Dominant Strategy（支配戦略）

Prisoner's Dilemma

Player 2

Player 1

	DC	C
DC	-2, -2	-10, -1
C	-1, -10	-5, -5

Player 1: "Confess" is the best strategy regardless of what 2 plays.

Player 2: Same. $\rightarrow$ strictly dominant strategy（狭義支配戦略）

Definition 8.B.1: (strictly dominant strategy)

In $\Gamma_N = [N = \{0,1,\ldots,I\}, \{S_i\}, \{u_i\}]$,

$s_i \in S_i$ is a strictly dominant strategy for i

$$\text{if } u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i}) \quad \forall s'_i \in S_i - \{s_i\}, \ \forall s_{-i} \in S_{-i}.$$

Domination

<u>Definition 8.B.2</u>: (strictly dominated strategy)

Let s_i, $s'_i \in S_i$. s'_i strictly dominates（狭義に支配する）s_i if

$$u_i(s'_i, s_{-i}) > u_i(s_i, s_{-i}) \ \forall\, s_{-i} \in S_{-i}.$$

If there exists at least one s'_i that strictly dominates s_i,

s_i is said to be strictly dominated.

<u>Note</u>: s_i is a strictly dominant strategy if it strictly dominates all other strategies in S_i.

<u>Example 8.B.1:</u>

1: U, M strictly dominates D

1 can eliminate D.

2: no domination

Player 2

Player 1

	L	R
U	1, -1	-1, 1
M	-1, 1	1, -1
D	-2, 5	-3, 2

Weakly Dominant Strategy

Definition 8.B.3:

Let s_i, $s'_i \in S_i$. s'_i weakly dominates（弱支配する） s_i if

$$u_i(s'_i, s_{-i}) \geq u_i(s_i, s_{-i}) \quad \forall\, s_{-i} \in S_{-i}$$

$$u_i(s'_i, s_{-i}) > u_i(s_i, s_{-i}) \quad \exists\, s_{-i} \in S_{-i}$$

If there exists at least one s'_i that weakly dominates s_i,

s_i is said to be weakly dominated.

s_i is a weakly dominant strategy（弱支配戦略）

if it weakly dominates all other strategies in S_i.

Example 8.B.2:

1: D weakly dominates U, M

2: no weak domination

1 can eliminate U and M ???

Player 2

Player 1

	L	R
U	5, 1	4, 0
M	6, 0	3, 1
D	6, 4	4, 4

Iterated Deletion

Example 8.B.3:

1 is DA's brother and

allow 1 to go free if both play DC.

Player 2

Player 1

	DC	C
DC	0, -2	-10, -1
C	-1, -10	-5, -5

No domination for 1.

2: C strictly dominates DC.

Payoffs and rationality of both players are common knowledge

→ 1 believes 2 eliminates DC and plays C

(1 knows 2's payoffs and rationality)

→ 1 plays C since -5 > -10. → (C, C)

Further iteration of deletion（逐次除去）is possible.

Note: Order of deletion does not affect the final outcome.

Iterated Deletion of Weakly Dominated Strategies

Deletion of weakly dominated strategies

→ other players play all strategies with positive probability

→ C! to iterated deletion

	L	R
U	5, 1	4, 0
M	6, 0	3, 1
D	6, 4	4, 4

<u>Example 8.B.2:</u>

1: D weakly dominates U, M

2: no weak domination

	L	R
U	5, 1	4, 0
D	6, 4	4, 4

Delete M → L w-dom R → (D, L)

Delete U → R w-dom L → (D, R)

	L	R
M	6, 0	3, 1
D	6, 4	4, 4

(Delete M & U → (D, L) or (D, R))

Domination with Mixed Strategies

Definition 8.B.4: (strictly dominated strategy with mixed strategies)

Let $\sigma_i, \sigma'_i \in \Delta(S_i)$. σ'_i strictly dominates σ_i if

$$u_i(\sigma'_i, \sigma_{-i}) > u_i(\sigma_i, \sigma_{-i}) \quad \forall \sigma_{-i} \in \Pi_{j\neq i} \Delta(S_j).$$

σ_i is said to be strictly dominated

if there exists at least one σ'_i that strictly dominates σ_i,

σ_i is a strictly dominant strategy

if it strictly dominates all other strategies in $\Delta(S_i)$.

No domination for 1 and 2

in pure strategies.

(1/2, 0, 1/2) strictly dominates M.

Pl. 2

Pl. 1		L	R
	U	10, 1	0, 4
	M	4, 2	4, 3
	D	0, 5	10, 2

Domination with Mixed Strategies

Proposition 8.B.1:

$s_i \in S_i$ is strictly dominated in $\Gamma_N = [N = \{0,1,\ldots,I\}, \{\Delta(S_i)\}, \{u_i\}]$ iff there exists $\sigma'_i \in \Delta(S_i)$ such that

$$u_i(\sigma'_i, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall\, s_{-i} \in S_{-i} = \Pi_{j \neq i} S_j.$$

Note: $u_i(\sigma'_i, \sigma_{-i}) > u_i(\sigma_i, \sigma_{-i}) \quad \forall\, \sigma_{-i} \in \Pi_{j \neq i} \Delta(S_j)$

iff $u_i(\sigma'_i, s_{-i}) > u_i(\sigma_i, s_{-i}) \quad \forall\, s_{-i} \in \Pi_{j \neq i} S_j$.

$\downarrow$

Delete all strictly dominated pure strategies in Γ_N.

$\downarrow$

How do we eliminate mixed strategies ?

Domination with Mixed Strategies

Exercise 8.B.6:

$s_i \in S_i$ is strictly dominated in $\Gamma_N = [N = \{0,1,\ldots,I\}, \{\Delta(S_i)\}, \{u_i\}]$

$\Rightarrow$ any strategy that plays s_i with positive probability is also strictly dominated.

$\downarrow$

Can eliminate some dominated mixed strategies.

Can eliminate further.

Neither U nor D strictly dominated;

But (1/2,0,1/2) is strictly dominated

By M.

Pl. 2

Pl. 1

	L	R
U	10, 1	0, 4
M	6, 2	6, 3
D	0, 5	10, 2

Domination with Mixed Strategies

<u>Elimination</u> of dominated strategies in

$$\Gamma_N = [N=\{0,1,\ldots,I\}, \{\Delta(S_i)\}, \{u_i\}]$$

1. Iteratively eliminate strictly dominated pure strategies.
2. Let S^u_i be the remaining pure strategy set of i
3. Eliminate strictly dominated mixed strategies in $\Delta(S^u_i)$

Rationalizable Strategies

Definition 8.C.1:

In $\Gamma_N=[N=\{0,1,\ldots,I\}, \{\Delta(S_i)\}, \{u_i\}]$, $\sigma_i \in \Delta(S_i)$ is a best response

for i to σ_{-i} if $u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma'_i, \sigma_{-i}) \quad \forall \sigma'_i \in \Delta(S_i)$.

Strategy σ_i is never a best response

if there is no σ_{-i} to which σ_i is a best response.

Note: Strictly dominated $\rightarrow$ never be a best response

never be a best response even if not strictly dominated

Rationalizable Strategies（合理化可能戦略）

Pl. 2

Pl. 1

	b_1	b_2	b_3	b_4
a_1	0, $\underline{7}$	2, 5	$\underline{7}$, 0	0, 1
a_2	5, 2	$\underline{3}$, $\underline{3}$	5, 2	0, 1
a_3	$\underline{7}$, 0	2, 5	0, $\underline{7}$	0, 1
a_4	0, $\underline{0}$	0, -2	0, $\underline{0}$	$\underline{10}$, -1

_ denotes the best response

b_4 is <u>not</u> strictly dominated.

But b_4 is never the best response.

$a_1 \rightarrow b_1$

$a_2 \rightarrow b_2$

$a_3 \rightarrow b_3$

$a_4 \rightarrow b_1, b_3$

Rationalizable Strategies

Iterated elimination of "never be a best response" strategies

Definition 8.C.2:

In $\Gamma_N=[\{0,1,\ldots,I\}, \{\Delta(S_i)\}, \{u_i\}]$, the strategies in $\Delta(S_i)$ that survives the iterated deletion of strategies that are never be a best response are called i's rationalizable strategies.

Note: Order of deletion does not affect

Rationalizable Strategies

Pl. 2

Pl. 1

	b_1	b_2	b_3	b_4
a_1	0, $\underline{7}$	2, 5	$\underline{7}$, 0	0, 1
a_2	5, 2	$\underline{3}$, $\underline{3}$	5, 2	0, 1
a_3	$\underline{7}$, 0	2, 5	0, $\underline{7}$	0, 1
a_4	0, $\underline{0}$	0, -2	0, $\underline{0}$	$\underline{10}$, -1

$\underline{\quad}$ denotes best response

b_4 is never a best response $\rightarrow$ eliminate b_4

$\rightarrow$ a_4 is never a best response $\rightarrow$ eliminate a_4

rationalizable strategies $\rightarrow$ $\{a_1, a_2, a_3\}$ for 1, $\{b_1, b_2, b_3\}$ for 2

<u>Chain of justification</u>:

$(a_2, b_2, a_2, b_2, a_2, \ldots)$, $(a_1, b_3, a_3, b_1, a_1, b_3, \ldots)$

$(a_4, b_4,$ nothing $)$

Rationalizable Strategies

Existence of rationalizable strategies $\leftarrow$ existence of Nash eq.

<u>many</u> rationalizable strategies.

set of rationalizable str.

$\subseteq$ remaining strategies after iterative deletion of strictly dominated strategies

strictly dominated $\rightarrow$ never be a best response

<u>Two-person games</u>:

set of rationalizable str.

$=$ remaining strategies after iterative deletion of strictly dominated strategies

<u>Note</u>: Three or more person games $\rightarrow$ <u>not</u> true

(OK for correlated str.)

Assignments

Problem Set 3 (due May 16):

Exercises (p.262) 8.B.1, 8.B.3, 8.B.6, 8.B.7

Reading Assignments:

Text Chapter 8, pp.246-253