光画像工学
Optical imaging and image processing (III)

1.6 2D Discrete Fourier transform

2D DFT

$$F[k,l] = \mathbf{DFT}\{f[m,n]\} = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f[m,n]\exp\{-j2\pi(\frac{mk}{M}+\frac{nl}{N})\}$$

Inverse 2D DFT

$$f[m,n] = \mathbf{DFT}^{-1}\{F[k,l]\} = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} F[k,l]\exp\{j2\pi(\frac{mk}{M}+\frac{nl}{N})\}$$

2D Fourier transform in continuous space

$$F(u,v) = \iint f(x,y)\exp\{-j2\pi(ux+vy)\}dxdy$$

Inverse 2D Fourier transform in continuous space

$$f(x,y) = \iint F(u,v)\exp\{j2\pi(ux+vy)\}dudv$$

Dirac Delta function array

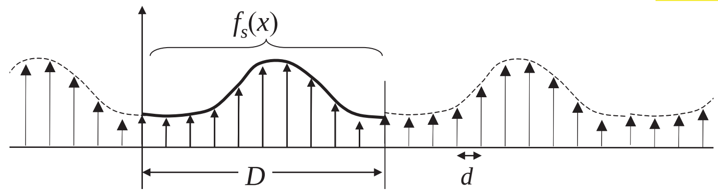
- Fourier transform of comb function (period = d)
  $\Rightarrow$ comb function (period = $1/d$)

- Modulated delta function array
  - Fourier transform of a modulated delta function array ($T = d$)
    $\Rightarrow$ Periodic function ($T = 1/d$)
  - Fourier transform of a periodic function ($T = D$)
    $\Rightarrow$ A modulated delta function array ($T = 1/D$)

Sampling and periodicity in DFT, Number of pixels = N

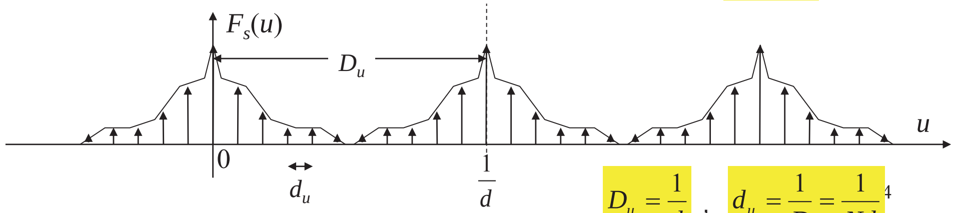
Spatial domain:

Sampling pitch (intervals of delta functions) = d,
Image size (period) = D → $N = \frac{D}{d}$



Fourier domain (Frequency domain):

Sampling pitch (intervals of delta functions) = d_u,
Period in frequency domain = D_u → $N = \frac{D_u}{d_u}$



$D_u = \frac{1}{d}$, $d_u = \frac{1}{D} = \frac{1}{Nd}$

The frequency components obtained by DFT and those in the continuous space

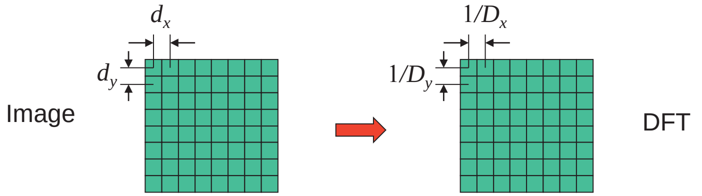
$$F[k,l] = \mathbf{DFT}\{f[m,n]\} = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f[m,n]\exp\{-j2\pi(\frac{mk}{M}+\frac{nl}{N})\}$$

$$F(u,v) = \iint f(x,y)\exp\{-j2\pi(ux+vy)\}dxdy$$

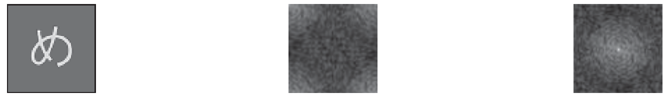
$$u = \frac{k}{D_x} = \frac{k}{Nd_x}$$

$$k = 0,\ 1,\ \dots\ N-1 \longrightarrow u = 0,\ \frac{1}{Nd_x},\ \dots\ \frac{N-1}{Nd_x}$$

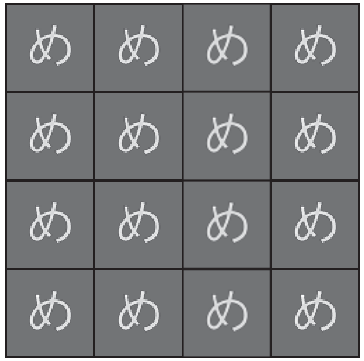
or, $$u = 0,\ \frac{1}{D_x},\ \dots\ \frac{N-1}{D_x}$$



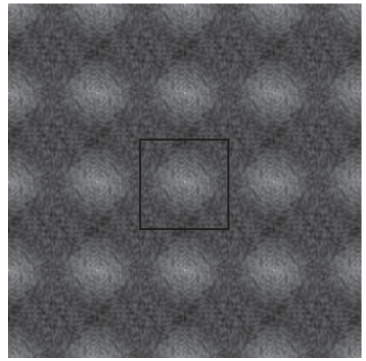
The periodicity in DFT



(a) Original image (b) DFT of (a) (c) zero Frequency centered.

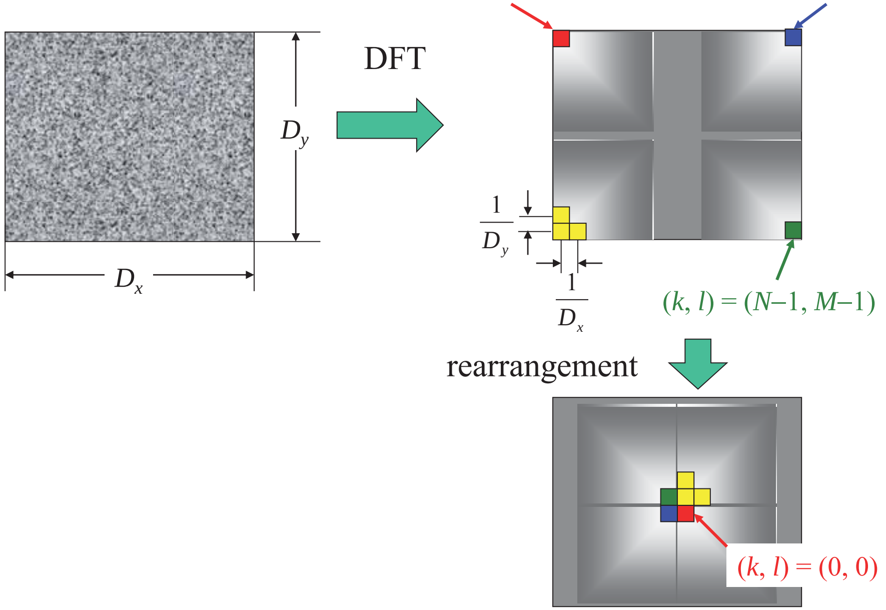


(d) DFT is considered as the Fourier transform of the periodic function like this figure.



(e) DFT of (d). The frequency spectra are also periodic. The square region surrounded by □ corresponds to (c).

For example, for an image in $N \times M$ pixels, we have $N \times M$ Fourier coefficients; $F[\,k,\,l\,]$



1.7 Fourier analysis of linear shift-invariant imaging system

- 2-D linear system in continuous space

  $$g(x,y) = \iint h(x,y;x',y')\,f(x',y')dx'dy'$$

- Shift-invariant (space-invariant)

  $$g(x,y) = \iint h(x-x',y-y')\,f(x',y')dx'dy'$$
  $$= f(x,y) * h(x,y)$$

  → Convolution

  $h(\,x,y\,)$: Impulse response, point spread function (PSF) インパルス応答 点像分布関数

- 2-D linear shift-invariant imaging system with additive noise

  $$g(x,y) = \iint_{-\infty}^{\infty} h(x-x',y-y')\,f(x'y')dx'dy' + n(x,y)$$

- Fourier transform of 2-D shift-invariant imaging system

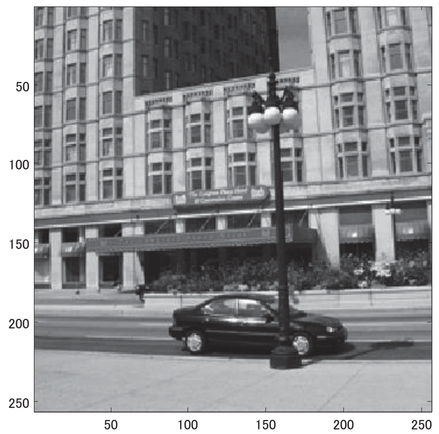
  $$G(u,v) = H(u,v)F(u,v)$$
  $$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

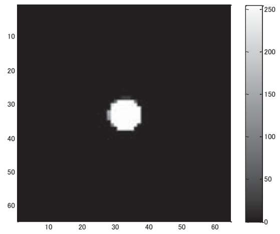
  $H(u,v)$: Transfer function 伝達関数、周波数特性

2-D Linear, shift-invariant system in discrete space

Discrete convolution 離散たたみ込み

$$g[m,n] = \mathbf{S}\{f[m,n]\} = \sum_{m',n'} h[m-m', n-n'] f[m',n']$$
$$= h[m,n] * f[m,n]$$





$h_D[m,n]$

$M_h \times N_h$ pixels

Discrete signals within a finite region

$f_D[m,n]$

$M \times N$ pixels

imagesc(img);
imagesc(ci);

$0 \le m' \le M-1 \qquad 0 \le m - m' \le M_h - 1$

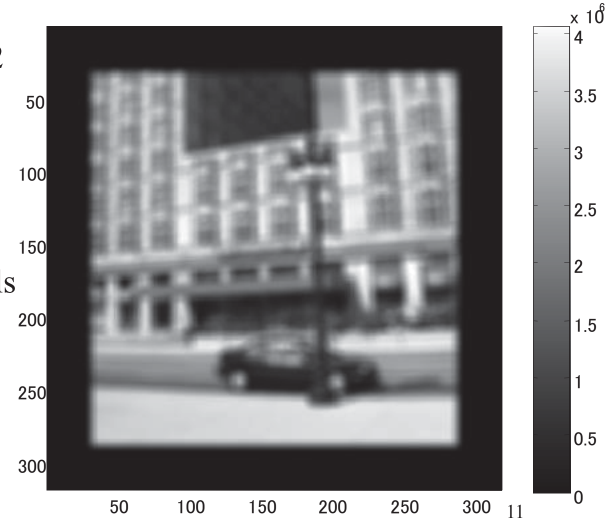
$0 \le n' \le N-1 \qquad 0 \le n - n' \le N_h - 1$

⇒ $0 \le m \le M + M_h - 2$

$0 \le n \le N + N_h - 2$

$g[m,n]$

$(M+M_h-1)\times(N+N_h-1)$ pixels



cres = conv2(double(img), double(ci));
imagesc(cres);

Discrete convolution by using DFT

Circulant convolution

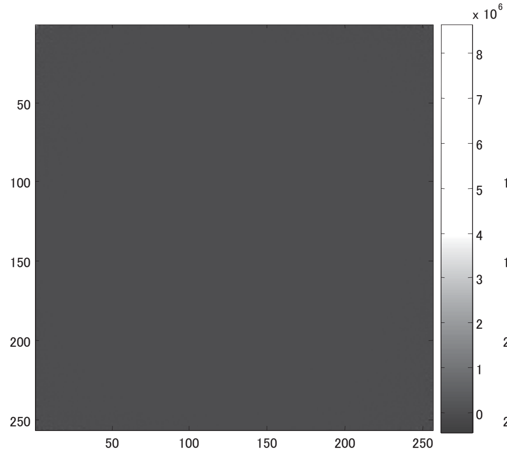
$F_D[k,l] = \mathbf{DFT}\{f_D[m,n]\}$
$H_D[k,l] = \mathbf{DFT}\{h_D[m,n]\}$
} Discrete signals within a finite interval

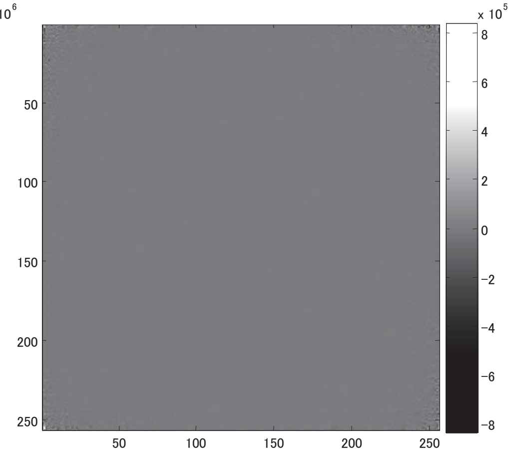
$g_D[m,n] = \mathbf{DFT}^{-1}\{G_D[k,l]\} = \mathbf{DFT}^{-1}\{H_D[k,l]\ F_D[k,l]\}$

Consider

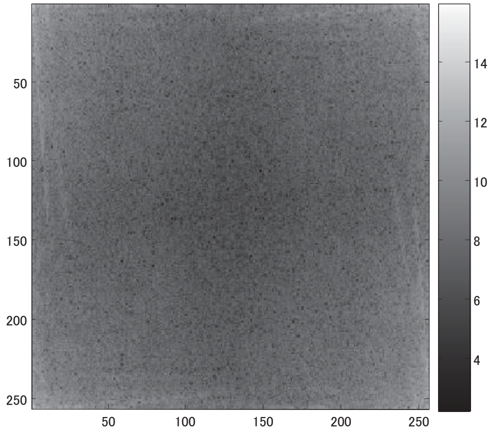
$f_p[m,n] = f_D[m-kM, n-lN]$
$h_p[m,n] = h_D[m-kM, n-lN]$
$g_p[m,n] = g_D[m-kM, n-lN]$
} Periodic functions

- fim = fft2(double(img));
- imagesc(real(fim));
- imagesc(imag(fim));

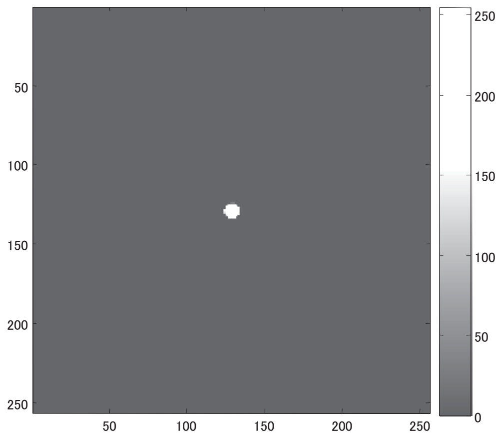


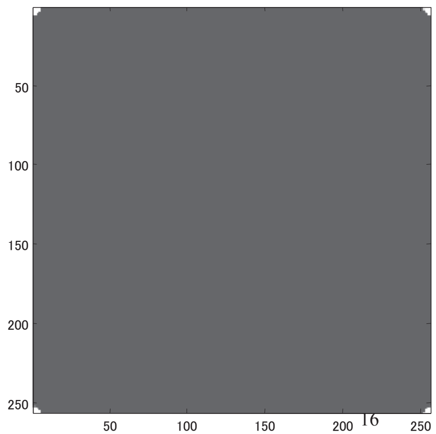


- imagesc(log(abs(fim)));

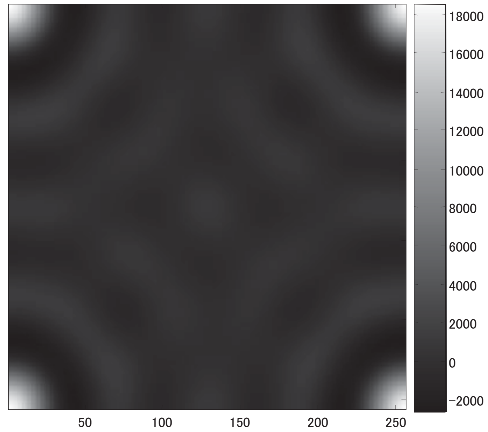


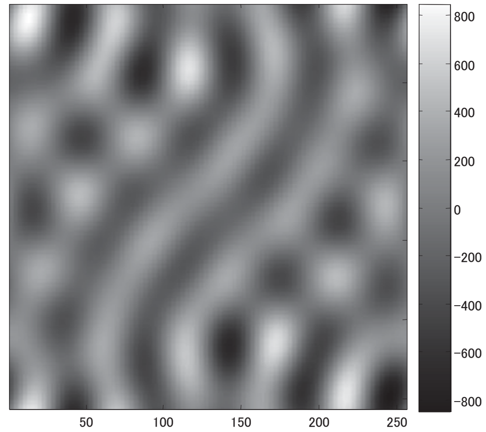
- cil = zeros(256,256);
- cil([97:160],[97:160])=ci;
- imagesc(cil);
- scil = fftshift(cil);
- imagesc(scil);



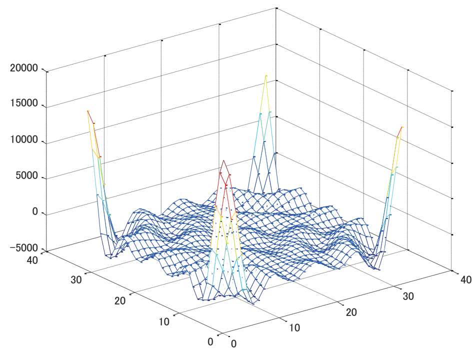


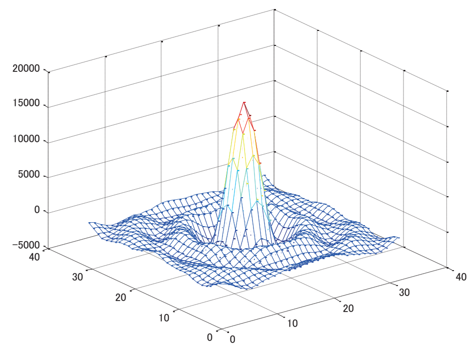
- fscil = fft2(scil);
- imagesc(real(fscil));
- imagesc(imag(fscil));



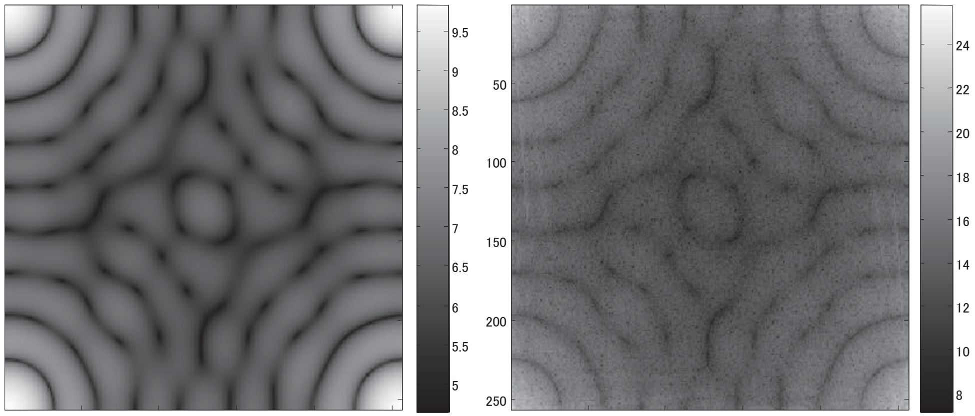


- mesh(real(fscil([1:8:256],[1:8:256])));
- mesh(fftshift(real(fscil([1:8:256],[1:8:256]))));

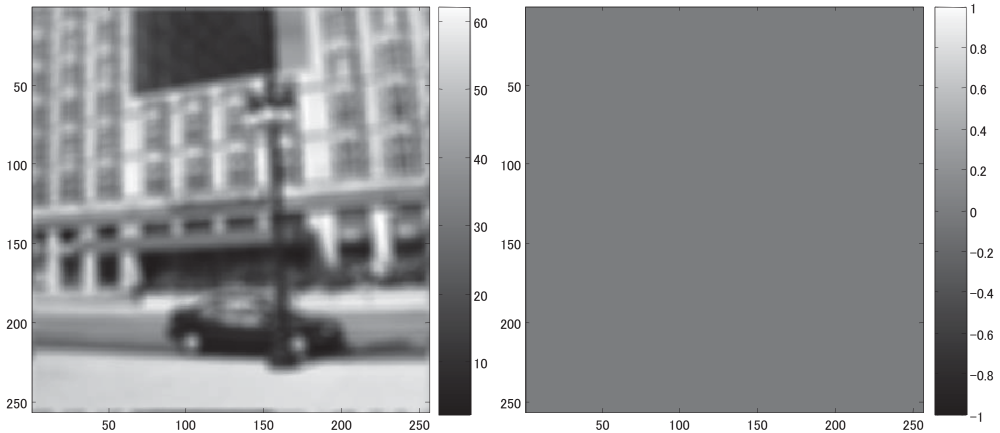




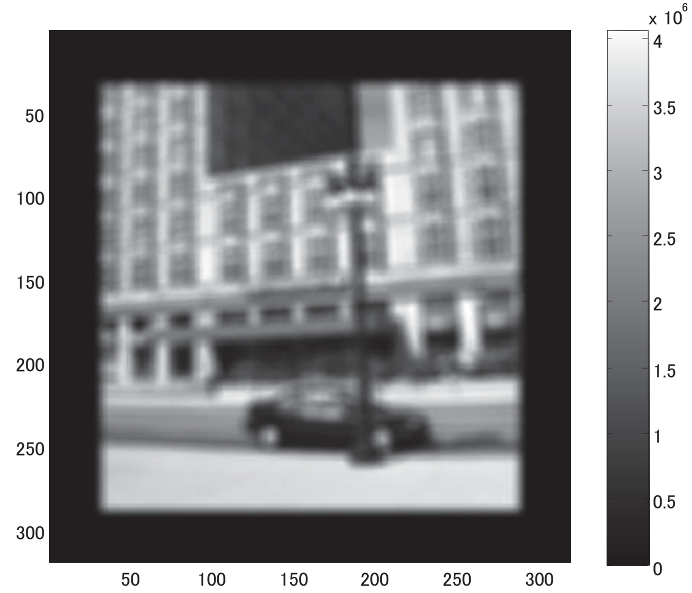
- imagesc(log(abs(fscil) + 100));
- fres = fim .* fscil;
- imagesc(log(abs(fres) + 1));



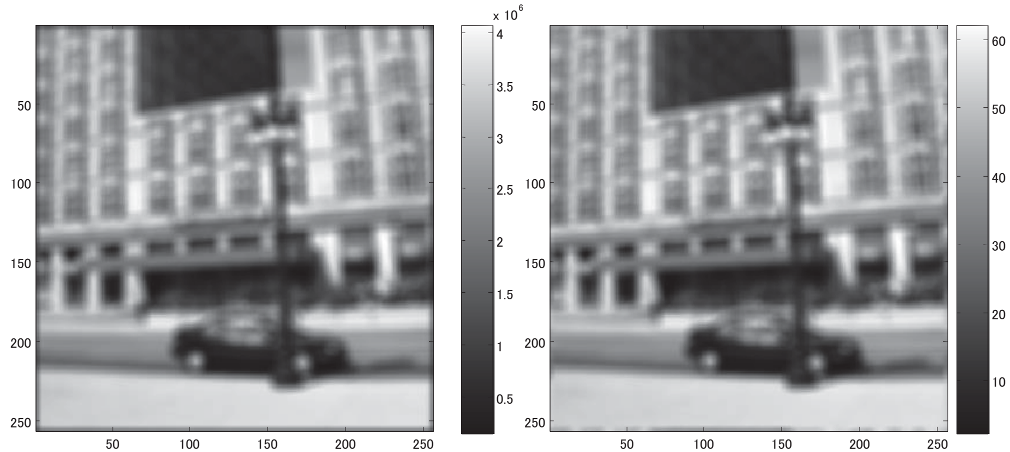
- res = ifft2(fres) / (256*256);
- imagesc(real(res));
- imagesc(imag(res));

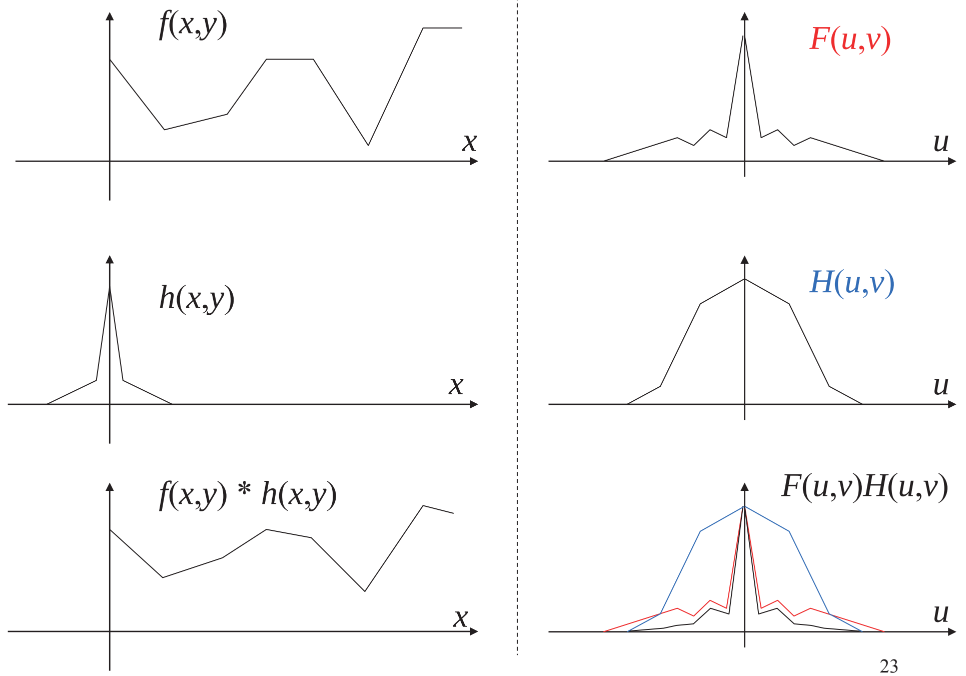


- cres = conv2(double(img), double(ci));
- imagesc(cres);



- cres = conv2(double(img), double(ci), 'same');
- imagesc(cres);
- imagesc(real(res));





Modulation transfer function

Transfer function : H
Modulation transfer function : | H |
Phase transfer function: arg{ H }

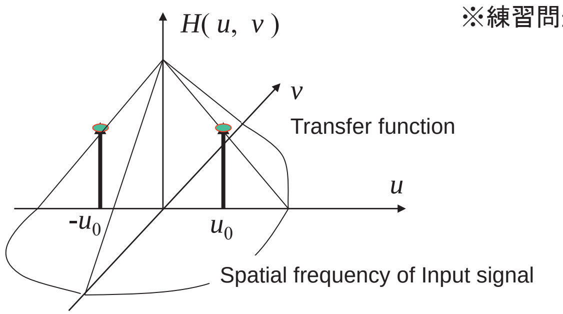
MTFの求め方（1）

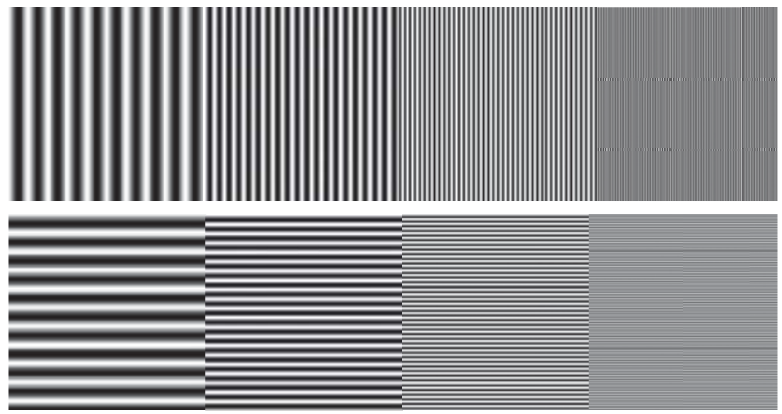
入力信号 $A_I + A_I \cos 2\pi u_0 x$

出力信号 $A_I' + A_O \cos(2\pi u_0 x + \phi)$

$$| H(u_0,\ 0) | = \frac{A_O}{A_I}$$

※練習問題：上式が正しいことを確かめよ





1.8 Causes of image degradation 画像劣化とその原因

- Defocus 焦点はずれ
- Lens aberration 収差
  - Spherical aberration
  - Coma
  - Astigmatism
  - Curvature of field
  - Distortion
- Diffraction limit 回折限界
- Sampling aperture サンプリング開口
- Sampling サンプリング
- Distortion 歪み
- Noise ノイズ

Image blur (approximately shift-invariant)

Image blur (shift-variant)

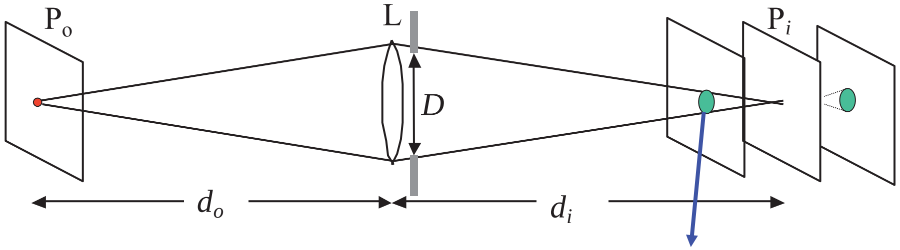
Resolution degradation

Distortion

Aliasing artifact

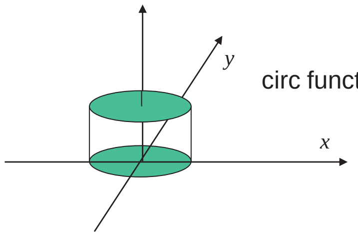
Noise

Impulse response of a defocused optical imaging system



- Incoherent model
  (Geometrical model)

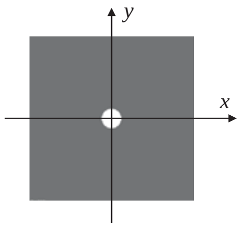
  2D object: (Approximately) shift-invariant
  3D object: Shift-variant



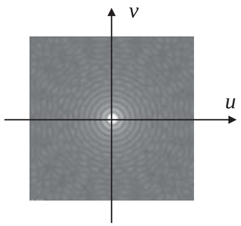
Coherent (wave optics model) case: see “diffraction and image formation.”

Transfer function of a defocused optical imaging system

- $\boldsymbol{F}\{ \text{circ}(r) \} = J_1(2 \pi \rho) / \rho$
  (Fourier-Bessel transform)
  - J_1 : Bessel function of the first kind, order 1.



Impulse response
Point spread function



Transfer function

Size	Large	Small
	Small	Large