

Game (Matching Pennies)

Players: Who is involved ?

Rules: Who moves when ? What do they know ?
What can they do ?

Outcomes: For each possible set of actions by the players,
what is the outcome of the game ?

Payoffs (利得): What are the players' preferences (選好)
(utility functions (効用関数)) over the possible outcomes ?

Example 7.B.1: Matching Pennies

Players: two players, denoted 1 and 2

Rules: Each player simultaneously puts a penny down,
either heads up or tails up.

Outcomes: If the pennies match, 1 pays 1 dollar to 2;
otherwise 2 pays 1 dollar to 1.

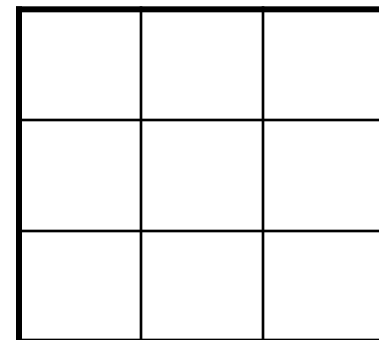
Game (Tick-Tack-Toe)

Players, Rules, Outcomes, Payoffs

Example 7.B.2: Tick-Tack-Toe

Players: two players, denoted X and O

Rules: The players take turns putting their marks (an X or an O) into as-yet-unmarked square. X moves first. All previous choices are observed.



Outcomes: The first player to have three of her marks in a row (horizontally, vertically, or diagonally) wins and receives 1 dollar from the other player. If no one succeeds in doing so after all nine boxes are marked, the game is a tie. No payments are made.

Preferences

Preference are described by a utility function
which assigns a utility level for each outcome.

Von Neumann-Morgenstern utility function

→ can take expected values (mixed strategy)
(→ Chapter 6)

payoff, payoff function

Examples 7.B.1 and 7.B.2 → payoff = amount of money

zero-sum games

Game (Meeting in New York)

Players, Rules, Outcomes, Payoffs

Example 7.B.3: Meeting in New York

Players: two players, Mr. T and Mr. S

Rules: The two players cannot communicate, and supposed to meet in N.Y. City. They have forgotten where.
Each must decide where to go.

Outcomes: If they meet each other, they get to enjoy each other's company. Otherwise, they are alone.

Payoffs: 100 dollars if they meet; 0 dollars otherwise

Each player's optimal action depends on what he thinks the other will do.

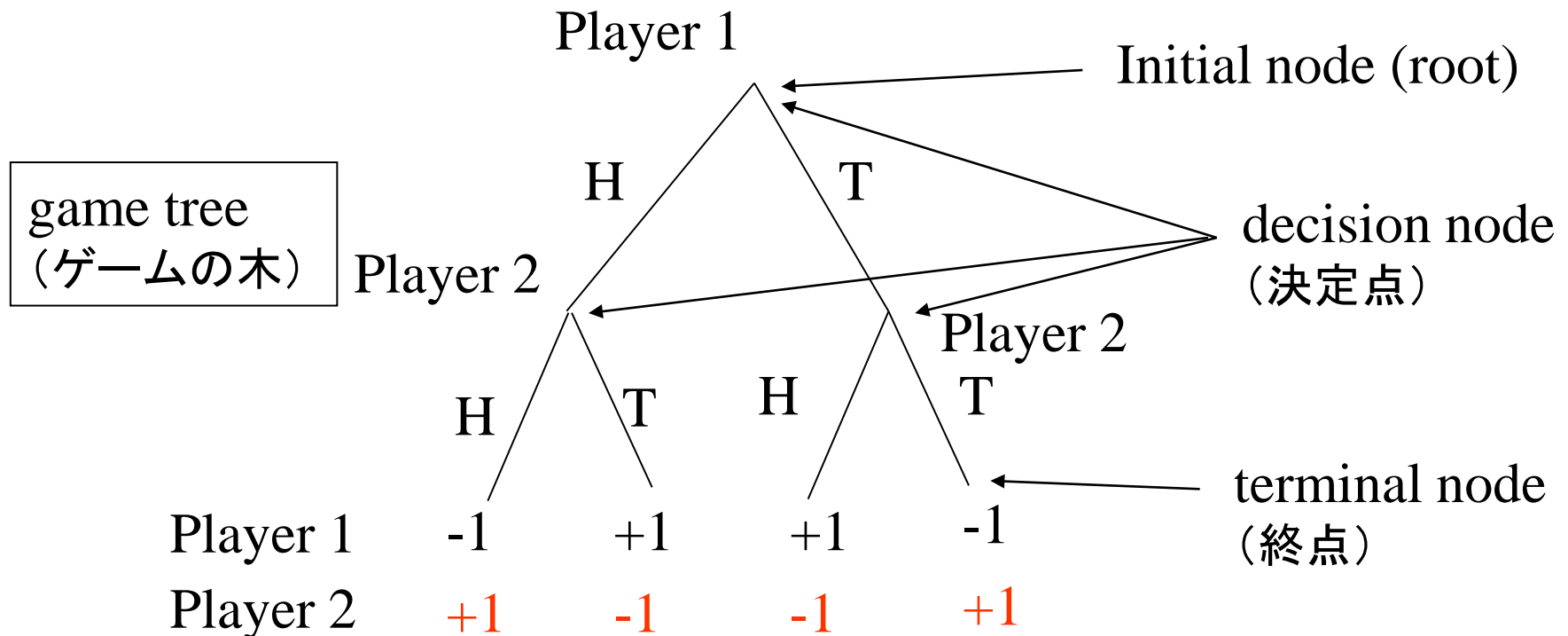
Extensive Form Game(展開形) (Matching Pennies B)

Example 7.C.1: Matching Pennies Version B

Players: two players, denoted 1 and 2

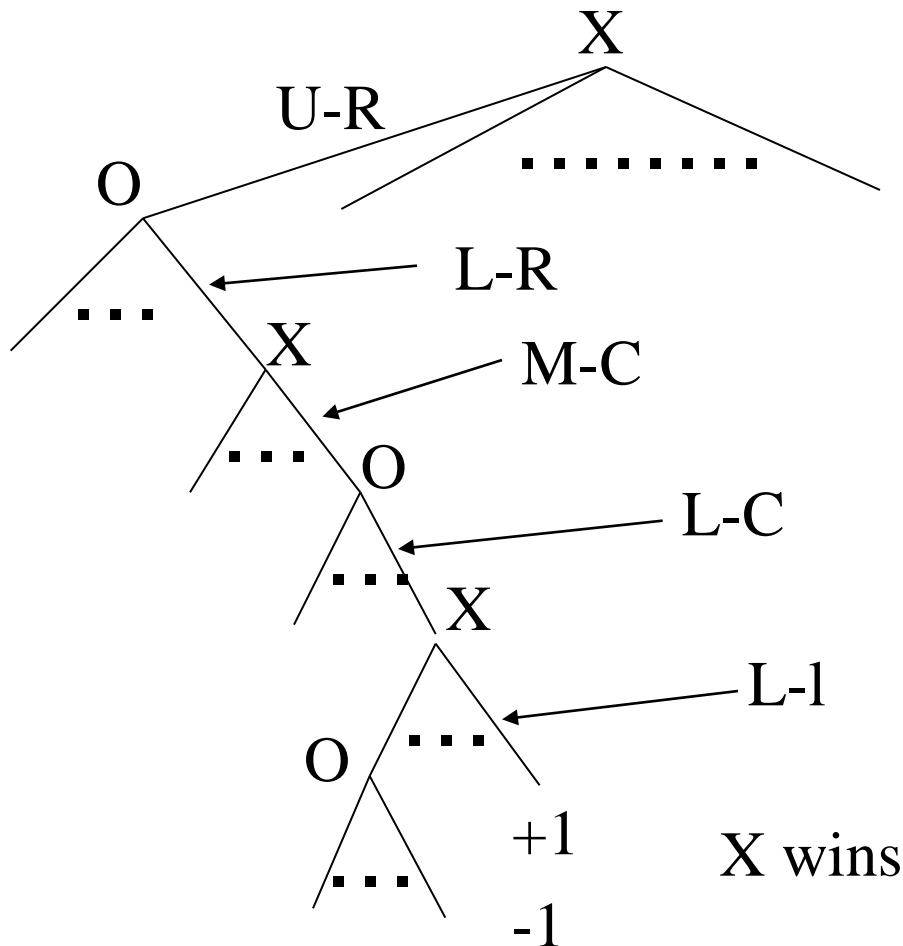
Rules: Player 1 puts her penny down **first; then after seeing her choice (head or tail)**, player 2 puts her penny down.

Outcomes: match, $1 \rightarrow 2$ 1dollar; otherwise $2 \rightarrow 1$ 1dollar



Extensive Form Game (Tick-Tack-Toe)

Example 7.C.2: The Extensive Form of Tick-Tack-Toe



	1	C	R
U			X1
M		X2	
L	X3	O2	O1

Perfect Information (完全情報)

Matching pennies version B, Tick-Tack-Toe

Each player knows whole history of the game when she moves

→ perfect information

Information set (情報集合)

→ subset of particular player's decision nodes

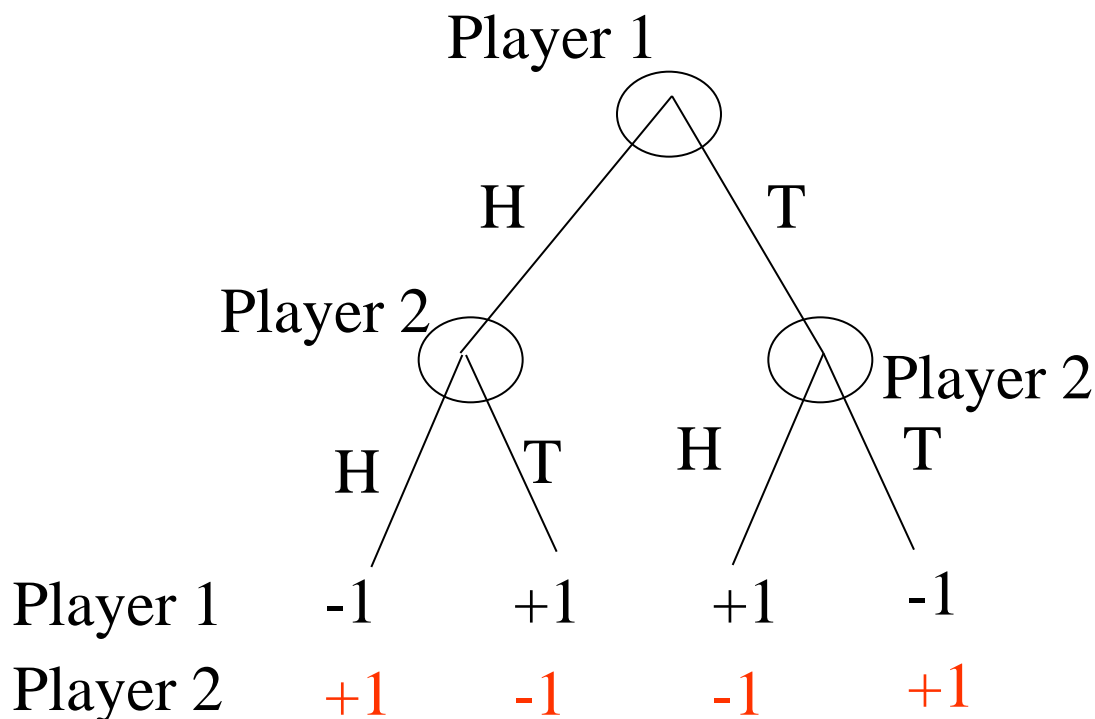
When play has reached one of the decision nodes in the information set of a player, she does not know which one of the nodes in the information set she is actually at.

Perfect information → every information set is a **singleton**.

Extensive Form Game (Matching Pennies B)

Example 7.C.1: Matching Pennies Version B

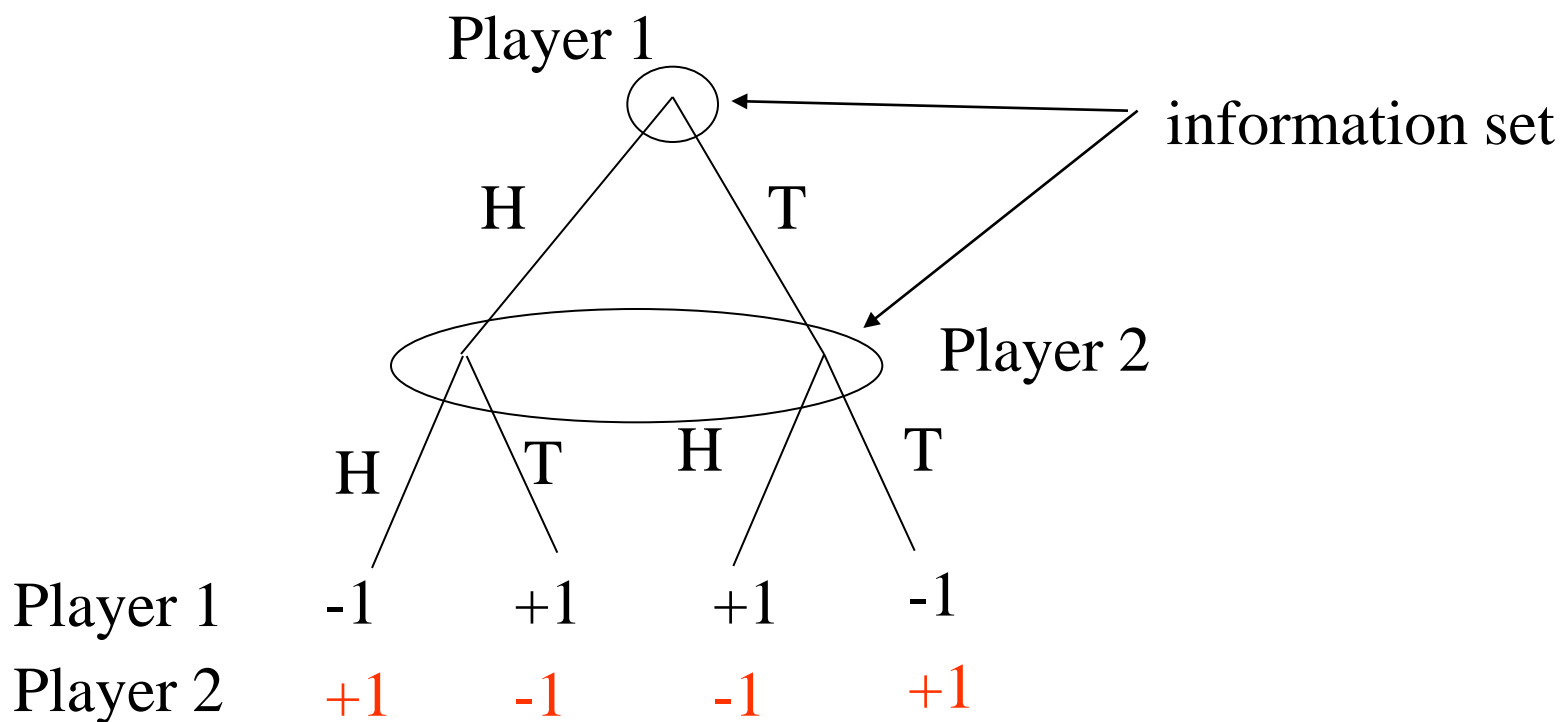
Rules: Player 1 puts her penny down **first; then after seeing her choice (head or tail)**, player 2 puts her penny down.



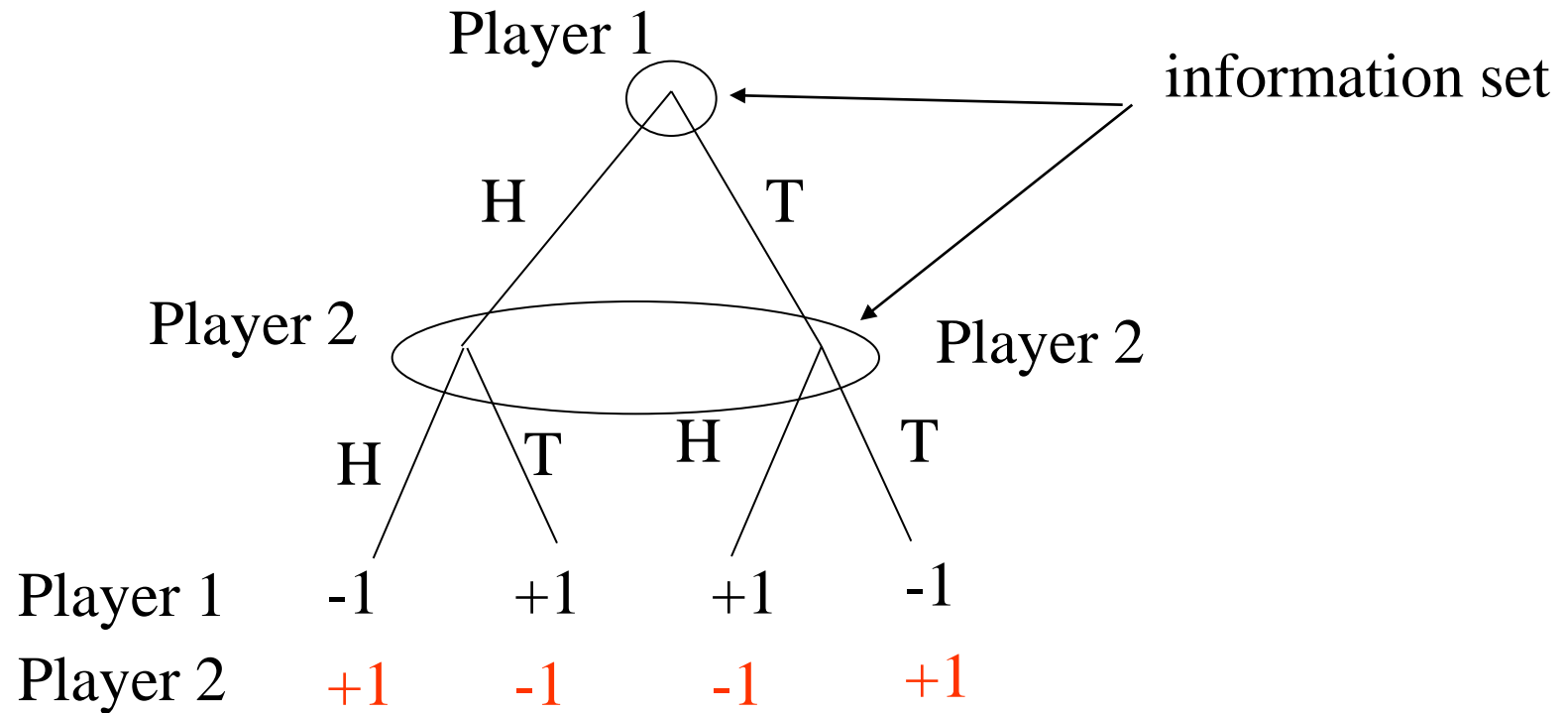
Extensive Form Game (Matching Pennies C)

Example 7.C.3: Matching Pennies Version C

Rules: Player 1 puts her penny down first; then player 2 puts her penny down **without knowing 1's choice**.



Restrictions on Information Sets



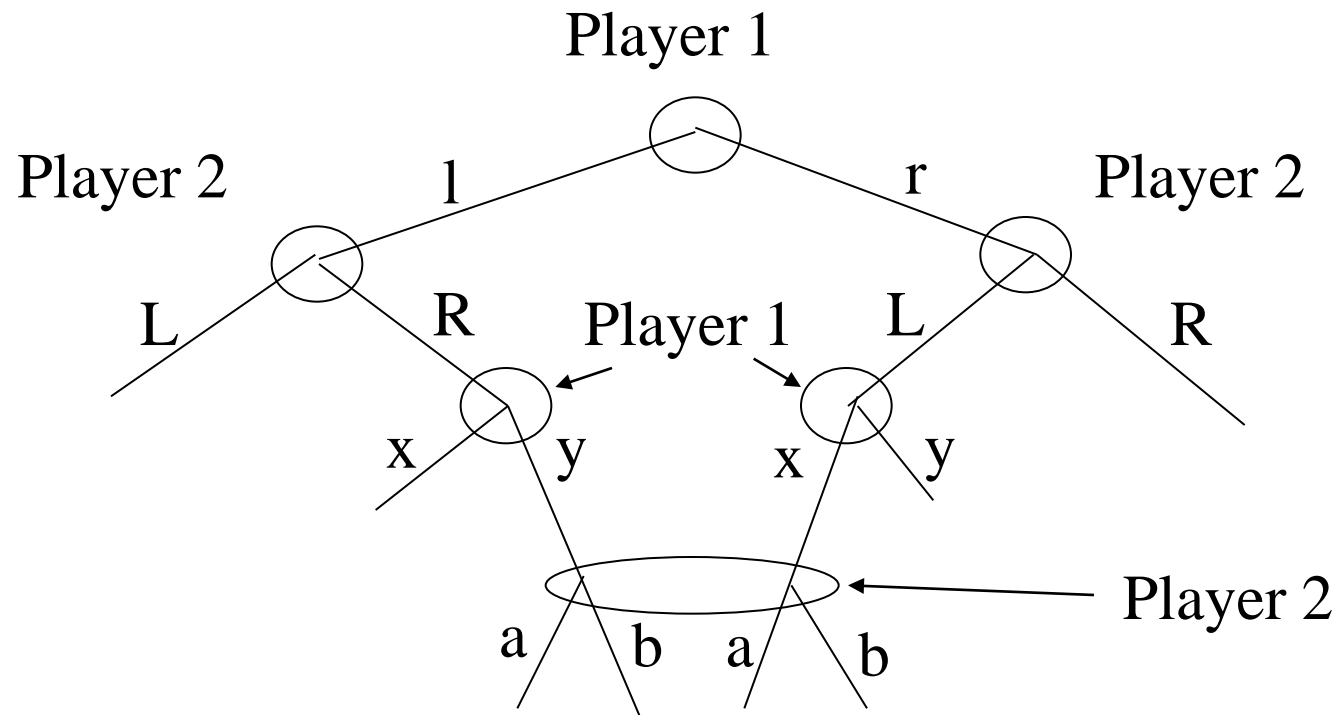
information set

→ every node has the same set of possible actions

Restrictions on Information Sets – Perfect Recall(完全記憶)

Perfect Recall

→ A player does not forget what she knew, including her own actions

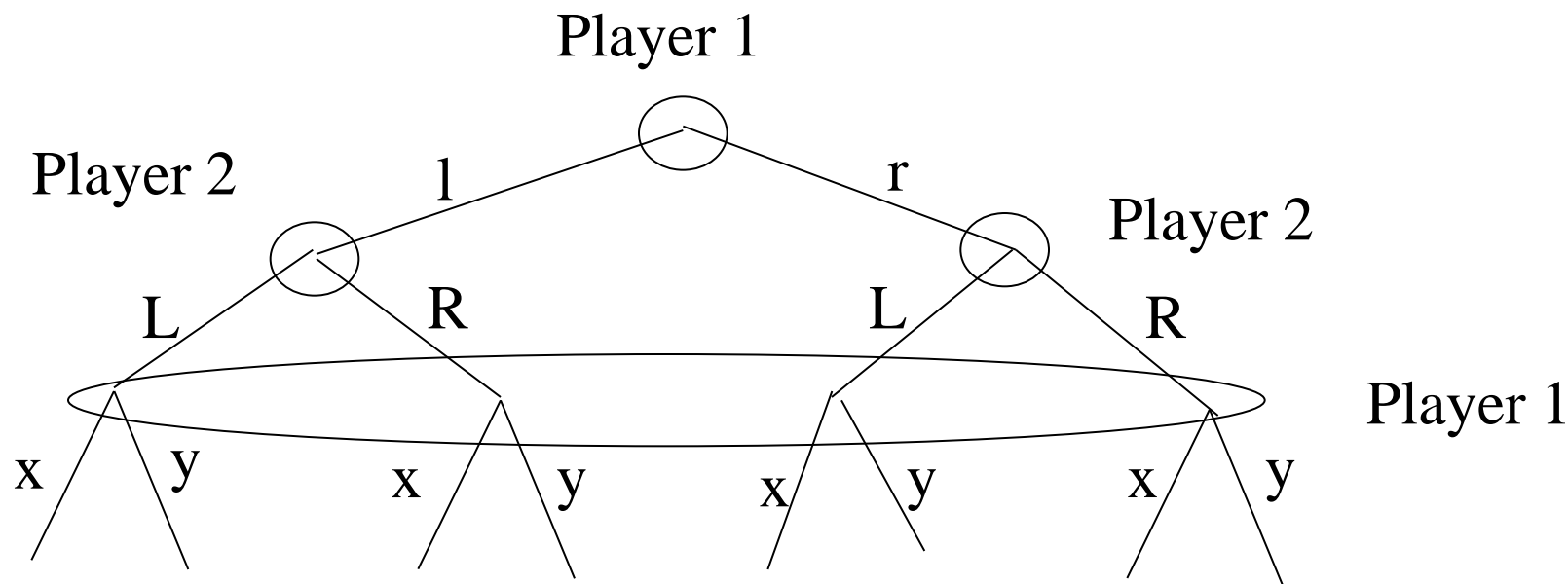


Not : Player 2 forgets 1's first action which she knows before.

Restrictions on Information Sets – Perfect Recall

Perfect Recall

→ A player does not forget what she knew, including her own actions



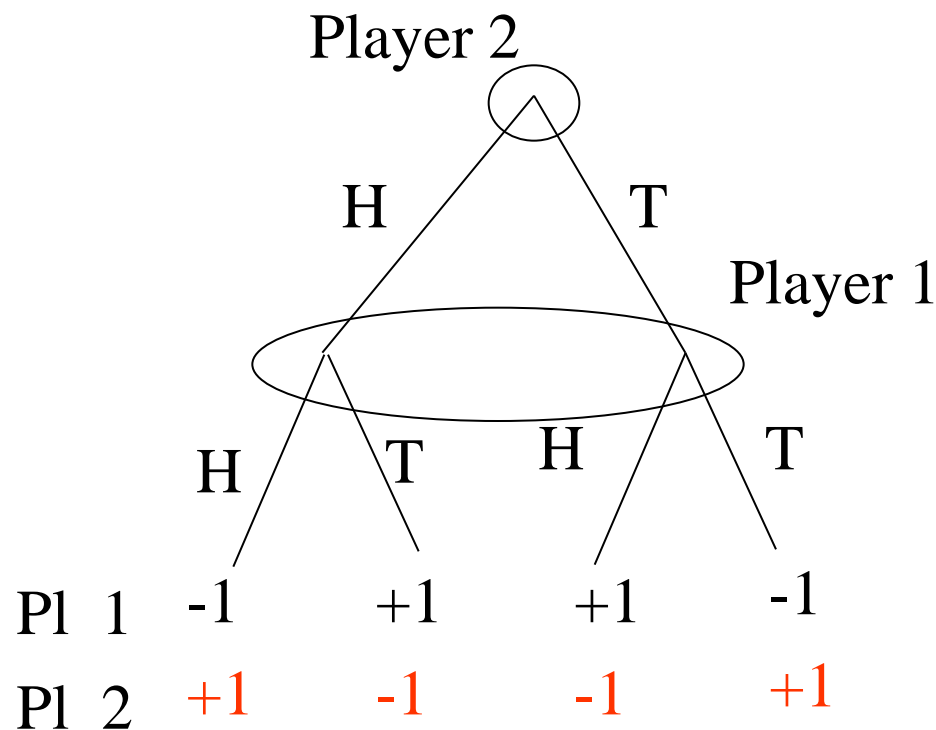
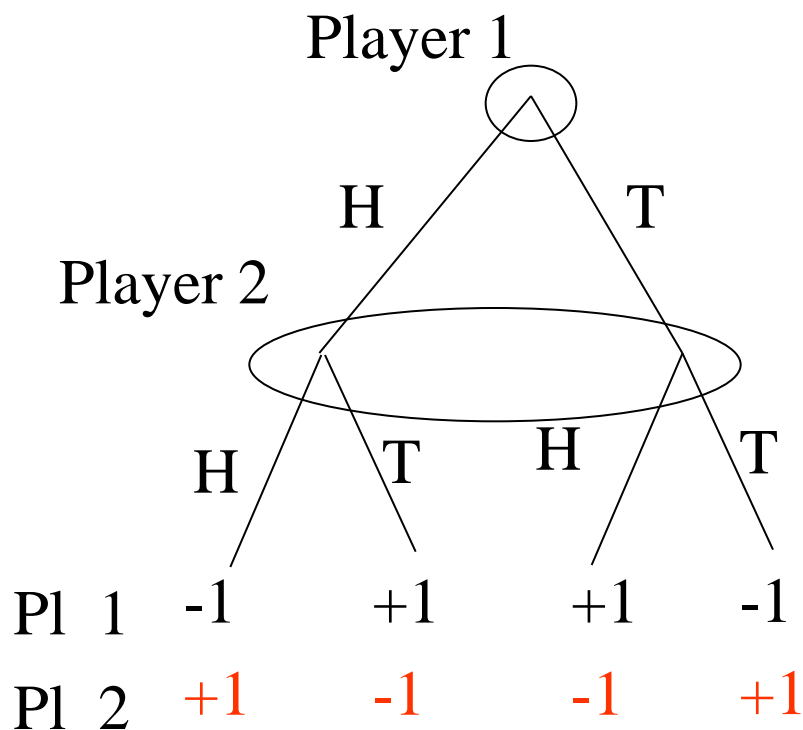
Not : Player 1 forgets her first action.

Extensive Form Game (Matching Pennies)

Example 7.C.4: Matching Pennies

Players 1 and 2 move simultaneously.

Equivalent to Version C.



Perfect Information

Perfect information:

Each information set contains a single decision node.

(\Leftrightarrow imperfect information)

Perfect information:

Example 7.C.1 (Matching Pennies Ver.B),

Example 7.C.2 (Tick-Tack-Toe),

Imperfect information:

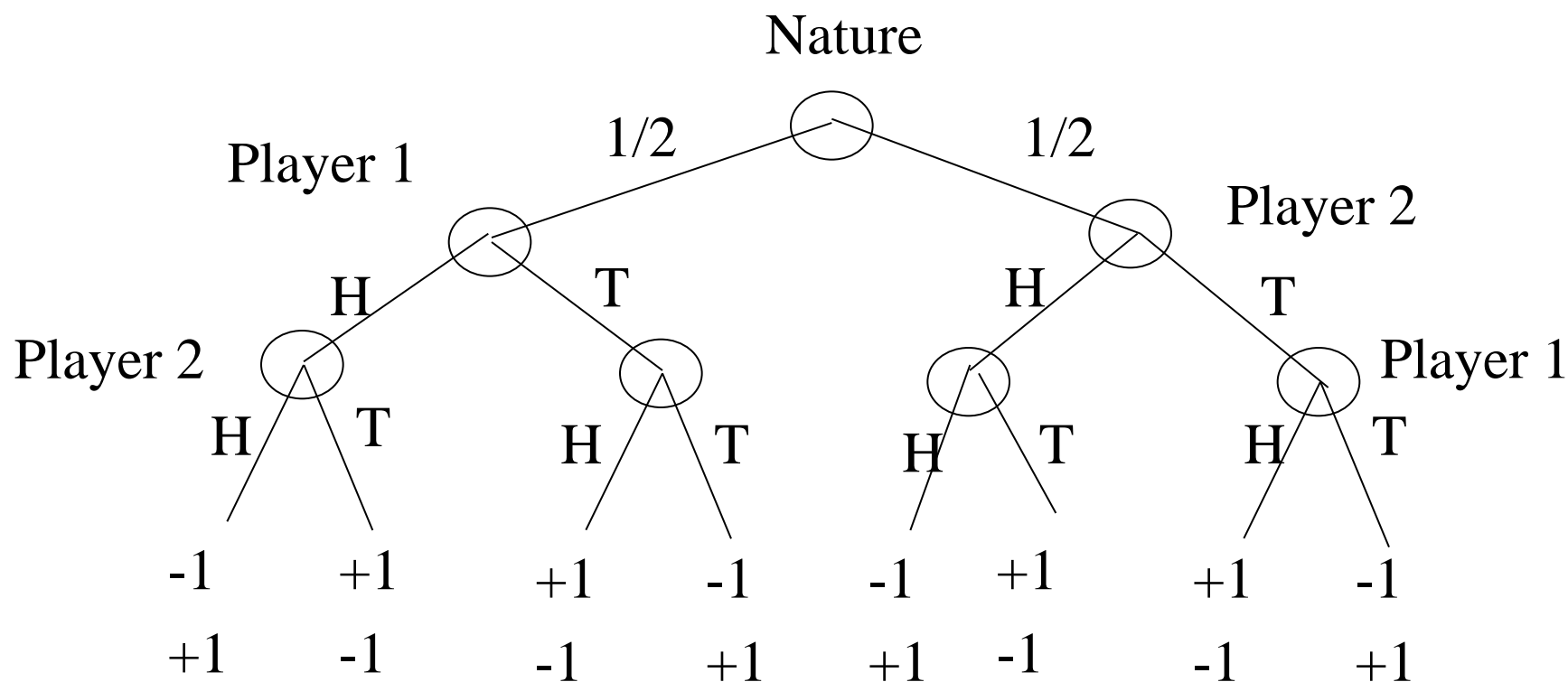
Example 7.B.1 (Matching Pennies),

Example 7.C.3 (Matching Pennies Ver.C),

Chance Moves (偶然手番)

Example 7.C.5: Matching Pennies Version D

Players 1 and 2 flip a coin to decide who will play first.
Then play Version B.



Common Knowledge(共有知識, 共通知識)

Basic postulate:

All players know the structures of the game

know that their rivals know it

know that their rivals know that their rivals know it

and so on

→ common knowledge

Game structure is common knowledge

→ complete information game(情報完備ゲーム)

Assignments

Problem Set 1 (due April 26):

Exercise (page 233) : 7.C.1

Reading Assignment:

Text Chapter 7, pp.219-233