

## WPBE in Ex.9.C. 3


$\underline{\mu}_{1}>2 / 3$
I plays $\mathrm{F}\left(\sigma_{\mathrm{F}}=1\right)$
$\rightarrow$ E plays $\mathrm{I}_{2}$ since $\gamma>0>-1$
$\rightarrow \mu=(0,1) \quad \mathrm{C}$ ! to $\mu_{1}>2 / 3$

## WPBE in Ex.9.C. 3


$\underline{\mu}_{1}<2 / 3$
I plays $\mathrm{A}\left(\sigma_{\mathrm{F}}=0\right)$
$\rightarrow$ E plays $\mathrm{I}_{1}$ since $3>2>0$
$\rightarrow \quad \mu=(1,0) \quad \mathrm{C}$ ! to $\mu_{1}<2 / 3$

## WPBE in Ex.9.C. 3



E: $\sigma_{1}=2 / 3, \sigma_{2}=1 / 3$

$$
\text { since } \sigma_{0}=0, \quad \mu_{1}=2 / 3 \text { and } 1-\mu_{1}=1 / 3
$$

$\rightarrow \mathrm{E}: \mathrm{I}_{1}$ and $\mathrm{I}_{2}$ are indifferent under $\left(\sigma_{\mathrm{F}}, 1-\sigma_{\mathrm{F}}\right)$ since $\sigma_{1}, \sigma_{2}>0$

## WPBE in Ex.9.C. 3

$\mathrm{E} \rightarrow 0$
I $\rightarrow 2$


$$
\sigma_{0}=0
$$

$\sigma_{0}=0$
$\rightarrow 0$
$\rightarrow 2$

E

$$
\underline{y} \geq 0
$$

$$
\mu_{1}>2 / 3 \rightarrow \mathrm{~F}
$$

$$
\mu_{1}<2 / 3 \rightarrow \mathrm{~A}
$$

$$
\mu_{1}=2 / 3 \rightarrow \mathrm{~F} \text { or } \mathrm{A}
$$

$\underline{\mu}_{1}=2 / 3$
E: $I_{1}$ and $I_{2}$ are indifferent under $\left(\sigma_{F}, 1-\sigma_{F}\right)$ since $\sigma_{1}, \sigma_{2}>0$.
E's payoff: $\mathrm{I}_{1} \rightarrow-\sigma_{\mathrm{F}}+3\left(1-\sigma_{\mathrm{F}}\right), \mathrm{I}_{2} \rightarrow \gamma \sigma_{\mathrm{F}}+2\left(1-\sigma_{\mathrm{F}}\right)$

$$
-\sigma_{\mathrm{F}}+3\left(1-\sigma_{\mathrm{F}}\right)=\gamma \sigma_{\mathrm{F}}+2\left(1-\sigma_{\mathrm{F}}\right) \rightarrow \sigma_{\mathrm{F}}=1 /(\gamma+2)
$$

I's strategy : $(1 /(\gamma+2),(\gamma+1) /(\gamma+2))$

## WPBE in Ex.9.C. 3



WPBE
$((0,2 / 3,1 / 3),(1 /(\gamma+2),(\gamma+1) /(\gamma+2)), \mu=(2 / 3,1 / 3))$


P2 has an arbitrary belief since his information set is not reached in equilibrium. ???

## Sequential Equilibrium (motivation, Ex.9.C.5)



## Nash eq $\rightarrow$ (A, A) <br> ((O,A),F) is not SPNE

## Sequential Equilibrium (definition)

Def. 9.C.4: $(\sigma, \mu)$ is a sequential equilibrium (SE) if
(i) $\sigma$ is sequentially rational given $\mu$;
(ii) $\exists$ a sequence of completely mixed strategies $\left\{\sigma^{\mathrm{k}}\right\}_{\mathrm{k}=1}{ }^{\infty}$
with $\lim _{\mathrm{k} \rightarrow \infty} \sigma^{\mathrm{k}}=\sigma$ such that $\mu=\lim _{\mathrm{k} \rightarrow \infty} \mu^{\mathrm{k}}$
where $\mu^{\mathrm{k}}$ is the set of beliefs derived from $\sigma^{k}$ using Bayes' rule.

## Sequential Equilibrium (Ex. 9.C.4)



For any comp. mixed strategy $\left(\sigma_{x}, \sigma_{y}\right), \mathrm{P} 2$ 's belief $=(.5, .5)$
P 2 's choice must be " r " since $5<2 \times .5+10 \times .5=6$
P1's choice must be " y " since $2<5$
$\mathrm{SE} \rightarrow(\mathrm{y}, \mathrm{r},(.5, .5),(.5, .5))$

## Sequential Equilibrium (Ex. 9.C.5)



SE must contain (A, A). ( $\rightarrow$ next slide)

Sequential Equilibrium (Ex. 9.C.5)


$$
\begin{aligned}
& \underline{\sigma}_{E}(\mathrm{O})=1, \sigma_{\mathrm{E}}(\mathrm{I})=0, \sigma_{\mathrm{E}}(\mathrm{~F})=0, \sigma_{E}(\mathrm{~A})=1, \sigma_{\mathrm{I}}(\mathrm{~F})=1, \sigma_{\mathrm{I}}(\mathrm{~A})=0 \\
& \rightarrow \sigma_{\mathrm{E}}^{\mathrm{k}}(\mathrm{O})=1-\varepsilon, \sigma_{\mathrm{E}}^{\mathrm{k}}(\mathrm{I})=\varepsilon, \sigma_{\mathrm{E}}^{\mathrm{k}}(\mathrm{~F})=\varepsilon^{\prime}, \sigma_{\mathrm{E}}^{\mathrm{k}}(\mathrm{~A})=1-\varepsilon^{\prime}, \\
& \sigma_{\mathrm{I}}^{\mathrm{k}}(\mathrm{~F})=1-\varepsilon^{\prime \prime}, \sigma_{\mathrm{I}}^{\mathrm{k}}(\mathrm{~A})=\varepsilon^{\prime \prime}
\end{aligned}
$$

$\operatorname{Prob}\left(\mathrm{H} \mid \sigma^{k}\right)=\sigma_{E}^{k}(\mathrm{I})=\varepsilon, \operatorname{Prob}\left(\mathrm{x} \mid \sigma^{k}\right)=\sigma_{E}^{k}(\mathrm{I}) \times \sigma_{\mathrm{E}}^{\mathrm{k}}(\mathrm{F})=\varepsilon \times \varepsilon{ }^{\prime}$

$$
\mu^{\mathrm{k}}(\mathrm{x})=\varepsilon^{\prime} \rightarrow \underline{\mu(\mathrm{x})=0} \quad \mu^{\mathrm{k}}(\mathrm{y})=1-\varepsilon^{\prime} \rightarrow \mu(\mathrm{y})=1
$$

Sequential Equilibrium (Ex. 9.C.5)


## Sequential Equilibrium and SPNE

Prop. 9.C.2: In every SE $(\sigma, \mu), \sigma$ is an SPNE.

## Assignments

Problem Set 10 (due July 18)
Exercises (pp.301-305)
9.C.2, 9.C.6(only 9.C.3 part)

Reading Assignment:
Text, Chapter 9, pp.292-300

