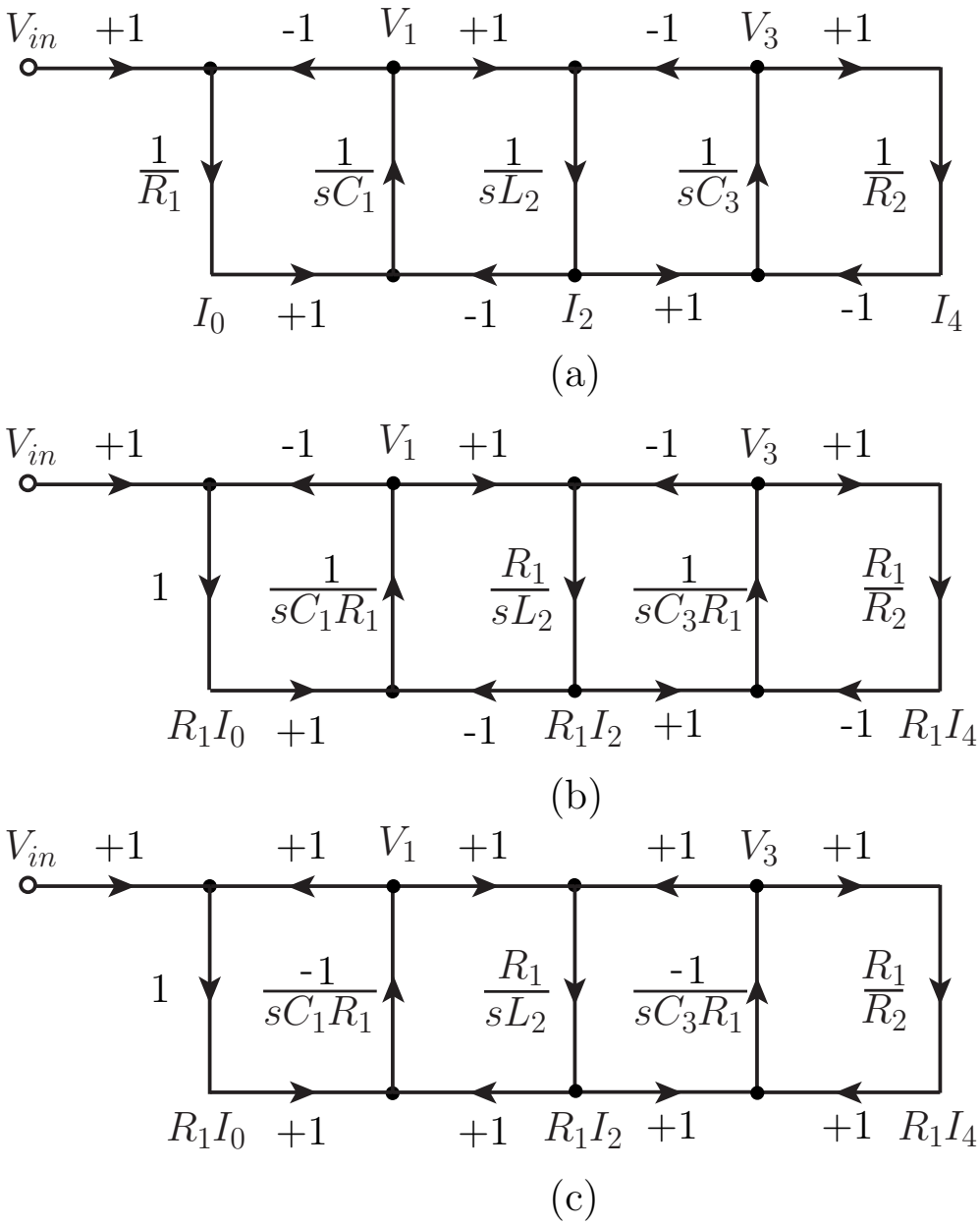
$$I_0=\frac{V_{in}-V_1}{R_1}$$

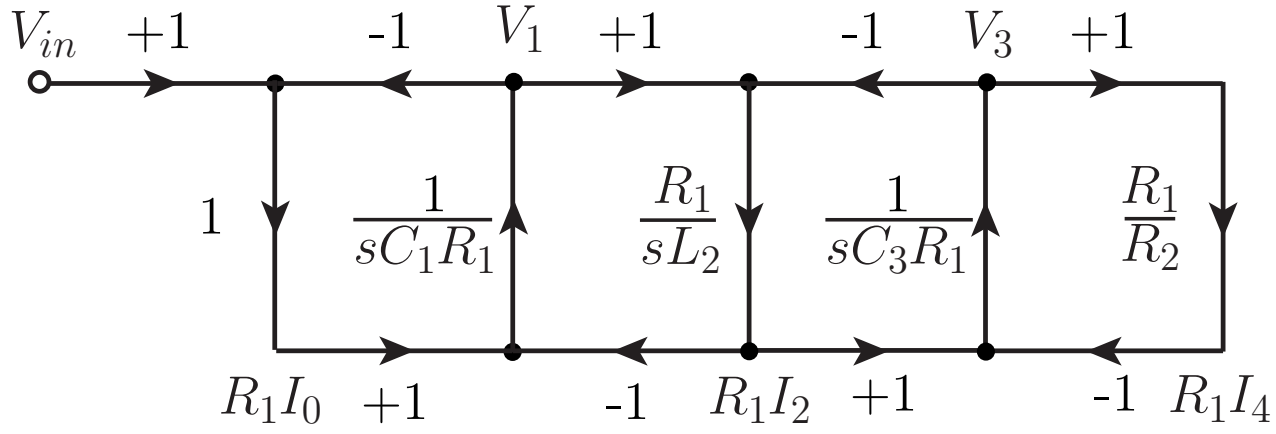
$$V_1=\frac{I_0-I_2}{sC_1}$$

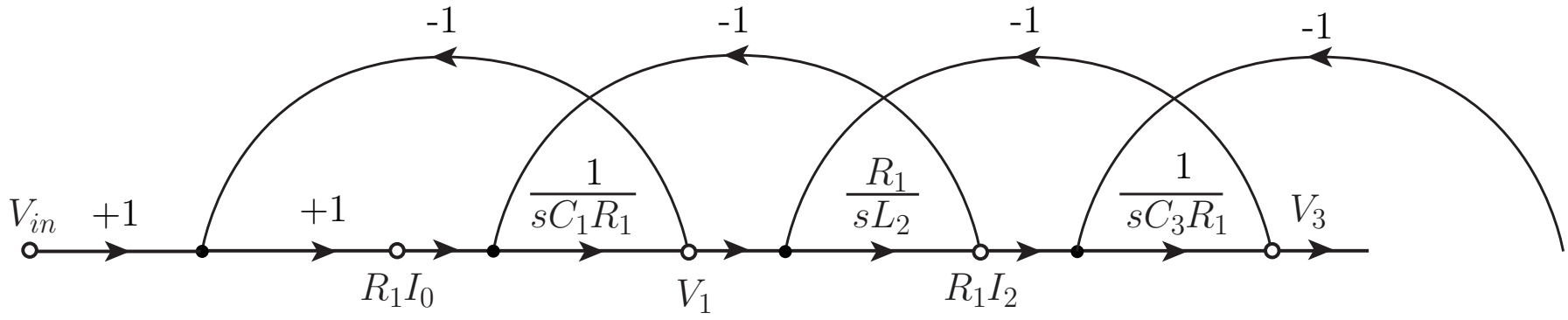
$$I_2=\frac{V_1-V_3}{sL_2}$$

$$V_3=\frac{I_2-I_4}{sC_3}$$

$$I_4=\frac{V_3}{R_2}$$

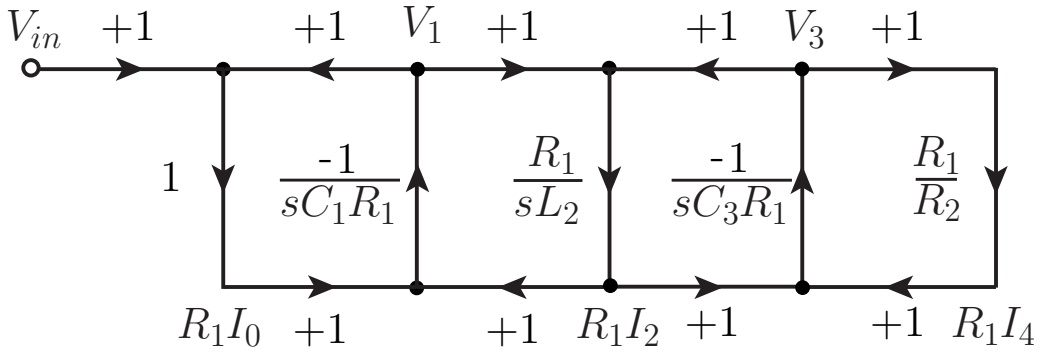


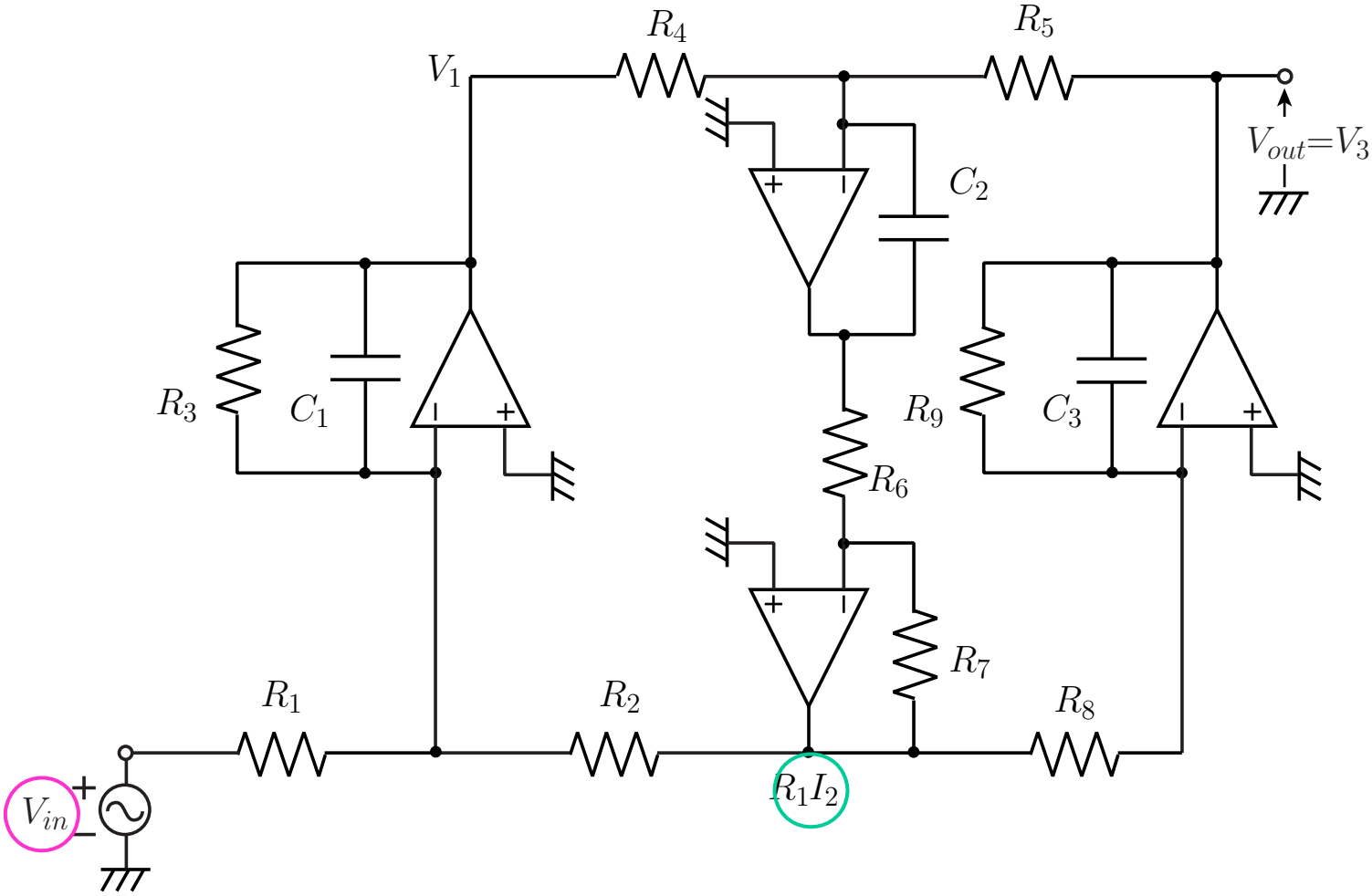

V_{in}
+1
-1
V_1
+1
-1
V_3
+1
1
$\frac{1}{sC_1R_1}$
$\frac{R_1}{sL_2}$
$\frac{1}{sC_3R_1}$
$\frac{R_1}{R_2}$
R_1I_0
+1
-1
R_1I_2
+1
-1
R_1I_4

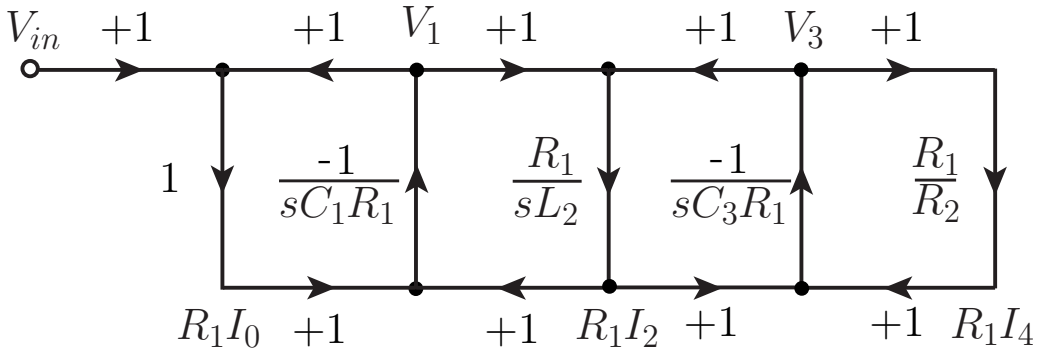

-1
-1
-1
-1
V_{in}
+1
+1
$\frac{1}{sC_1R_1}$
$\frac{R_1}{sL_2}$
$\frac{1}{sC_3R_1}$
V_3
R_1I_0
V_1
R_1I_2

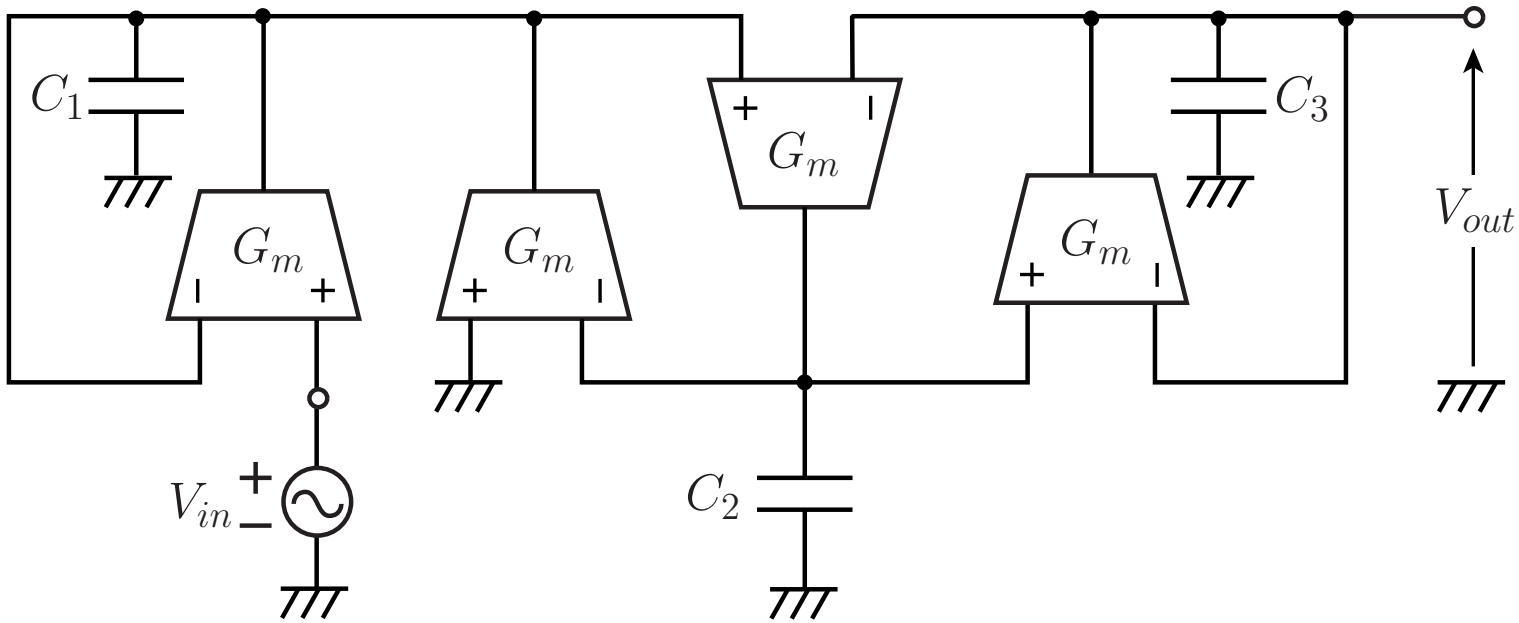
馬跳び

3次低域通過型フィルタの構成例





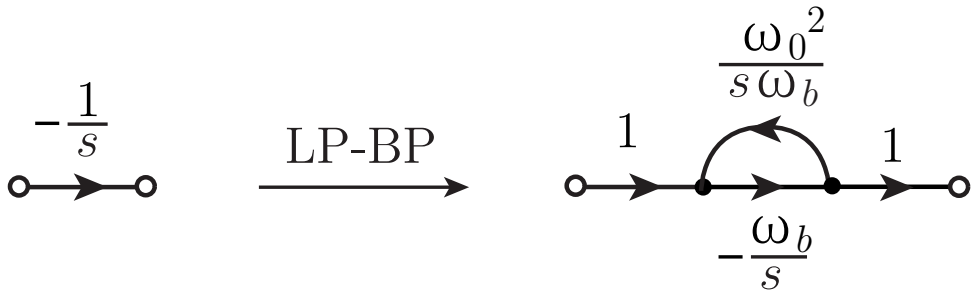

V_{in}
+1
+1
V_1
+1
+1
V_3
+1
1
$\frac{-1}{sC_1R_1}$
$\frac{R_1}{sL_2}$
$\frac{-1}{sC_3R_1}$
$\frac{R_1}{R_2}$
R_1I_0
+1
+1
R_1I_2
+1
+1
R_1I_4

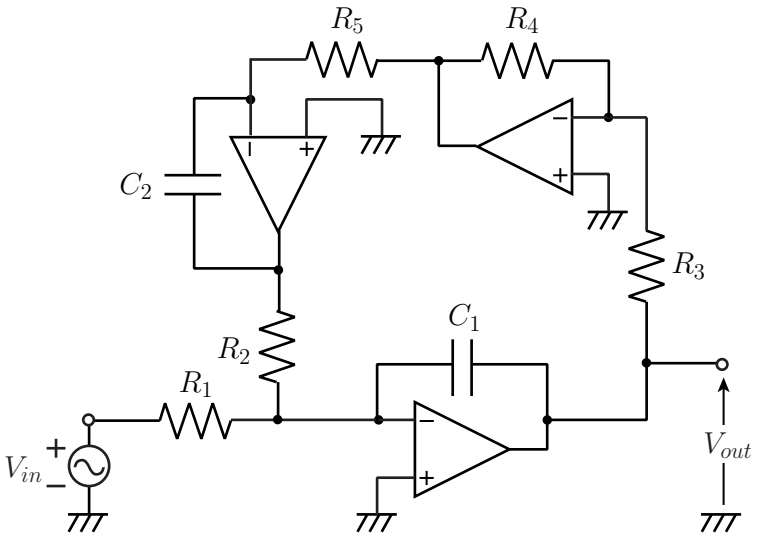

C_1
G_m
G_m
G_m
G_m
C_3
V_{out}
V_{in}
C_2

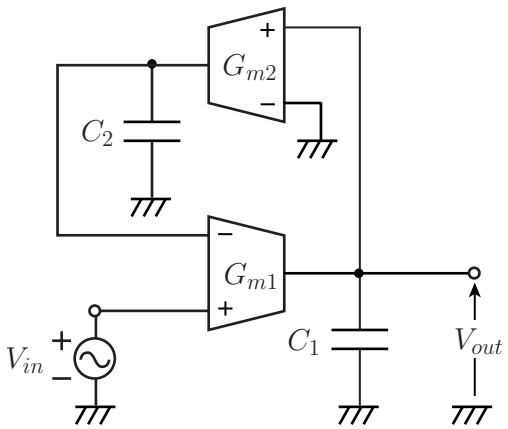
帯域通過フィルタの構成

低域-帯域通過変換:

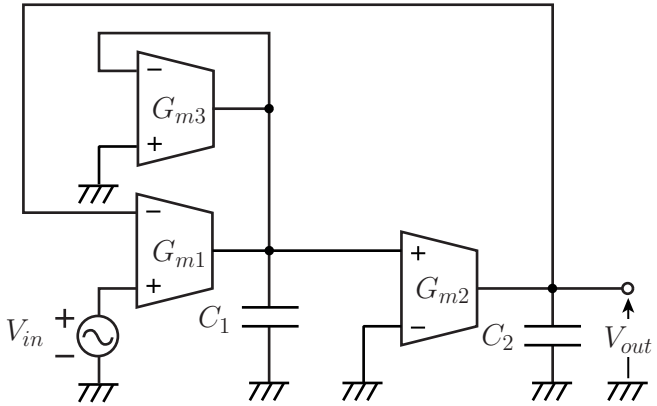
$$s \rightarrow \frac{\omega_0}{\omega_b}\left(\frac{s}{\omega_0}+\frac{\omega_0}{s}\right)$$

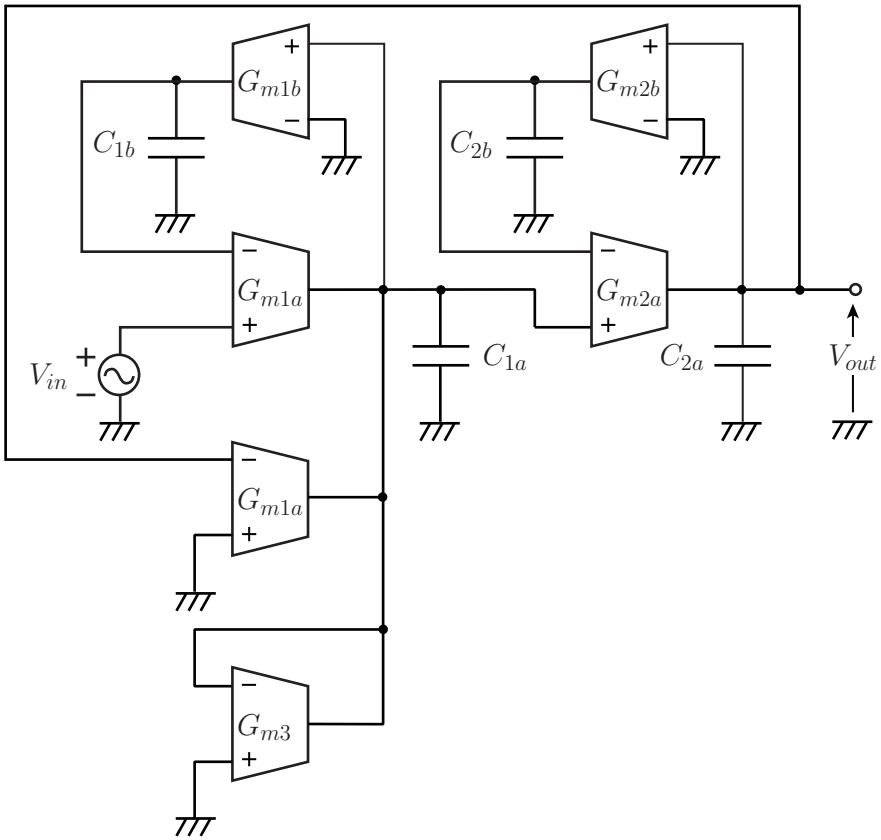






4次帯域通過フィルタ


G_{m3}
G_{m1}
G_{m2}
V_{in}
C_1
C_2
V_{out}


G_{m1b}
G_{m2b}
C_{1b}
C_{2b}
G_{m1a}
G_{m2a}
V_{in}
C_{1a}
C_{2a}
V_{out}
G_{m1a}
G_{m3}

周波数特性の自動調整

素子値の一様偏差の影響

$$T_0(s)=F[sC_i,\quad G_j]$$

一様偏差:　$C_i \rightarrow k_C C_i \qquad G_j \rightarrow k_G G_j$

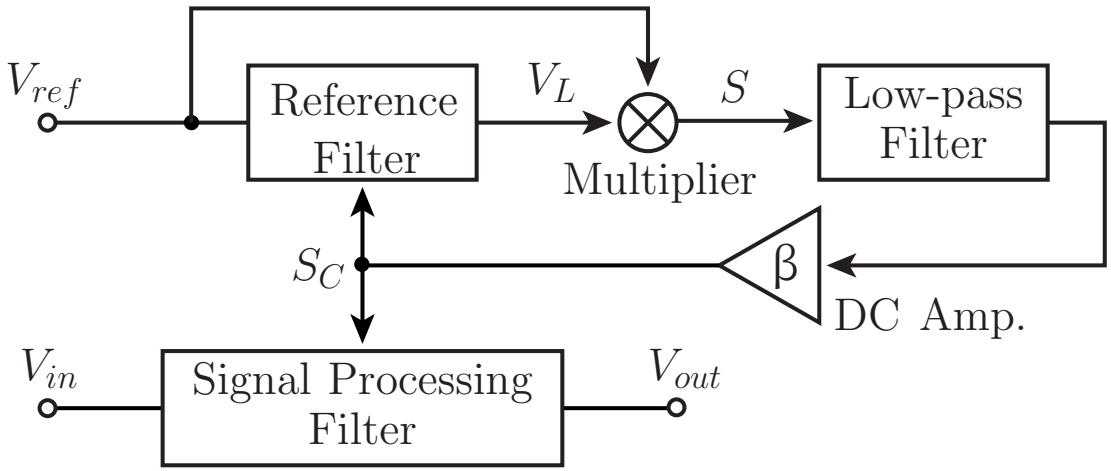
$$T(s)=F[sk_C C_i,\quad k_G G_j]$$

インピーダンス・スケーリング

$$T(s)=F\left[s\frac{k_C}{k_G}C_i,\quad G_j\right]=T_0\left(s\frac{k_C}{k_G}\right)$$

周波数スケーリング

周波数偏差自動調整回路

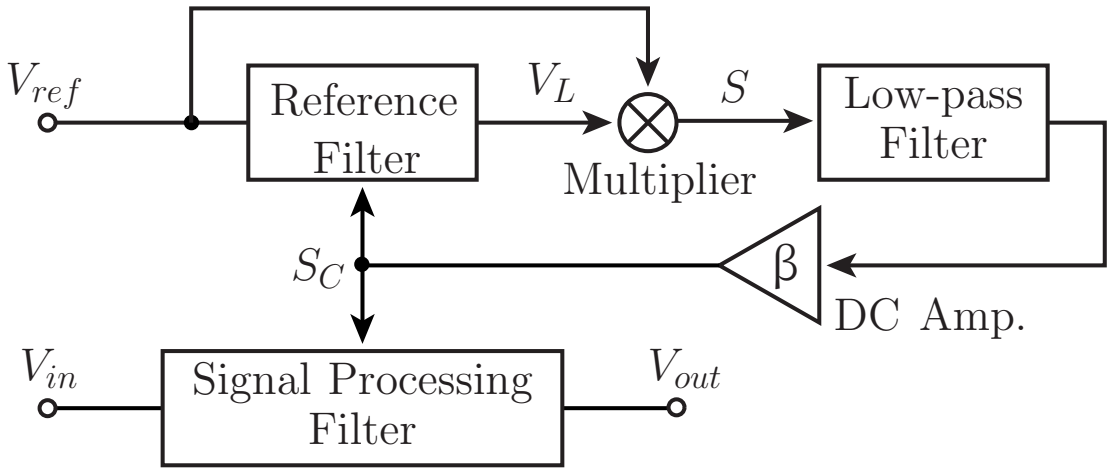


$$V_{ref}=V_m \sin \omega_{ref} t \qquad V_L=AV_m \sin(\omega_{ref} t+\theta)$$

$$S=M(V_{ref}\times V_L)=M\{V_m \sin \omega_{ref} t\times AV_m \sin(\omega_{ref} t+\theta)\}$$

$$=\frac{1}{2}MAV_m{}^2\{\cos\theta-\cos(2\omega_{ref} t+\theta)\}$$

$$\boxed{S_C=\frac{1}{2}\beta MAV_m{}^2\cos\theta}$$

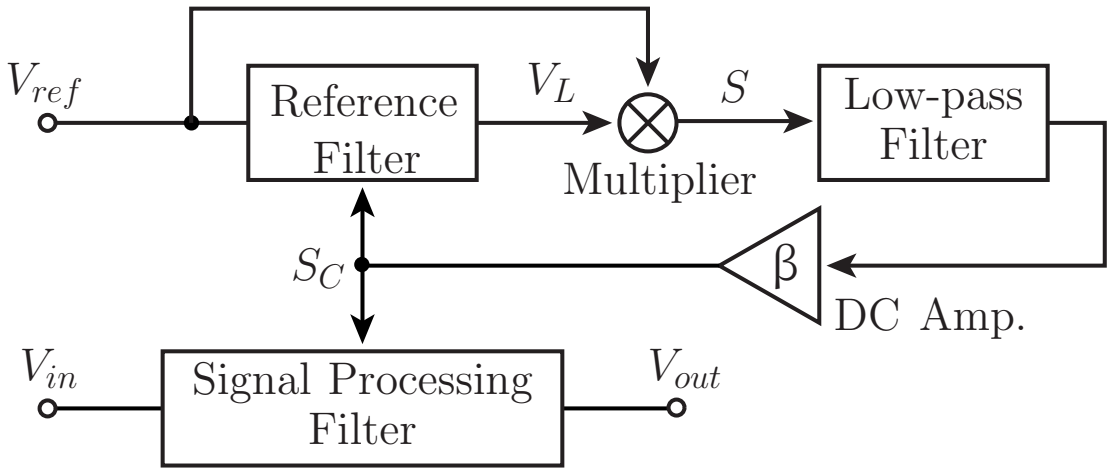


$$S_C = \frac{1}{2}\beta M A V_m{}^2 \cos\theta$$

$\beta \to \infty$:有限値に確定

$$\cos\theta = \frac{2S_C}{\beta M A V_m{}^2}$$

$$\cos\theta \to 0$$



参照用フィルタの伝達関数:

$$T_{L2}(s)=\frac{K\omega_0{}^2}{s^2+\frac{\omega_0}{Q}s+\omega_0{}^2}$$

$$\theta=-\tan^{-1}\frac{\omega_{ref}\omega_0}{Q(\omega_0{}^2-\omega_{ref}{}^2)}$$

$$\theta=-90^\circ\rightarrow\omega_0=\omega_{ref}$$

参照用フィルタの素子偏差の割合

||

信号処理用フィルタの素子偏差の割合

例題

参照用低域通過フィルタ

$$T_{L2}(s)=\frac{K\omega_0{}^2}{s^2+\frac{\omega_0}{Q}s+\omega_0{}^2}$$

$K=1,\quad \omega_0=2\pi\times1000\,\mathrm{rad/s},\quad Q=5$

乗算回路 $M=1\mathrm{V}^{-1}$

直流増幅回路 $\beta=100$

参照入力信号振幅 $V_m=0.2\mathrm{V}$

$$cos\theta=\frac{2S_C}{\beta MAV_m{}^2}$$

$A=?$

$S_C=?$

$\omega_{ref} \approx \omega_0$とすれば

$$A = \left| T_{L2}(j\omega_{ref}) \right| = \left| \frac{K\omega_0{}^2}{-\omega_{ref}{}^2 + j\frac{\omega_0}{Q}\omega_{ref} + \omega_0{}^2} \right| \approx Q$$

$g_m = \alpha S_C$かつ$g_m \propto \omega_0$とすれば

$$\omega_{00} \rightarrow S_{C0}$$

$$\omega_0 \rightarrow S_C$$

$$\frac{\omega_0}{\omega_{00}} = \frac{S_C}{S_{C0}}$$

$$S_C = \frac{\omega_0}{\omega_{00}} S_{C0}$$

$M=1\mathrm{V}^{-1}$ $\beta=100$ $V_m=0.2\mathrm{V}$ $A\approx Q=5$

$\omega_{00}=2\pi\times 800\mathrm{rad/s}$, $S_{C0}=0.1\mathrm{V}\rightarrow S_C=0.125\mathrm{V}$

$$\cos\theta=\frac{2S_C}{\beta MAV_m{}^2}=0.03125$$

$\theta=-88.2$度

$$\theta=-\tan^{-1}\frac{\omega_{ref}\omega_0}{Q(\omega_0{}^2-\omega_{ref}{}^2)}$$

$$\boxed{\omega_0=\frac{-1+\sqrt{1+4Q^2\tan^2\theta}}{2Q\tan\theta}\omega_{ref}=0.992\omega_{ref}}$$