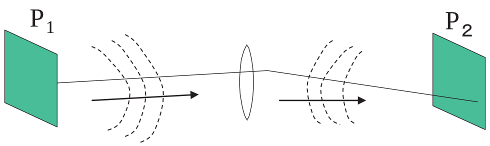
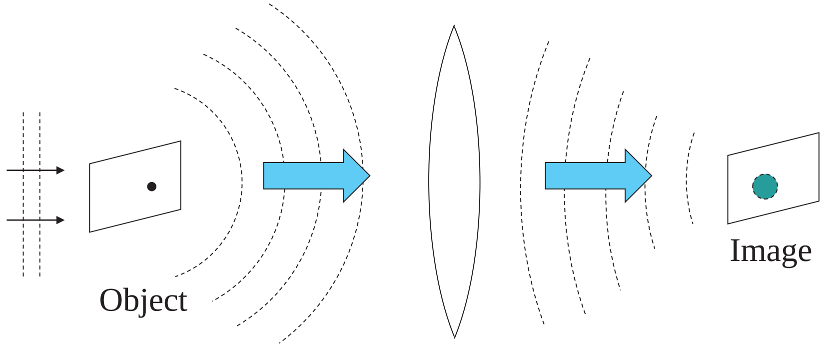
光画像工学

Optical imaging and image processing (V)

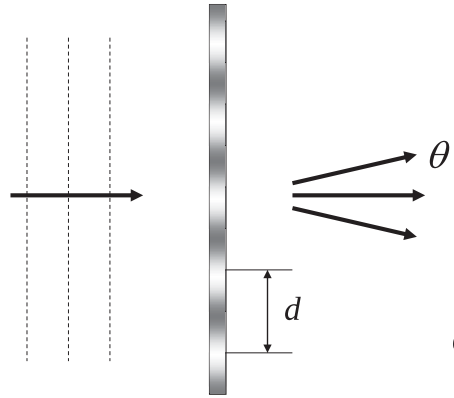
2. Optical imaging systems
2. 光学的イメージングシステム



2.3 Diffraction and wave propagation
2.3 光の回折と伝搬



Diffraction Grating
回折格子



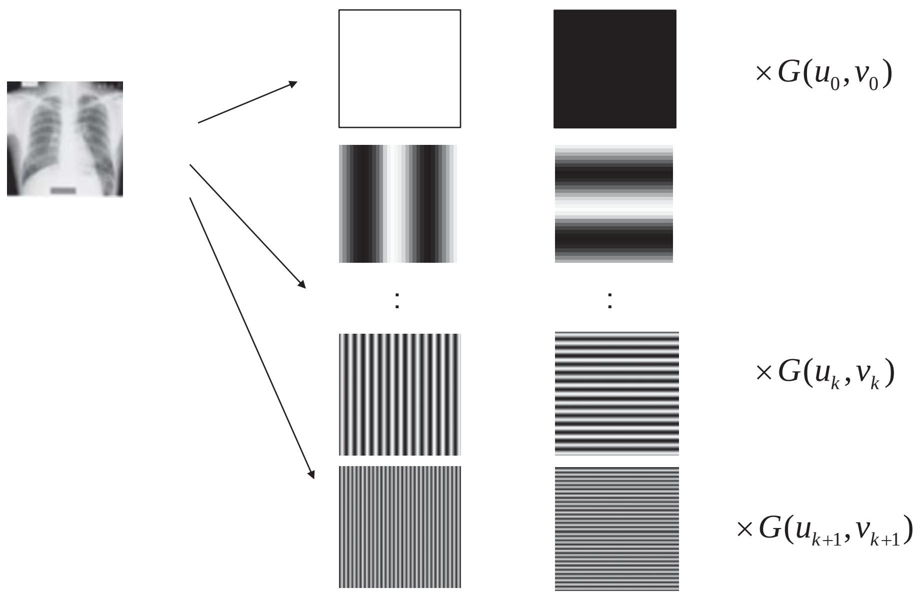
$$d \sin \theta = m \lambda$$

If sinusoidal grating,

$$d \sin \theta = 0、\pm\lambda$$

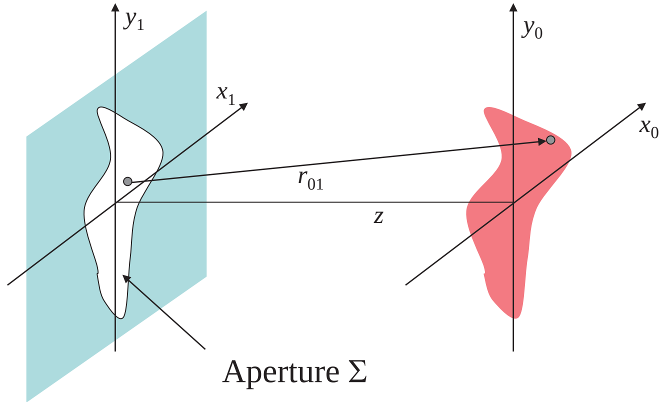
2-D Fourier transform

$$G(u,v) = \iint g(x,y)\exp\{-j2\pi(xu+yv)\}dxdy$$

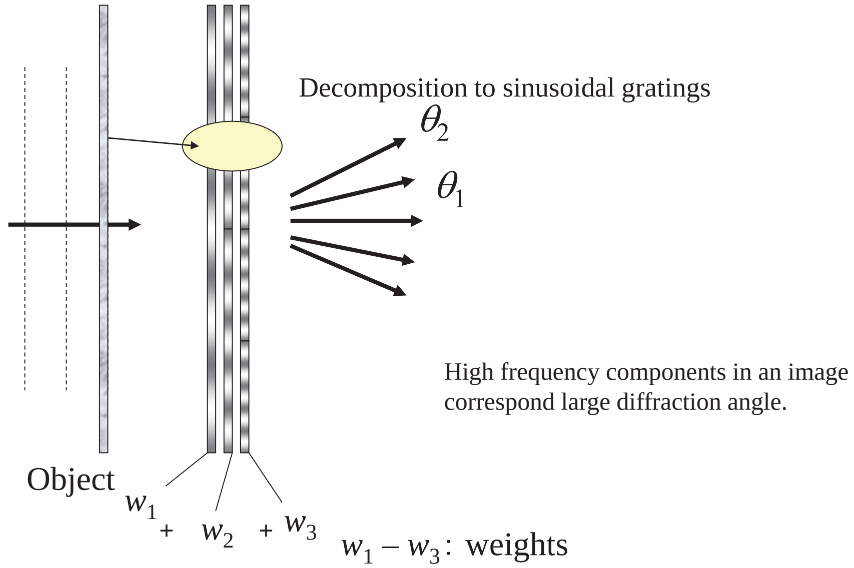


Scalar diffraction theory
スカラー回折理論

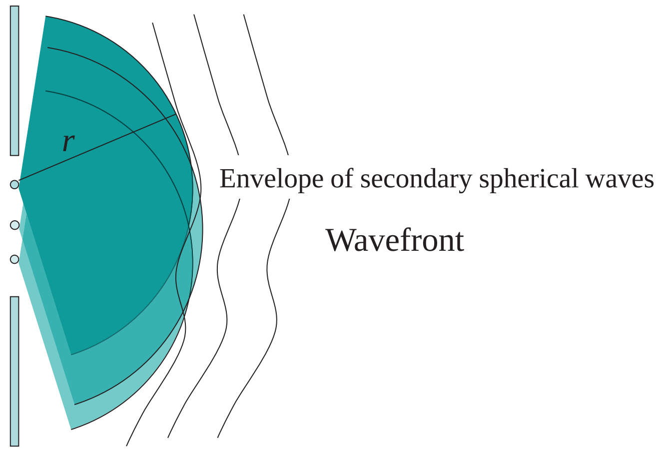
$$U(x_0, y_0) = \frac{1}{j\lambda z}\iint_{\Sigma} U(x_1, y_1)\exp(jkr_{01})dx_1 dy_1$$



Superposition of sinusoidal gratings



Huygens-Fresnel Principle



Fresnel approximation,　フレネル近似

If $|x_0 - x_1| << z$ and $|y_0 - y_1| << z$

$$r_{01} = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2}$$
$$= z\sqrt{1 + (\frac{x_0 - x_1}{z})^2 + (\frac{y_0 - y_1}{z})^2}$$
$$\cong z\left[1 + \frac{1}{2}(\frac{x_0 - x_1}{z})^2 + \frac{1}{2}(\frac{y_0 - y_1}{z})^2\right]$$

⇨ Paraxial approximation

Spherical wave is approximated by quadratic wave:

Spherical wave

$$U(x_0, y_0) = \frac{\exp(jkz)}{j\lambda z}\exp\{j\frac{k}{2z}[(x_0 - x_1)^2 + (y_0 - y_1)^2]\}$$

Fresnel diffraction

$$U(x_0, y_0) = \frac{\exp(jkz)}{j\lambda z}\iint_{\Sigma} U(x_1, y_1)\exp\{j\frac{k}{2z}[(x_0 - x_1)^2 + (y_0 - y_1)^2]\}dx_1 dy_1$$

Rewriting the Fresnel diffraction equation

$$g(x_0, y_0) = \iint h(x_0 - x_1, y_0 - y_1) f(x_1, y_1) dx_1 dy_1$$
$$= f(x_0, y_0) * h(x_0, y_0)$$

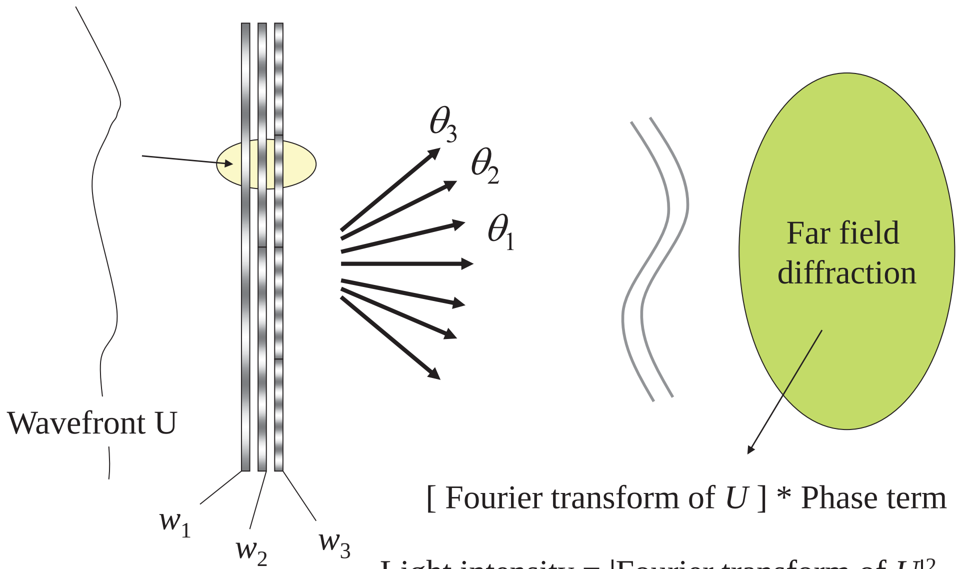
⟶ Convolution

$$h(x_0, y_0; x_1, y_1) = \frac{\exp\{jkz\}}{j\lambda z}\exp\{j\frac{k}{2z}[(x_0 - x_1)^2 + (y_0 - y_1)^2]\}$$

$$U(x_0, y_0) = \frac{\exp\{jkz\}}{j\lambda z}\exp\{j\frac{k}{2z}(x_0^2 + y_0^2)\}\int\int_{-\infty}^{\infty} U(x_1, y_1)$$
$$\exp\{j\frac{k}{2z}(x_1^2 + y_1^2)\}\exp\{-j\frac{2\pi}{\lambda z}(x_0 x_1 + y_0 y_1)\}dx_1 dy_1$$

Fourier Transform of $U(x_1, y_1)\exp\{j\frac{k}{2z}(x_1^2 + y_1^2)\}$ * Phase Term

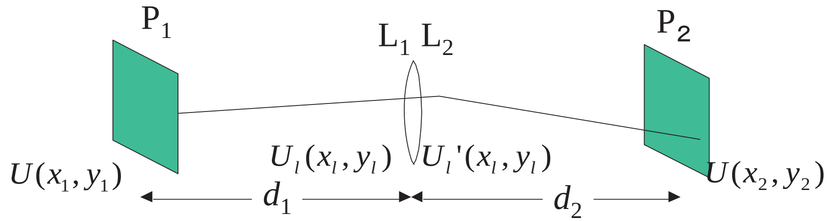
Fraunhofer Diffraction
フラウンホファー回折



[Fourier transform of U] * Phase term

Light intensity = |Fourier transform of U|^2

2.4 Imaging through a lens system
2.4 レンズ系による結像



$P_1 \rightarrow L_1$　Fresnel Diffraction

$$U_l(x_l, y_l) = \frac{\exp(jkd_1)}{j\lambda d_1}\int\int_{-\infty}^{\infty} U(x_1, y_1)\exp\{j\frac{k}{2d_1}[(x_l - x_1)^2 + (y_l - y_1)^2]\}dx_1 dy_1$$

$L_1 \rightarrow L_2$　Phase modulation by lens

Transformation of wavefront　Spherical wave → Plane wave
Spherical wave → Spherical wave

$$U_l'(x_l, y_l) = U_l(x_l, y_l)P(x_l, y_l)\exp\{-j\frac{k}{2f}(x_l^2 + y_l^2)\}$$

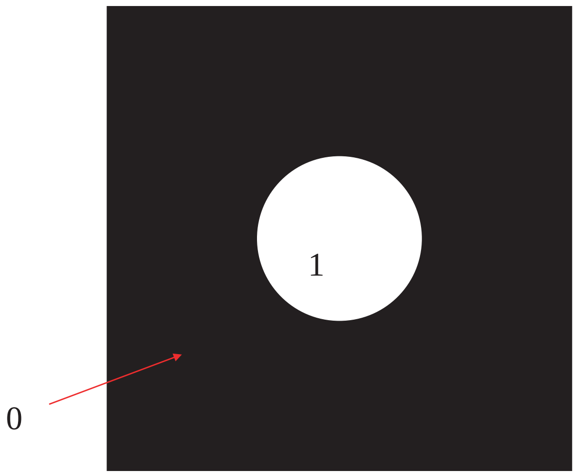
$L_2 \rightarrow P_2$　Fresnel Diffraction

$$U(x_2, y_2) = \frac{\exp(jkd_2)}{j\lambda d_2}\int\int_{-\infty}^{\infty} U_l'(x_l, y_l)\exp\{j\frac{k}{2d_2}[(x_2 - x_l)^2 + (y_2 - y_l)^2]\}dx_l dy_l$$

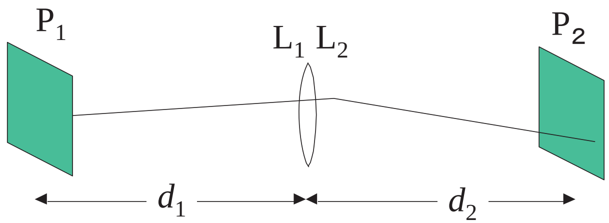
Lens aperture = Pupil function

瞳関数

$P(x_l, y_l)$



Wavefront at P_1 and P_2 planes



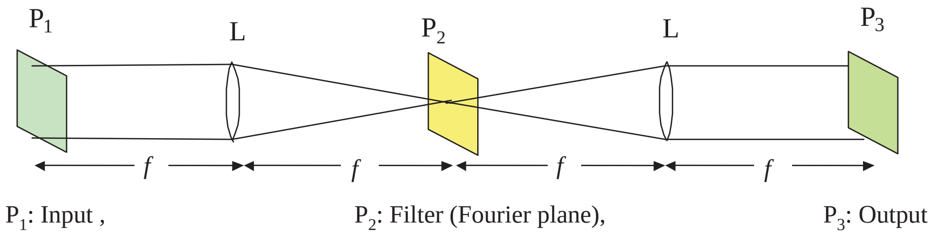
$$U(x_2,y_2) = A\int\int_{-\infty}^{\infty} h(x_2,y_2;x_1,y_1)U(x_1,y_1)dx_1dy_1$$

$$h(x_2,y_2;x_1,y_1) = \frac{1}{\lambda^2 d_1 d_2}\exp\left[j\frac{k}{2d_2}(x_2^2+y_2^2)\right]\exp\left[j\frac{k}{2d_1}(x_1^2+y_1^2)\right]$$

$$\int\int_{-\infty}^{\infty} P(x,y)\exp\left[j\frac{k}{2}(\frac{1}{d_1}+\frac{1}{d_2}-\frac{1}{f})(x^2+y^2)\right]$$

$$\exp\left[-jk\{(\frac{x_1}{d_1}+\frac{x_2}{d_2})x+(\frac{y_1}{d_1}+\frac{y_2}{d_2})y\}\right]dxdy$$

Optical Fourier transform and Coherent optical filtering



When $d_1 = f$ and $d_2 = f$,

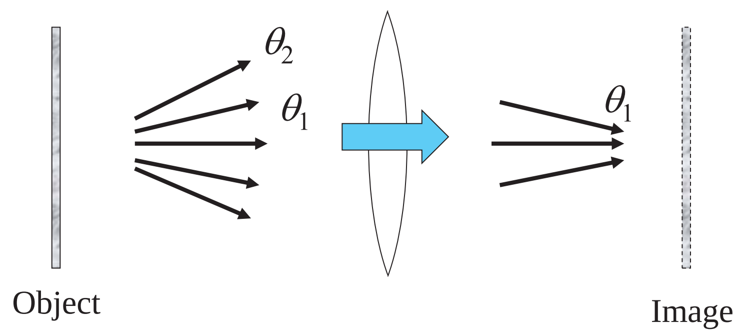
$$U(x_2,y_2) = A\int\int_{-\infty}^{\infty} U(x_1,y_1)\exp\{j\frac{2\pi}{\lambda f}[x_1x_2+y_1y_2]\}dx_1dy_1$$

$$u = x_2/\lambda f,\ v = y_2/\lambda f$$

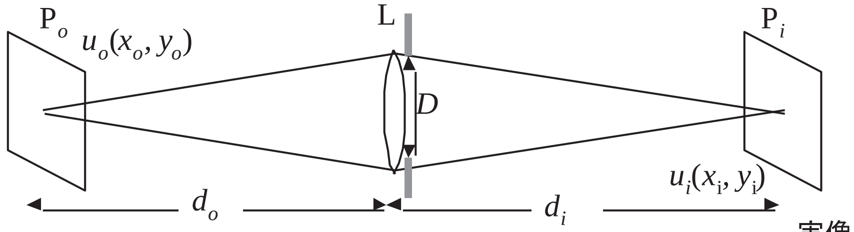
$$U(u,v) = C\,\boldsymbol{F}\{U(x_1,y_1)\}$$

2.5 Impulse response (PSF) and transfer function of a lens system
2.5 レンズ系の点像分布関数と伝達関数

Imaging by a lens system



Impulse response of an lens system



$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$ Lens formula

$$M=\frac{d_i}{d_o}$$ Magnification

$$h(x_i, y_i; x_o, y_o)=\frac{1}{\lambda^2 d_i d_o}\int\int_{-\infty}^{\infty} P(x,y)\exp\left[-j\frac{2\pi}{\lambda d_i}\{(x_i+Mx_o)x+(y_i+My_o)y\}\right]dxdy$$

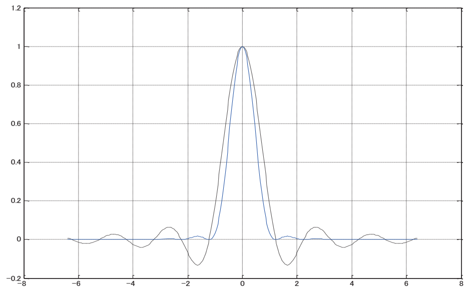
⇨ Impulse response = Fourier transform of the pupil function

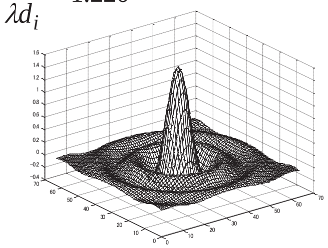
Impulse response of a circular aperture

$$h'(x_i, y_i)=|P_f(\frac{x_i}{\lambda d_i},\frac{y_i}{\lambda d_i})|^2$$

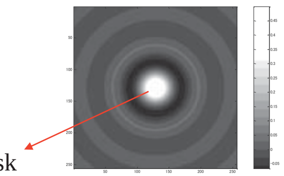
$$=|\frac{\pi D^2}{2}\cdot\frac{J_1(\frac{\pi D r_i}{\lambda d_i})}{\frac{\pi D r_i}{\lambda d_i}}|^2$$

$h'(r_i) = 0$ for $\frac{Dr_i}{\lambda d_i}=1.220$



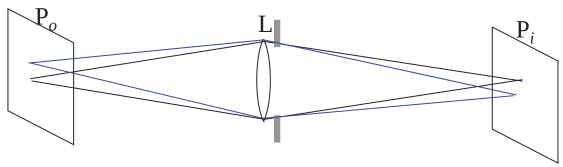


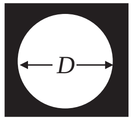
$2J_1(\pi z)/\pi z$, $|2J_1(\pi z)/\pi z|^2$



Transfer function of lens system

→ 11/6 HW: ex.5



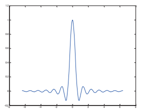


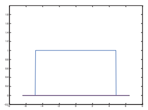
$P(x,y)$

- Coherent transfer function $H_c(u,v)$

PSF $u_i(x_i,y_i)=P_f(\frac{x_i}{\lambda d_i},\frac{y_i}{\lambda d_i})*u_o(-\frac{x_i}{M},-\frac{y_i}{M})=h_c(x_i,y_i)*u_o(-\frac{x_i}{M},-\frac{y_i}{M})$

$U_i(u,v)=H_c(u,v)U_o(u,v)$



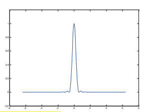


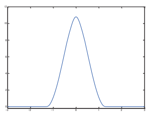
- Optical transfer function (Incoherent)

PSF $|u_i(x_i,y_i)|^2=|P_f(\frac{x_i}{\lambda d_i},\frac{y_i}{\lambda d_i})|^2*|u_o(-\frac{x_i}{M},-\frac{y_i}{M})|^2$

$g(x_i,y_i)=h'(x_i,y_i)*f(x_i,y_i)$

$G(u,v)=H'(u,v)F(u,v)$

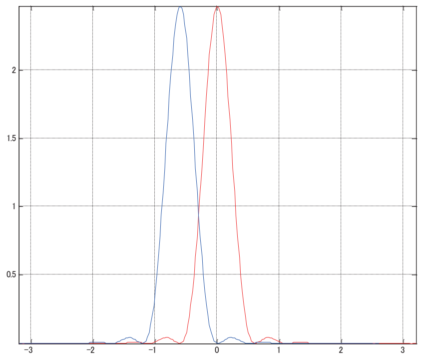


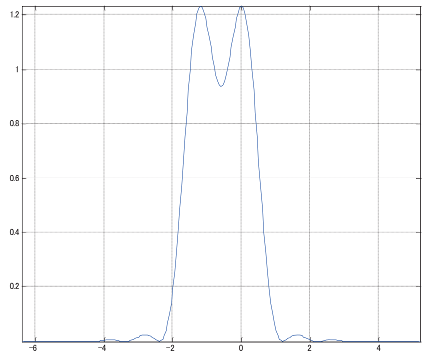


$H(u,v) = H'(u,v) / H'(0,0)$: Optical transfer function (OTF)

2.6 Resolution of a lens system
2.6 レンズ系の分解能

- Rayleigh criterion -



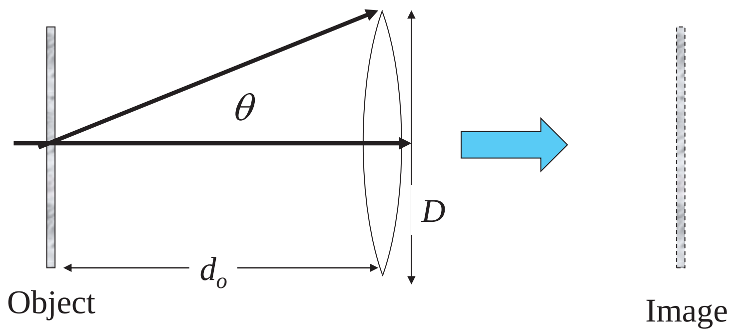


Rayleigh limit $L=1.22\frac{\lambda d_i}{D}$

回折限界・解像限界 (Diffraction limit)

Estimating the resolution of a lens system

$p \sin\theta = \lambda$



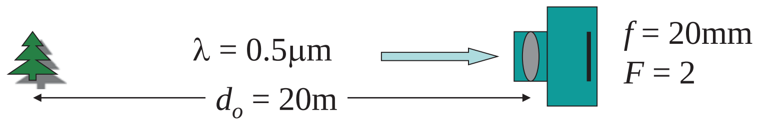
$\sin\theta \approx \theta = D/(2\,d_o)$

$U_{max} = 1/p_{min} = \sin\theta/\lambda \approx D/(2\lambda d_o)$

Rayleigh limit (image plane) $L = 1.22\dfrac{\lambda d_i}{D}$ $\longrightarrow$ Object plane $\dfrac{L}{M} = 1.22\dfrac{\lambda d_i}{DM} = 1.22\dfrac{\lambda d_o}{D}$

Example

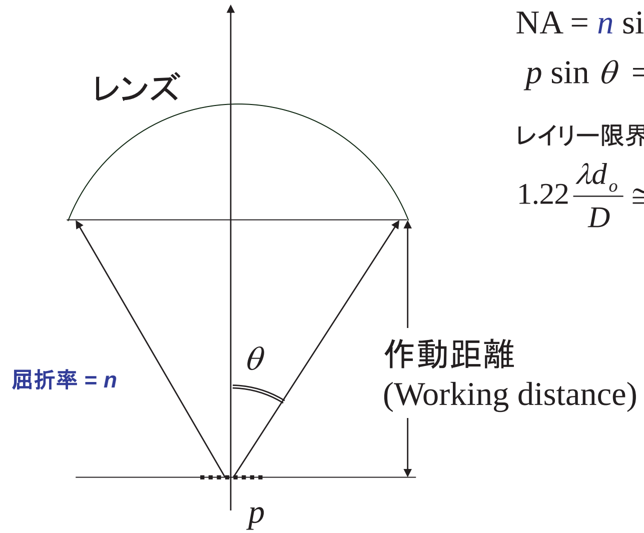
$$u_{o\,max} = \frac{D}{\lambda d_o}$$



- Lens diameter $D = f/F = 10\text{mm}$
- $u_{o\,max} = 10 / (0.5 \times 10^{-3} \times 20 \times 10^{3}) = 1.0\ (\text{mm}^{-1})$

- Object larger than ≅ 1mm can be resolved.

Numerical Aperture (開口数)



$\mathrm{NA} = n\sin\theta$
$p\sin\theta = \lambda$ $\longrightarrow$ $p \geq a\dfrac{\lambda}{NA}$

レイリー限界

$$1.22\frac{\lambda d_o}{D} \cong 1.22\frac{\lambda}{2\,NA} = 0.61\frac{\lambda}{NA}$$

NA = 0.25
λ = 500nm
a = 0.61

$p \geq 1.22\mu\text{m}$