Advanced Signal Processing

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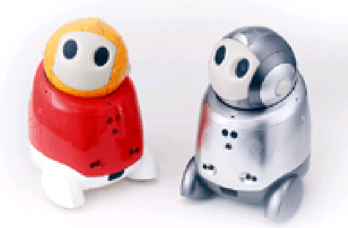
Why do we study Signal Processing?

- Important elemental technology
- especially in this digital era



Lecture Schedule

Apr.7 Overview

Apr.14 Filter design

Apr.21 Finite wordlength

Apr. 28 Multirate systems

May 7 Polyphase rep.

May 12 Filter banks

May 19 M-channel Filter banks

May 26 Adaptive filters

June 2 Exercise

June 9 Gradient algorithm

June 16 Recursive algorithm

June 23 DSP systems

June 30 Pipelining and parallel processing

July 7 Implementation of DSP algorithms

July 14 Exercise

July 28? Final Exam

Tokyo Tech OCW

http://www.ocw.titech.ac.jp/

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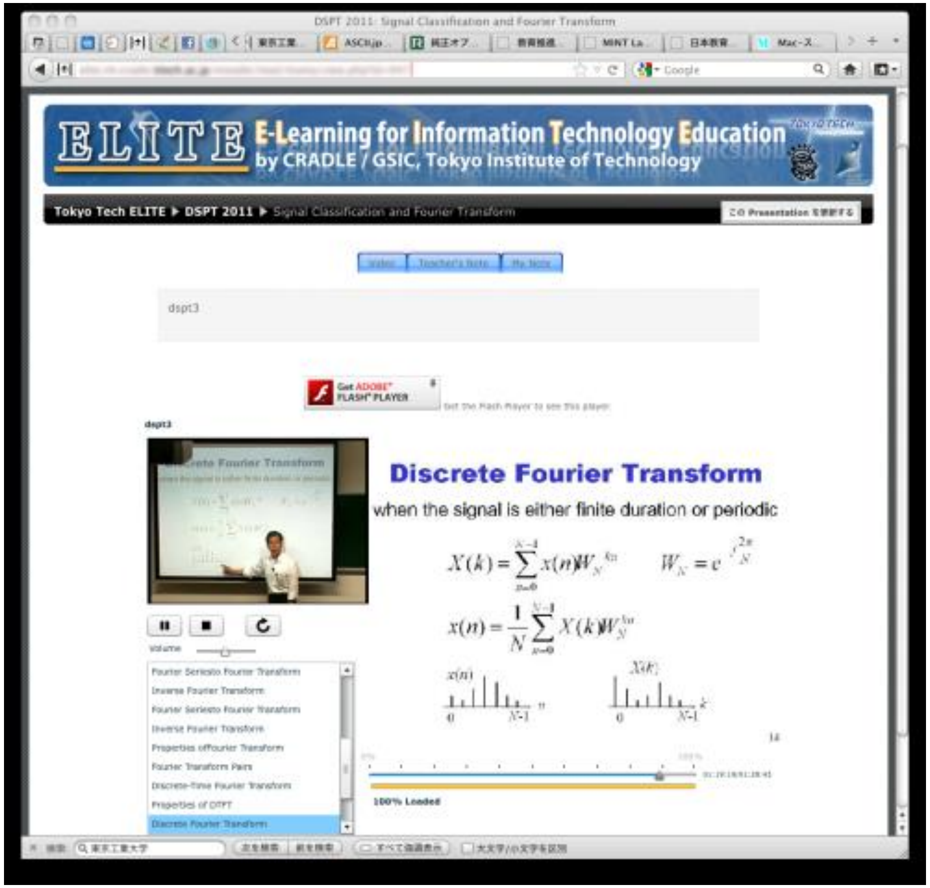


Registration information is collected today

Flipped Classroom: You have to study in advance

Reference Books

- **Alan V. Oppenheim & Ronald W. Schafer**
  Discrete-Time Signal Processing, Prentice Hall, 1989
- **Andreas Antoniou**
  Digital Filters: Analysis and Design, McGraw-Hill, 1979
- **P. P. Vaidynathan**
  Multirate Systems and Filter Banks, Prentice Hall, 1993
- **Maurice G. Bellanger**
  Adaptive Digital Filters and Signal Analysis, Dekker, 1987
- **Keshab K. Parhi**
  VLSI Digital Signal Processing Systems, Wiley, 1999

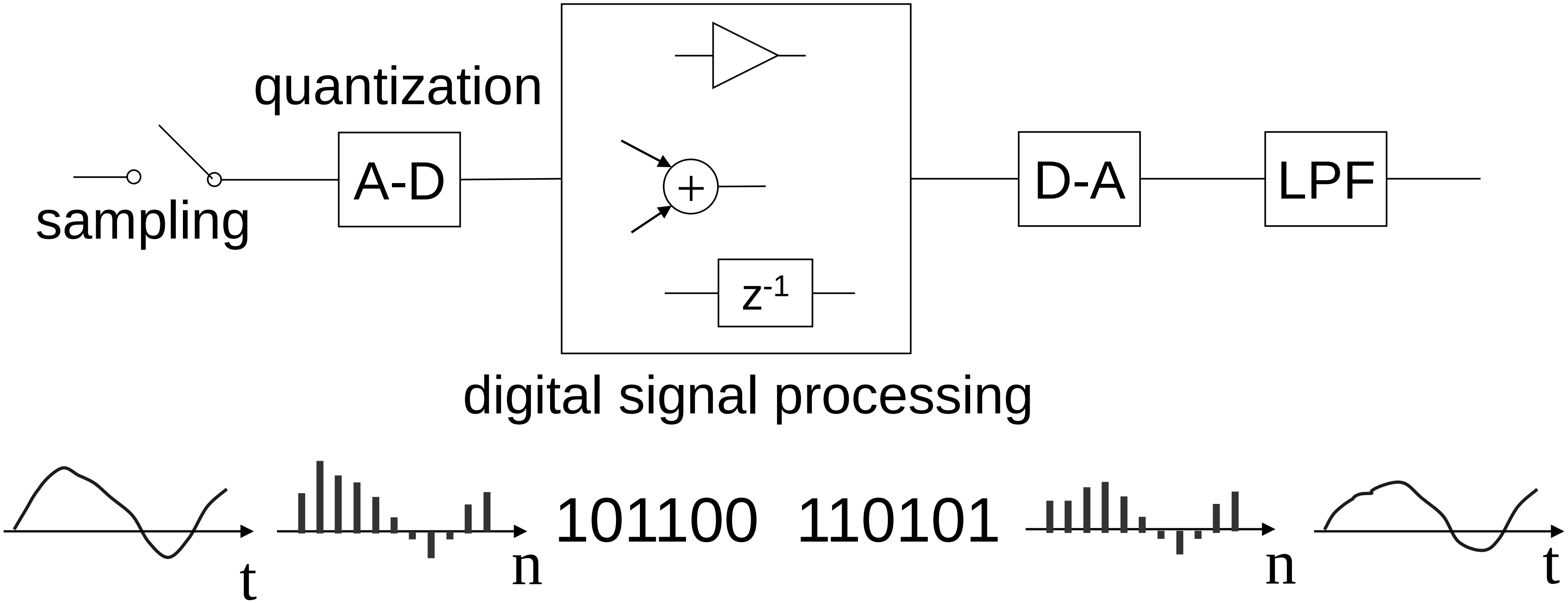
Signal Processing

Signals in real world : Analog

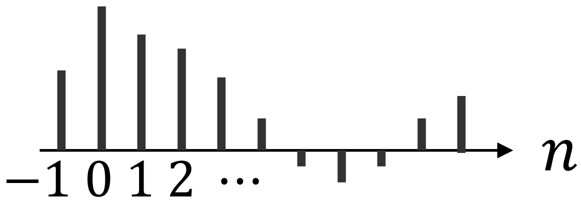
Digital processing has advantage in

- **VLSI implementation**
  - ✧**low component tolerance**
  - ✧**robust to environmental change**
- **flexibility**
  - ✧**processor architecture and**
  - ✧**software control**

Digital Processing of Analog Signals



Discrete-Time signal



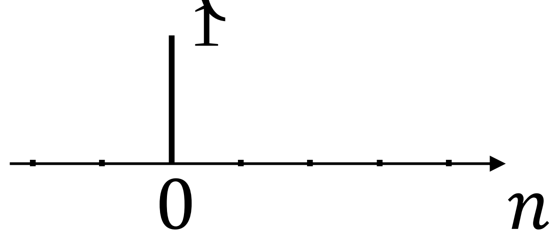
$$x(n), \qquad n = \cdots, -2, -1, 0, 1, 2, \cdots$$

digital : quantized amplitude

will be considered later

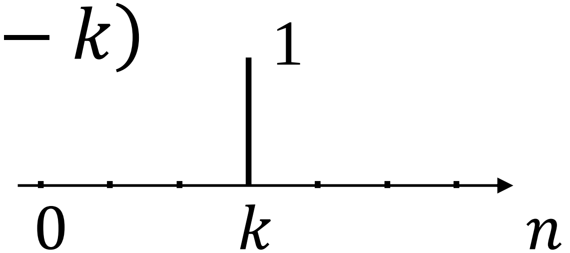
Unit Pulse Signal
(Impulse Signal)

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$



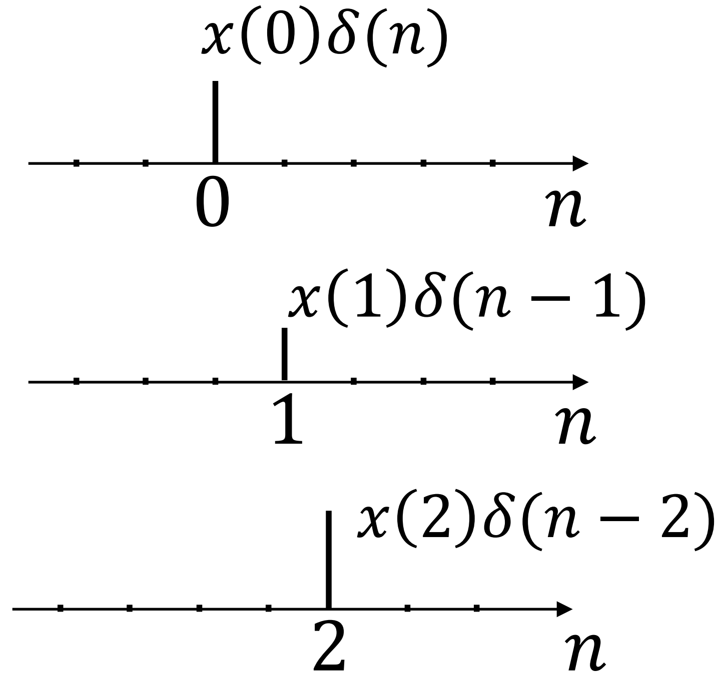
shifted (delayed) unit pulse

$$\delta(n-k)$$

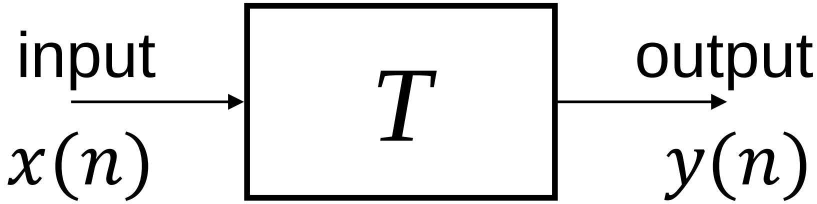


General Signal

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$



Discrete-Time System



Output $y(n)$ is a $\left\{\begin{matrix}\text{mapping}\\ \text{transform}\end{matrix}\right\}$ of input

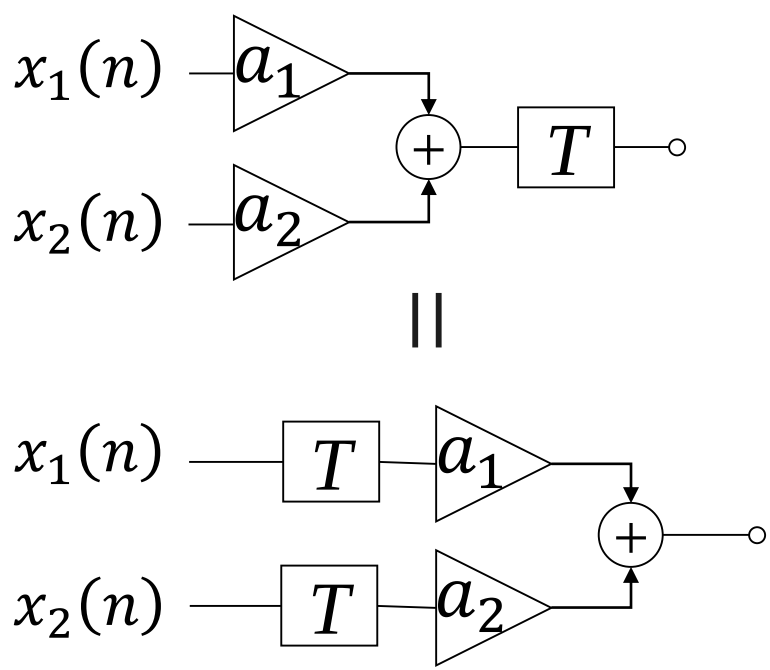
$$y(n) = T[x(n)]$$

Linearity

When $y_i(n) = T[x_i(n)]$

The system is **linear** if

$$\boxed{\sum_i a_i\, y_i(n) = T\left[\sum_i a_i\, x_i(n)\right]}$$

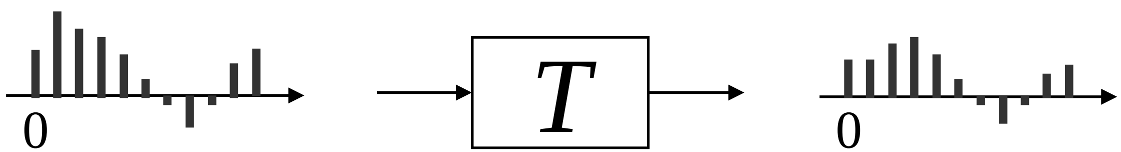


Shift-Invariance

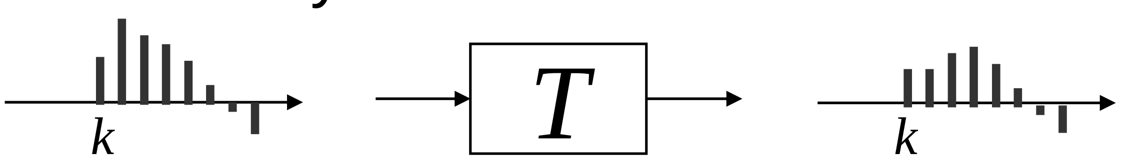
When $y(n) = T[x(n)]$

The system is **shift-invariant** if

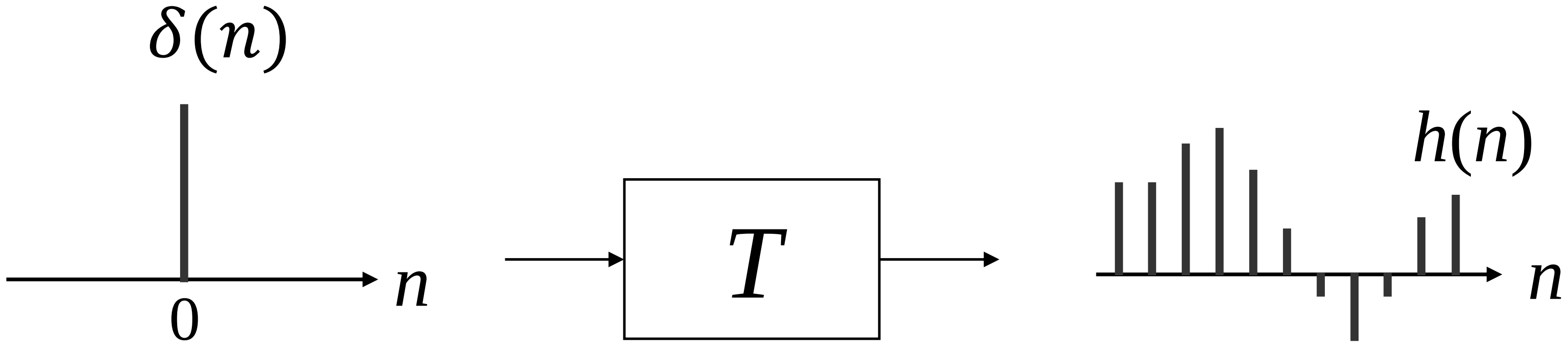
$$\boxed{y(n-k) = T[x(n-k)]}$$ for arbitrary k



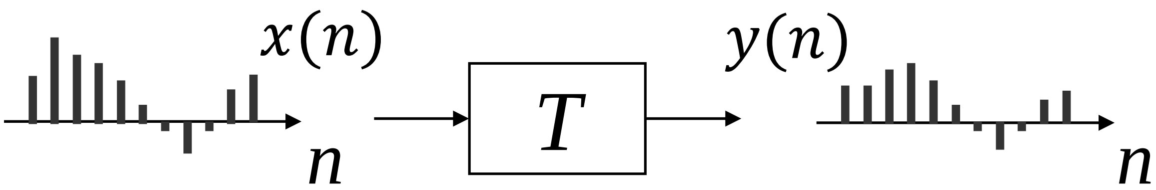
→ shifted by k



Impulse Response



Response to General Input



$$
\begin{aligned}
y(n) &= T[x(n)] \\
&= T\left[\sum_k x(k)\delta(n-k)\right] \\
&= \sum_k x(k)\,T[\delta(n-k)] \\
&= \sum_k x(k)h(n-k)
\end{aligned}
$$

linearity

shift-invariance

<u>convolution</u>

Convolution

$$y(n) = \sum_{k} x(k)h(n-k) = x(n) * h(n)$$
$$= \sum_{k} h(k)x(n-k) = h(n) * x(n)$$

$$a(n) * \big(b(n) * c(n)\big) = \big(a(n) * b(n)\big) * c(n)$$

BIBO Stability

Bounded Input Bounded Output Stability

$|x(n)| \le M < \infty$ bounded input

$$|y(n)| = \left| \sum_k h(k) x(n-k) \right|$$

$$\le \sum_k |h(k)||x(n-k)|$$

$$\le \sum_k |h(k)| M < \infty$$ bounded output

$$\boxed{\sum_k |h(k)| < \infty}$$ BIBO stability

Causality

causal $h(n) = 0 \quad n < 0$

anticausal $h(n) = 0 \quad n > 0$

if n is a time index

causality $\Leftrightarrow$ physical realizability

z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

convergence depends on

$$\begin{cases} \text{shape of sequence } x(n) \\ \text{value of complex variable } z \end{cases}$$

Properties of z-Transform

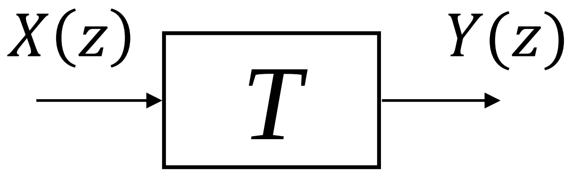
Linearity $a_1X_1(z) + a_2X_2(z) = \sum_{n=-\infty}^{\infty}(a_1x_1(n) + a_2x_2(n))z^{-n}$

Shift $X(z)z^{-k} = \sum_{n=-\infty}^{\infty} x(n-k)z^{-n}$

Inverse *z*-Transform

$$x(n) = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$$

Linear, Shift-Invariant System in *z*-Domain

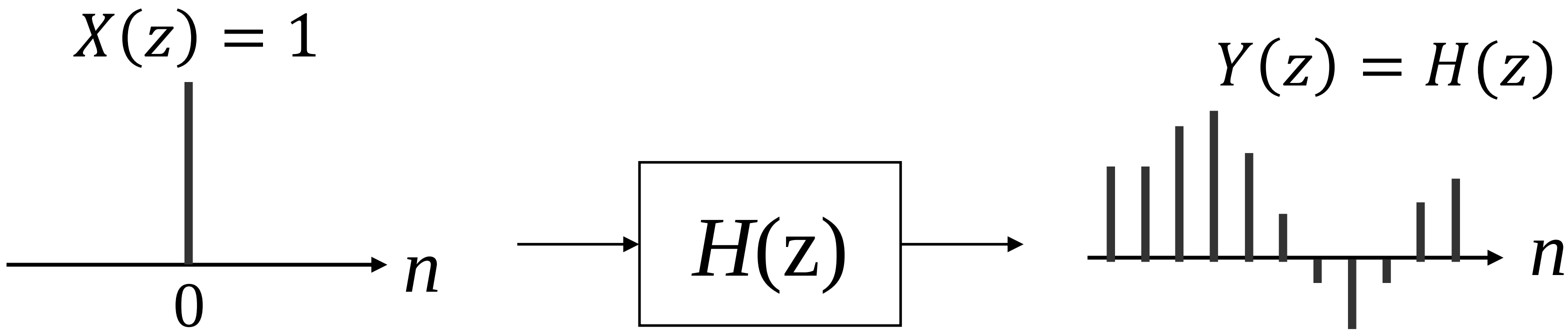


$$
\begin{aligned}
Y(z) &= \sum_{n=-\infty}^{\infty} y(n) z^{-n} \\
&= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x(k) h(n-k) z^{-n} \\
&= H(z) X(z)
\end{aligned}
$$

$H(z)$ **:** Transfer Function

z-transform of Impulse Response

Impulse Input



Complex Sinusoid Input

Input : $x(n) = e^{j\omega n}$

Output : $y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$

$$= \sum_{k=-\infty}^{\infty} h(k)e^{j\omega(n-k)}$$

$$= \sum_{k=-\infty}^{\infty} h(k)e^{-j\omega k}e^{j\omega n}$$

$$= \underline{H(e^{j\omega})}e^{j\omega n}$$

complex sinusiod with the same frequency

Frequency Response

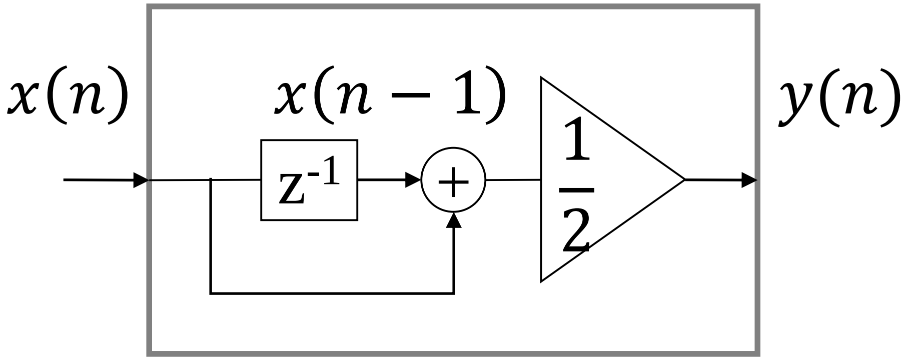
$$H\left(e^{j\omega}\right) = \sum_{k=-\infty}^{\infty} h(n)e^{-j\omega n}$$

$$= H(z)\Big|_{z=e^{j\omega}} \qquad 2\pi \text{ periodic}$$

$\left|\boldsymbol{H}(\boldsymbol{e}^{\boldsymbol{j\omega}})\right|$: Amplitude Response

$\arg H(\boldsymbol{e}^{\boldsymbol{j\omega}})$: Phase Response

Example 1

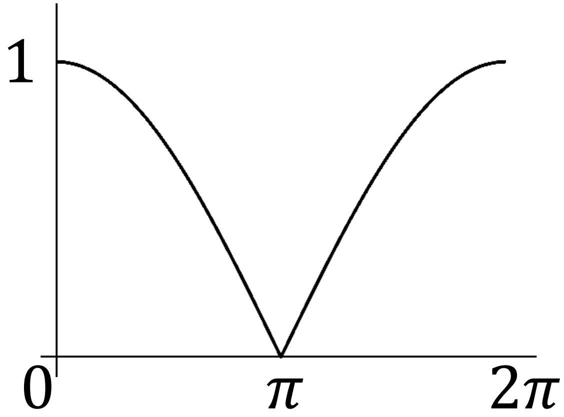


$$y(n) = \frac{1}{2}(x(n) + x(n-1))$$

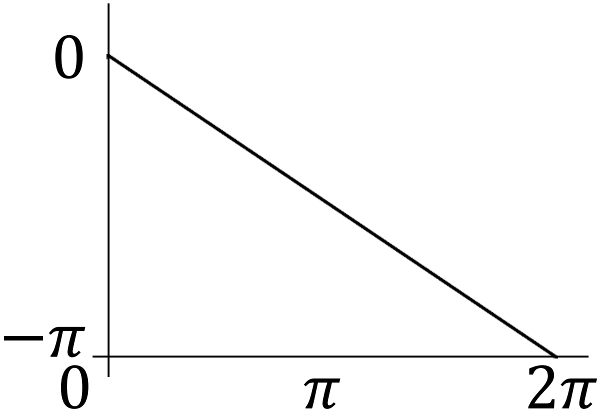
$$h(n) = \begin{cases} \frac{1}{2} & n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

$$H(z) = \frac{1}{2}(1 + z^{-1})$$

Frequency Response of Ex.1

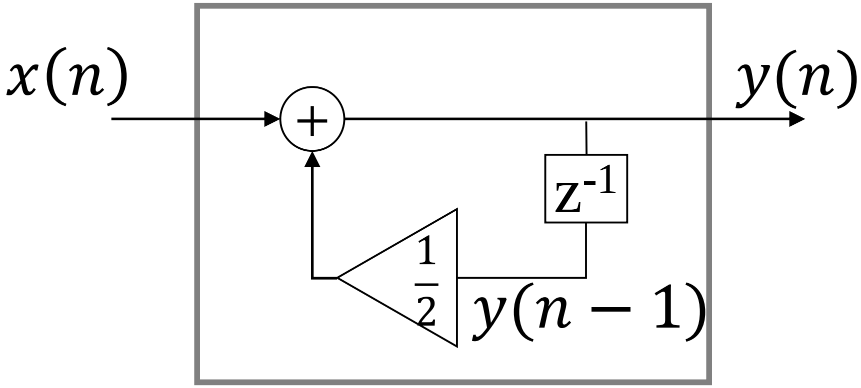


amplitude



phase

Example 2

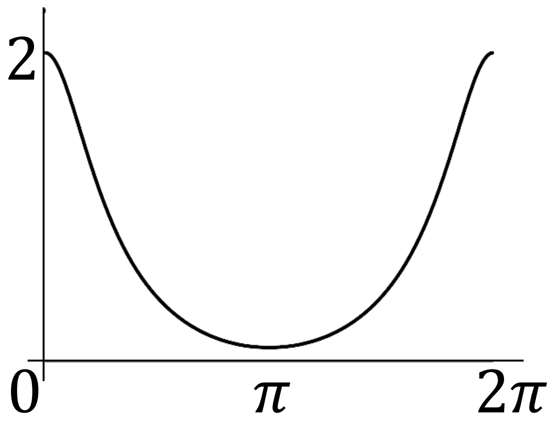


$$y(n) = x(n) + \frac{1}{2}y(n-1)$$

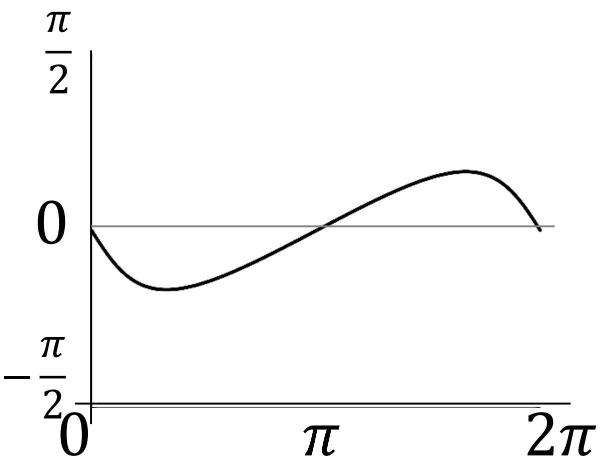
$$h(n) = \begin{cases} 0 & n < 0 \\ \left(\frac{1}{2}\right)^n & n \geq 0 \end{cases}$$

$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

Frequency Response of Ex.2



amplitude



phase

FIR/IIR, recursive/non recursive

$$\begin{cases} \text{FIR : Finite Impulse Response} \\ \text{IIR : Infinite Impulse Response} \end{cases}$$

$$\begin{cases} \text{non recursive : no feedback loops inside} \\ \text{recursive : feedback loop(s) exist} \end{cases}$$

Transfer Function

$$H(z) = \frac{a_0 + a_1 z^{-1} + \cdots + a_N z^{-N}}{1 + b_1 z^{-1} + \cdots + b_N z^{-N}}$$

N –th order rational function in z^{-1}

$$\frac{Y(z)}{X(z)} = \frac{a_0 + a_1 z^{-1} + \cdots + a_N z^{-N}}{1 + b_1 z^{-1} + \cdots + b_N z^{-N}}$$

$$(1 + b_1 z^{-1} + \cdots + b_N z^{-N})Y(z) = (a_0 + a_1 z^{-1} + \cdots + a_N z^{-N})X(z)$$

$$\Downarrow$$

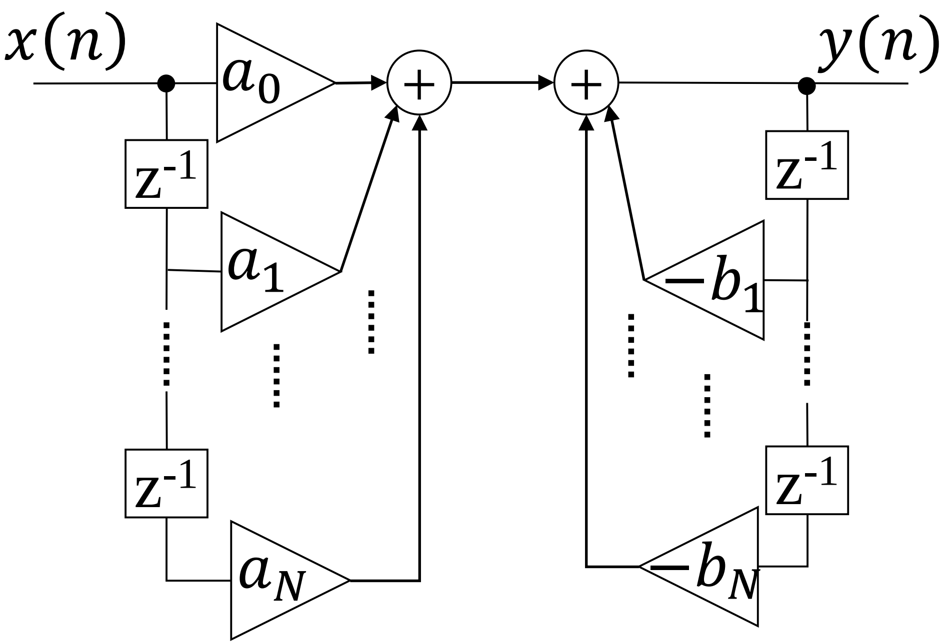
Inverse z-Transform

$$y(n) + b_1 y(n-1) + \cdots + b_N y(n-N) = a_0 x(n) + a_1 x(n-1) + \cdots + a_N x(n-N)$$

Standard Difference Equation

Realization of Standard Difference Equation

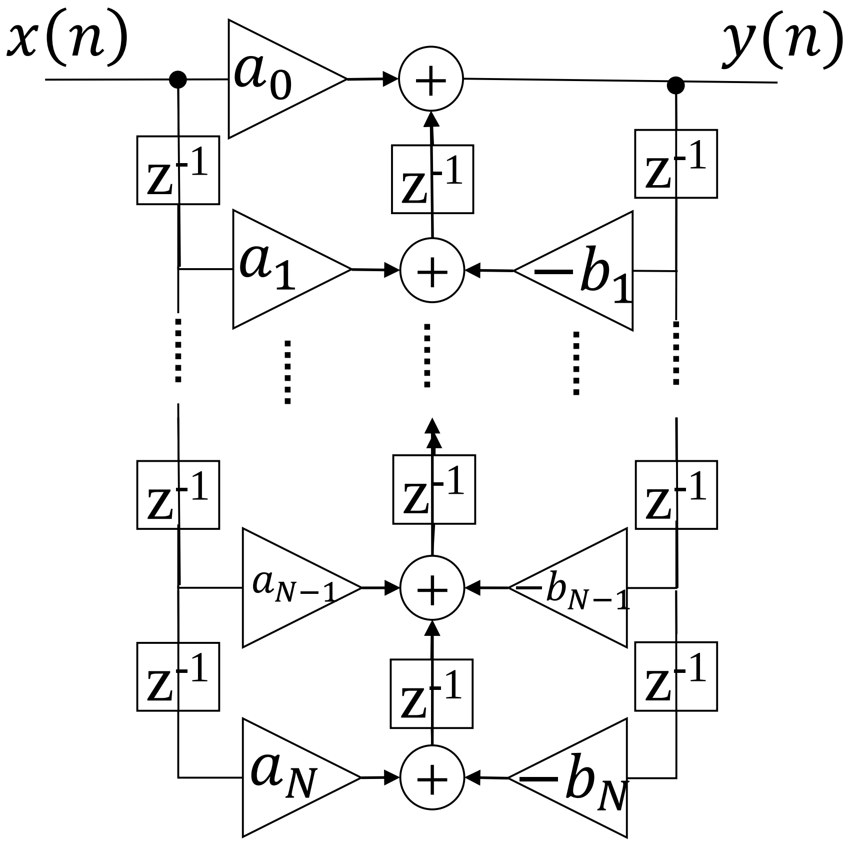
$$y(n) = a_0 x(n) + a_1 x(n-1) + \cdots + a_N x(n-N) - b_1 y(n-1) - \cdots - b_N y(n-N)$$



Direct Form

$2N$ delays
$2N+1$ multiplications
$2N$ additions

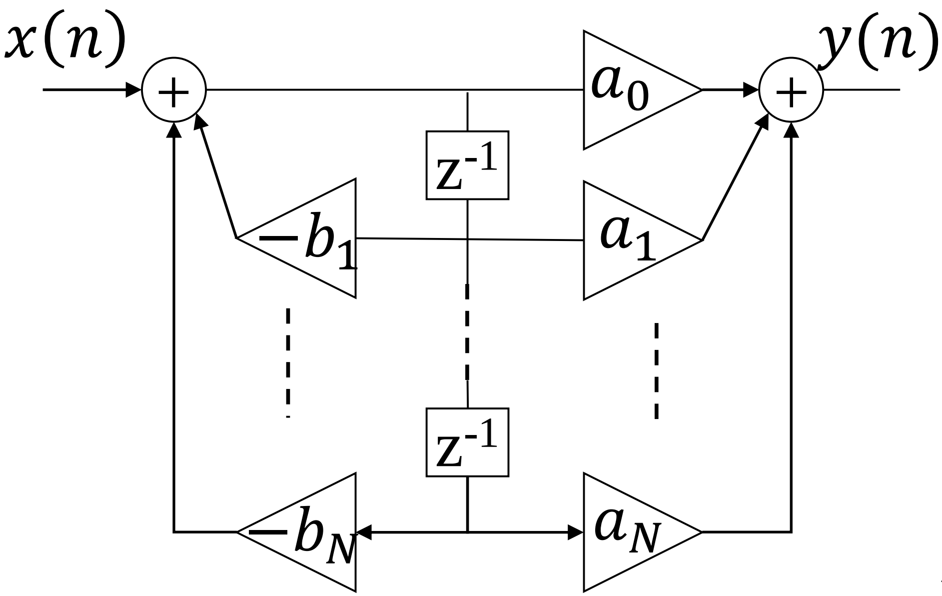
Equivalent Transformation



N delays
$2N+1$ multiplications
$2N$ additions

Another Equivalent Transformation

change in Order of subblocks



N delays
$2N$+1 multiplications
$2N$ additions

Flow Reversal ---- yet another equivalence

Exercise 1

1. Prove that the *z*-transform of the convolution of two sequences is the product of the respective *z*-transforms.
2. Obtain the transfer function for an *N*-tap moving average filter given by

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} x(n-k)$$

   and calculate its magnitude response.

3. Prove that the system of Example 2 is BIBO stable.
4. Show an example of recursive but FIR systems.