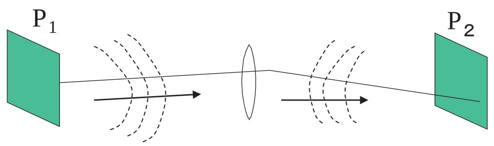
光画像工学
Optical imaging and image processing
(IV)

2. Optical imaging systems
2. 光学的イメージングシステム

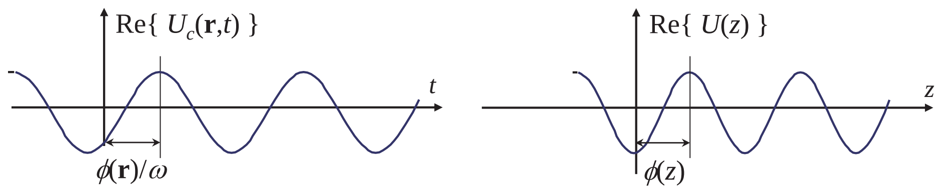
2. Optical imaging systems

- 2.1 Complex expression of waves
  - – Complex amplitude, Wavefront
  - – Plane wave, spherical wave
- 2.2 Interference
  - – Coherence, Interferometer
- 2.3 Diffraction and wave propagation
  - – Scalar wave propagation theory
  - – Fresnhel diffraction, Fraunhofer diffraction
- 2.4 Imaging through a lens system
  - – Optical Fourier transform, Coherent optical filtering
  - – Image formation
- 2.5 Impulse response (PSF) and transfer function of a lens system
  - – Pupil function, Point spread function
  - – Coherent transfer function, Optical transfer function, Modulation transfer function
- 2.6 Resolution of a lens system
  - – Diffraction limit, Rayleigh criterion, Numerical aperture

Appendix. Geometrical optics, ray-tracing, lens aberration

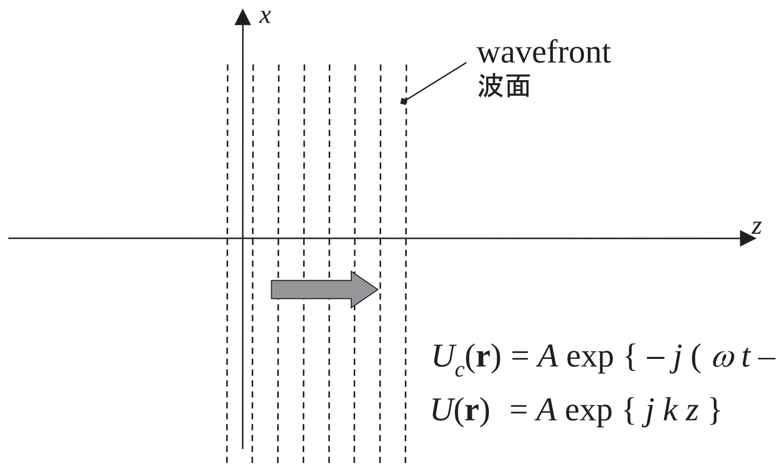


2.1 Complex expression of waves
2.1 波動の複素表示



- $U_c(\mathbf{r}, t) = A(\mathbf{r}) \exp\{-j(\omega t - \phi(\mathbf{r}))\}$
  $= U(\mathbf{r}) \exp(-j\omega t)$
- $U(\mathbf{r})$: Complex amplitude 複素振幅
  $U(\mathbf{r}) = A(\mathbf{r}) \exp\{j\,\phi(\mathbf{r})\}$
- $A(\mathbf{r})$: Amplitude 振幅
- $\phi(\mathbf{r})$: Phase 位相
- $I(\mathbf{r})$: Intensity 強度
  $I(\mathbf{r}) = \langle |U_c(\mathbf{r}, t)|^2 \rangle = |U(\mathbf{r})|^2 = A(\mathbf{r})^2$

Plane wave traveling in the *z*-direction



$$U_c(\mathbf{r}) = A \exp\{-j(\omega t - kz)\}$$
$$U(\mathbf{r}) = A \exp\{jkz\}$$

k: wave number 波数 $k = \dfrac{2\pi}{\lambda}$

Plane wave traveling along wave vector **k** 波数ベクトル

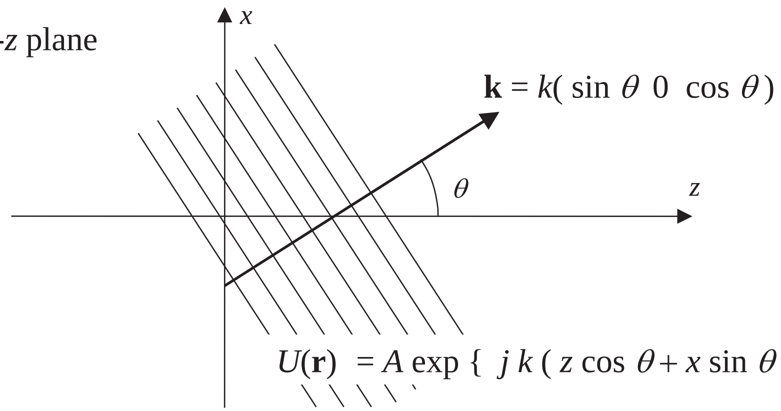
$$U_c(\mathbf{r}, t) = A(\mathbf{r}) \exp\{ -j (\omega t - \phi(\mathbf{r})) \}$$

$$\phi(\mathbf{r}) = \mathbf{k} \cdot \mathbf{r} = k_x x + k_y y + k_z z$$

$\mathbf{k} = (k_x \ \ k_y \ \ k_z)$: wave vector, $\cdot$: inner product

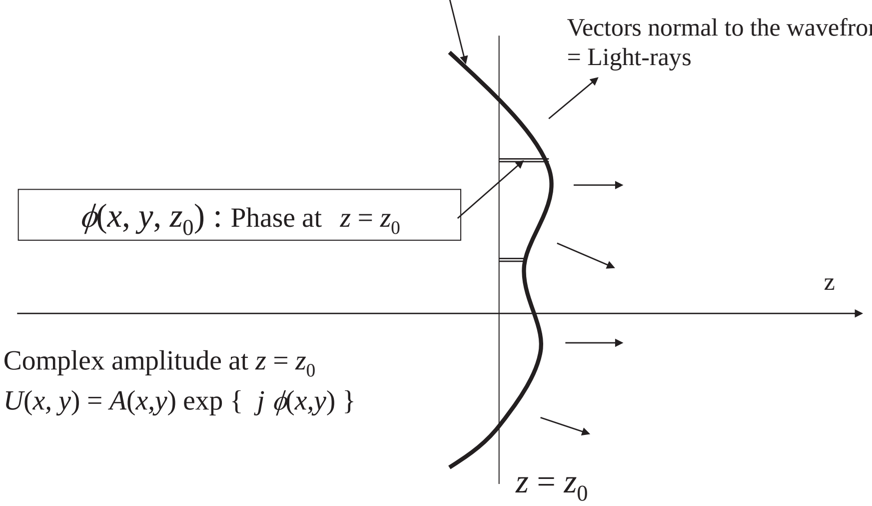
$$U(\mathbf{r}) = A(\mathbf{r}) \exp(\ j \, \mathbf{k} \cdot \mathbf{r}) = A(\mathbf{r}) \exp\{ \ j (k_x x + k_y y + k_z z) \}$$

x-*z* plane



$$U(\mathbf{r}) = A \exp\{ \ j k (z \cos\theta + x \sin\theta) \}$$

Wavefront
波面
(Surface of equal phase)

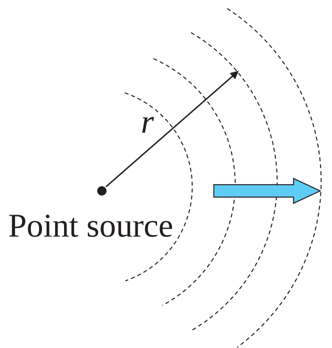


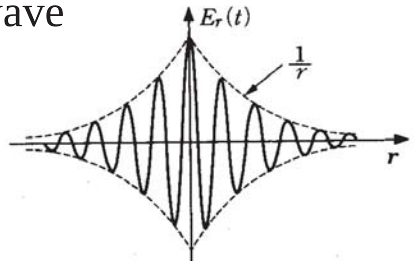
Complex amplitude at $z = z_0$

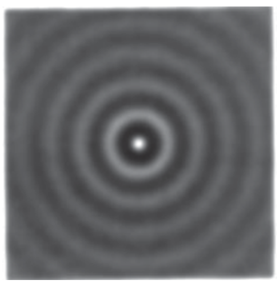
$$U(x, y) = A(x,y) \exp\{ \ j \ \phi(x,y) \}$$

Spherical wave
球面波

$$U(r) = \frac{U_0 \exp\{ j k r \}}{r}$$

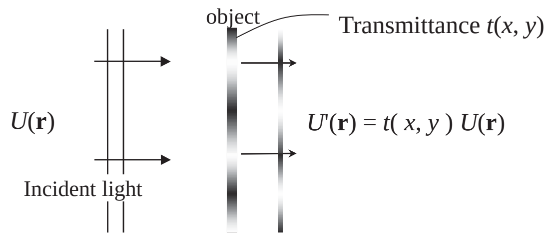




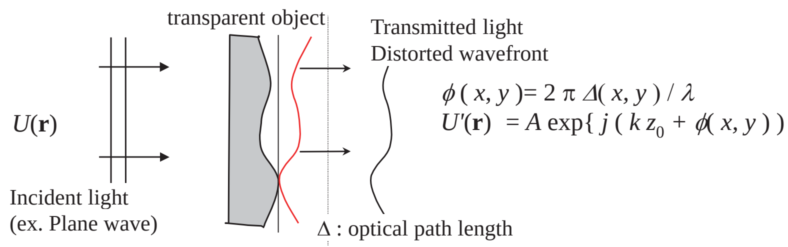


$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

Complex amplitude modulation 複素振幅変調



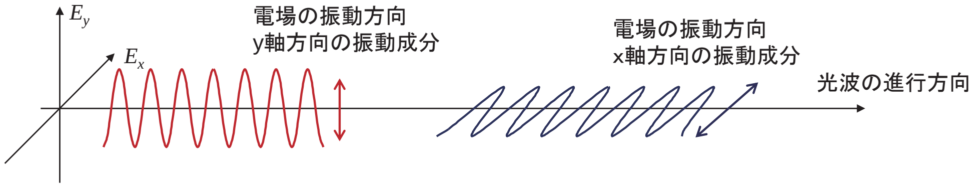
Amplitude modulation



Phase modulation

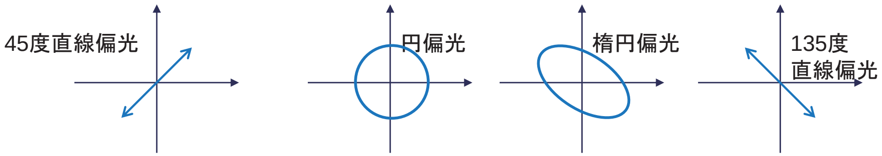
Amplitude transmittance $t(x,y)$ →→ $U'(x, y) = t(x, y) \ U(x, y)$

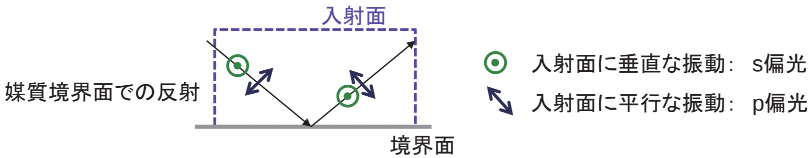
Polarization 偏光



x, y 方向の振幅が等しい場合を考える

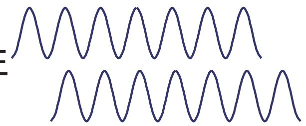
x, y 方向の振動の位相差 = 0, = π/2, = θ, = π



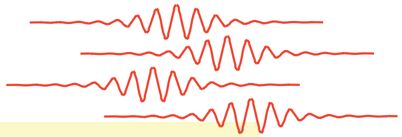


偏光板, 波長板, 複屈折

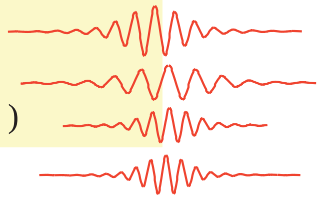
Coherence
コヒーレンス、干渉性



- if $\omega_1 \neq \omega_2$, $< \cos \{ (\omega_1 - \omega_2) t + \phi \} > = 0$
  for the observation time >> $2\pi / \omega$
  $I(\mathbf{r}) = A_1(\mathbf{r})^2 + A_2(\mathbf{r})^2$
  → incoherent (temporal)



- if $\omega_1 = \omega_2$, → coherent
  $I(\mathbf{r}) = < | U_{1c}(\mathbf{r}, t) + U_{2c}(\mathbf{r}, t) |^2 >$
  $= | U_1(\mathbf{r}) + U_2(\mathbf{r}) |^2$
  $= A_1(\mathbf{r})^2 + A_2(\mathbf{r})^2 +$
  $2 A_1(\mathbf{r}) A_2(\mathbf{r}) \cos (\phi_1(\mathbf{r}) - \phi_2(\mathbf{r}))$



- (Partially coherent)

2.2 Interference
2.2 光の干渉

- Interference of two wavefronts
  $U_{1c}(\mathbf{r}, t) = A_1(\mathbf{r}) \exp \{ -j (\omega_1 t - \phi_1(\mathbf{r})) \}$
  $U_{2c}(\mathbf{r}, t) = A_2(\mathbf{r}) \exp \{ -j (\omega_2 t - \phi_2(\mathbf{r})) \}$

$I(\mathbf{r}) = < | U_{1c}(\mathbf{r}, t) + U_{2c}(\mathbf{r}, t) |^2 >$
$= A_1(\mathbf{r})^2 + A_2(\mathbf{r})^2$
$+ 2 A_1(\mathbf{r}) A_2(\mathbf{r}) < \cos \{ (\omega_1 - \omega_2) t - (\phi_1(\mathbf{r}) - \phi_2(\mathbf{r})) \} >$

Interference of two plane waves

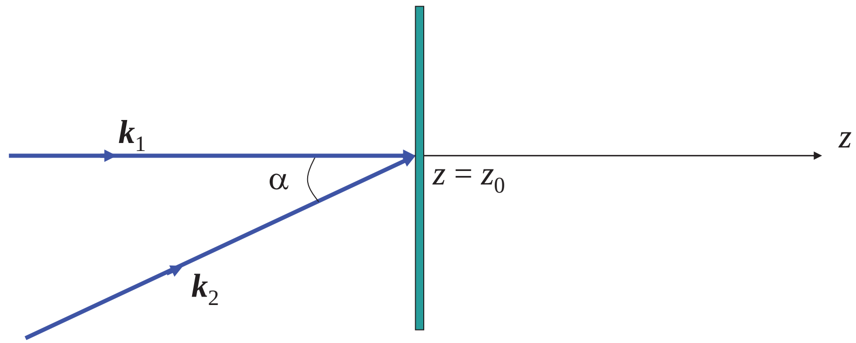
$$I = A_1^2 + A_2^2 + 2A_1A_2 \cos\{(\boldsymbol{k}_2 - \boldsymbol{k}_1)\boldsymbol{r} - (\phi_2 - \phi_1)\}$$

$$\boldsymbol{k}_1\boldsymbol{r} = kz_0$$

$$\boldsymbol{k}_2\boldsymbol{r} = kx\sin\alpha + kz_0\cos\alpha$$

$$(\boldsymbol{k}_2 - \boldsymbol{k}_1)\boldsymbol{r} = kx\sin\alpha + kz_0(1 - \cos\alpha)$$

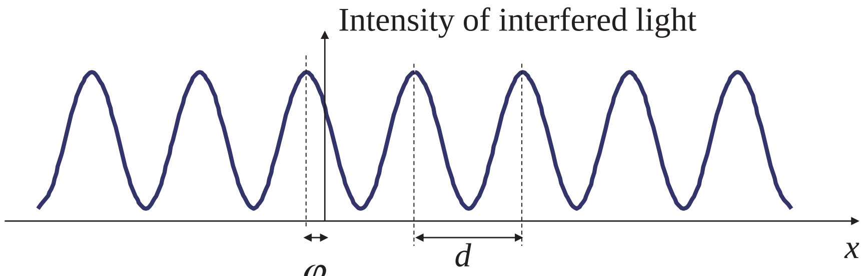
Constant



Interference fringe
干渉縞

$$I = A_1^2 + A_2^2 + 2A_1A_2\cos\{kx\sin\alpha - \varphi_0\}$$

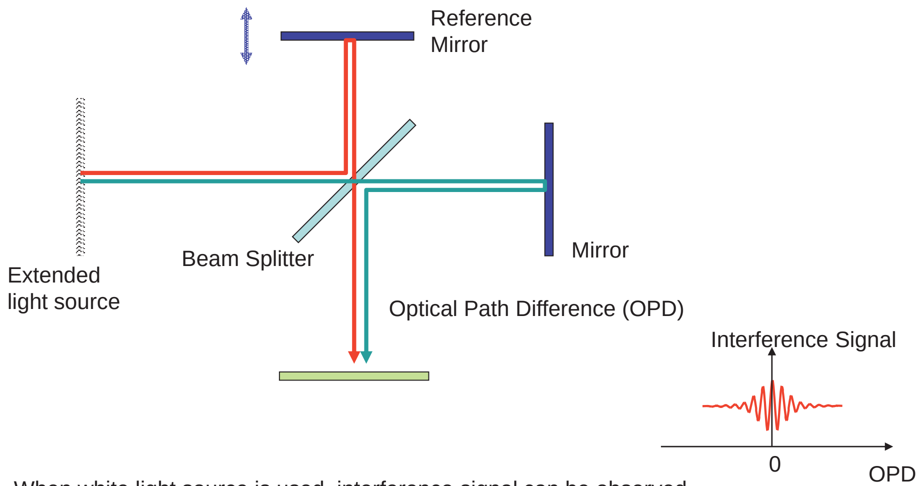
$$\varphi_0 = kz_0(1-\cos\alpha) - (\phi_2 - \phi_1)$$



$$\frac{\varphi_0}{k\sin\alpha}$$

$$kd\sin\alpha = \frac{2\pi d\sin\alpha}{\lambda} = 2\pi \;\rightarrow\; d\sin\alpha = \lambda$$

Michelson Interferometer



When white light source is used, interference signal can be observed only when OPD ≈ 0