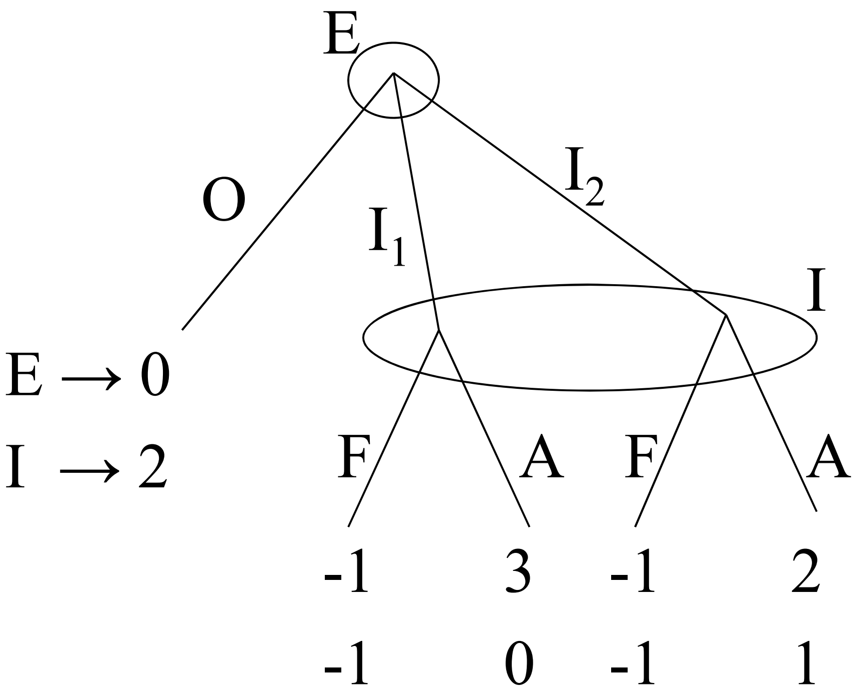
Weak Perfect Bayesian Nash Equilibrium (motivation)



		I	
		F	A
E	O	$\underline{0}$, $\underline{2}$	0, $\underline{2}$
	I_1	-1, -1	$\underline{3}$, $\underline{0}$
	I_2	-1, -1	2, $\underline{1}$

Nash eq (SPNE)

→ (O, F), (I_1, A)

For I : in either decision point, A > F ($-1 < 0$, $-1 < 1$)

→ I should play “A”.

→ introduce “belief”

Weak Perfect Bayesian Nash Eq (definition)

Def. 9.C.1: $\mu = (\mu(x))_{x \in X}$ is a system of beliefs (X: set of all nodes)

if $\sum_{x \in H} \mu(x) = 1 \quad \forall$ information set H

Def. 9.C.2: $\sigma = (\sigma_1, \dots, \sigma_I)$ is sequentially rational at H given μ

if $E(u_{i(H)} \mid H, \mu, \sigma_{i(H)}, \sigma_{-i(H)}) \geq E(u_{i(H)} \mid H, \mu, \sigma\hat{}_{i(H)}, \sigma_{-i(H)})$

$\forall \sigma\hat{}_{i(H)} \in \Delta(S_{i(H)})$ (i(H) : the player who moves at H)

$E(u_{i(H)} \mid H, \mu, \sigma_{i(H)}, \sigma_{-i(H)})$: expected payoff to i(H) from H

if he/she is in H according to the prob. given by μ

and he/she plays $\sigma_{i(H)}$, and rivals play $\sigma_{-i(H)}$.

$\sigma = (\sigma_1, \dots, \sigma_I)$ is sequentially rational given μ

if $\forall$ H, $\sigma = (\sigma_1, \dots, \sigma_I)$ is sequential rational at H given μ

Weak Perfect Bayesian Nash Eq (definition)

Def. 9.C.3.: (σ, μ) is a weak perfect Bayesian Eq (WPBE) if

(i) σ is sequential rational given μ

(ii) μ is derived from σ by Bayes' rule if possible, i.e.,

$\forall$ H such that $\text{Prob}(H \mid \sigma) > 0$

$\mu(x) = \text{Prob}(x \mid \sigma) / \text{Prob}(H \mid \sigma) \; \forall x \in H$

WPBE and Nash Equilibrium

Prop. 9.C.1: σ is a Nash Equilibrium

$\Leftrightarrow \quad \exists \quad \mu$ such that

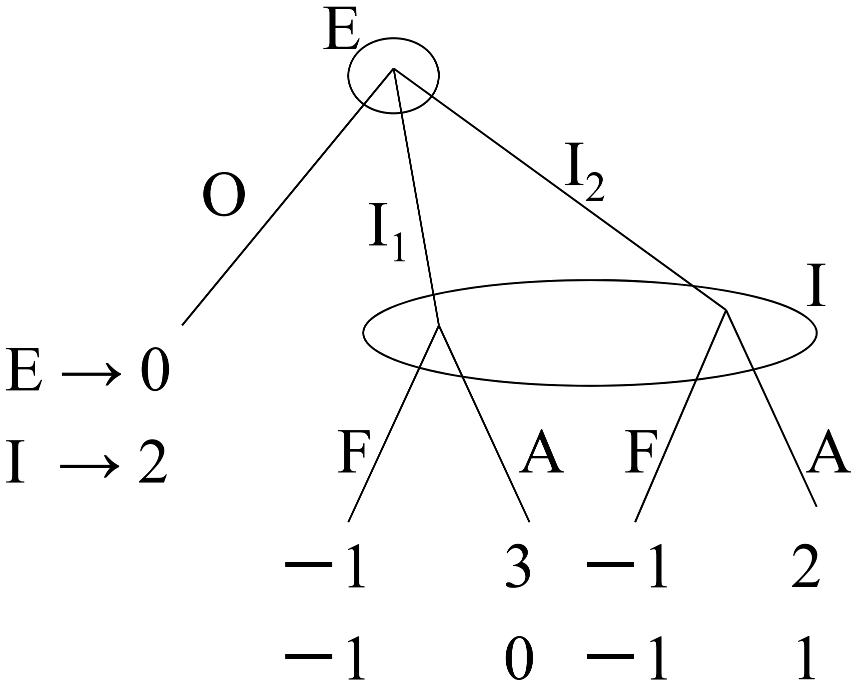
(i) σ is sequentially rational given μ

at H with $\text{Prob}(H \mid \sigma) > 0$.

(ii) μ is derived from σ by Bayes' rule whenever possible.

Cor.: (σ, μ) is a WPBE → σ is a Nash Equilibrium

WPBE in Ex.9.C.1



Nash eq (SPNE)

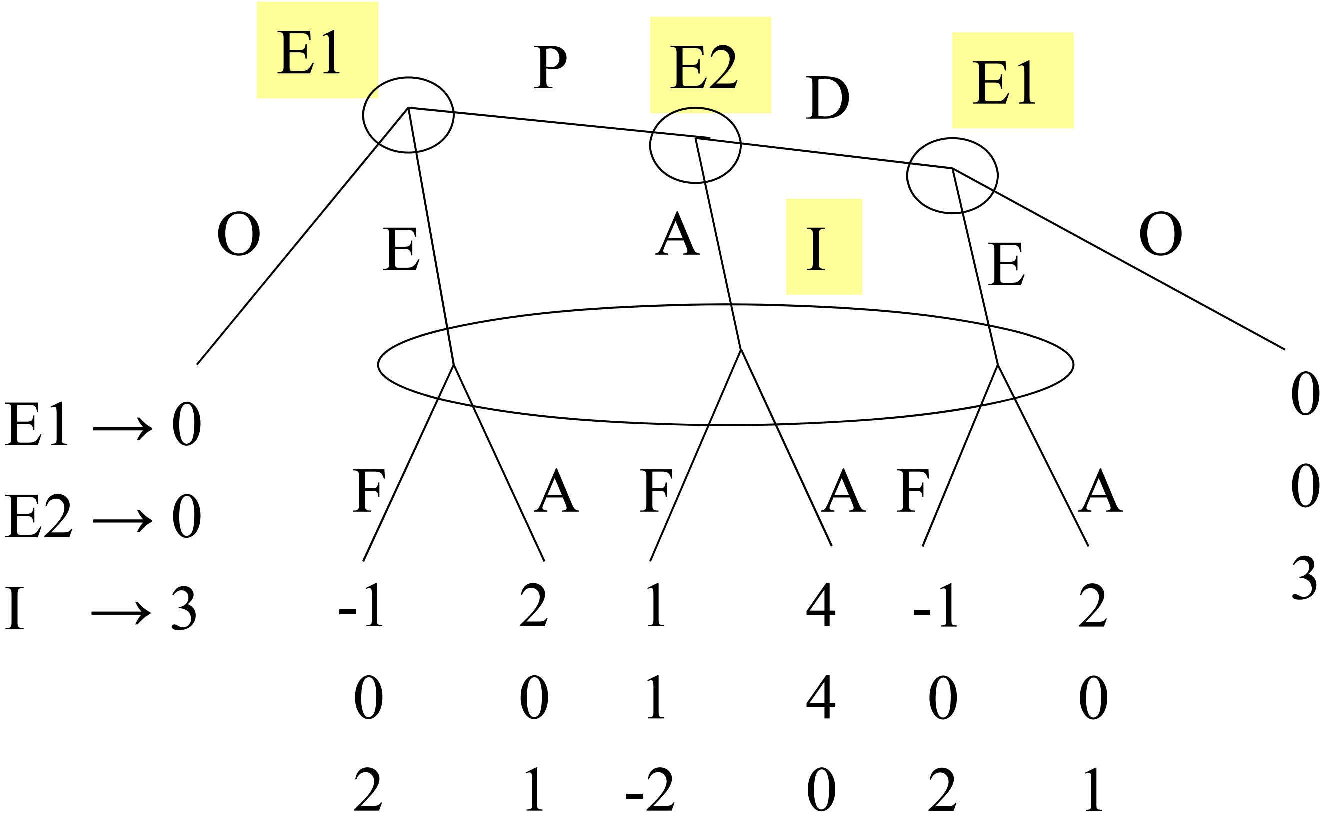
→ (O, F), (I_1, A)

"F" is <u>not</u> sequentially rational for any belief

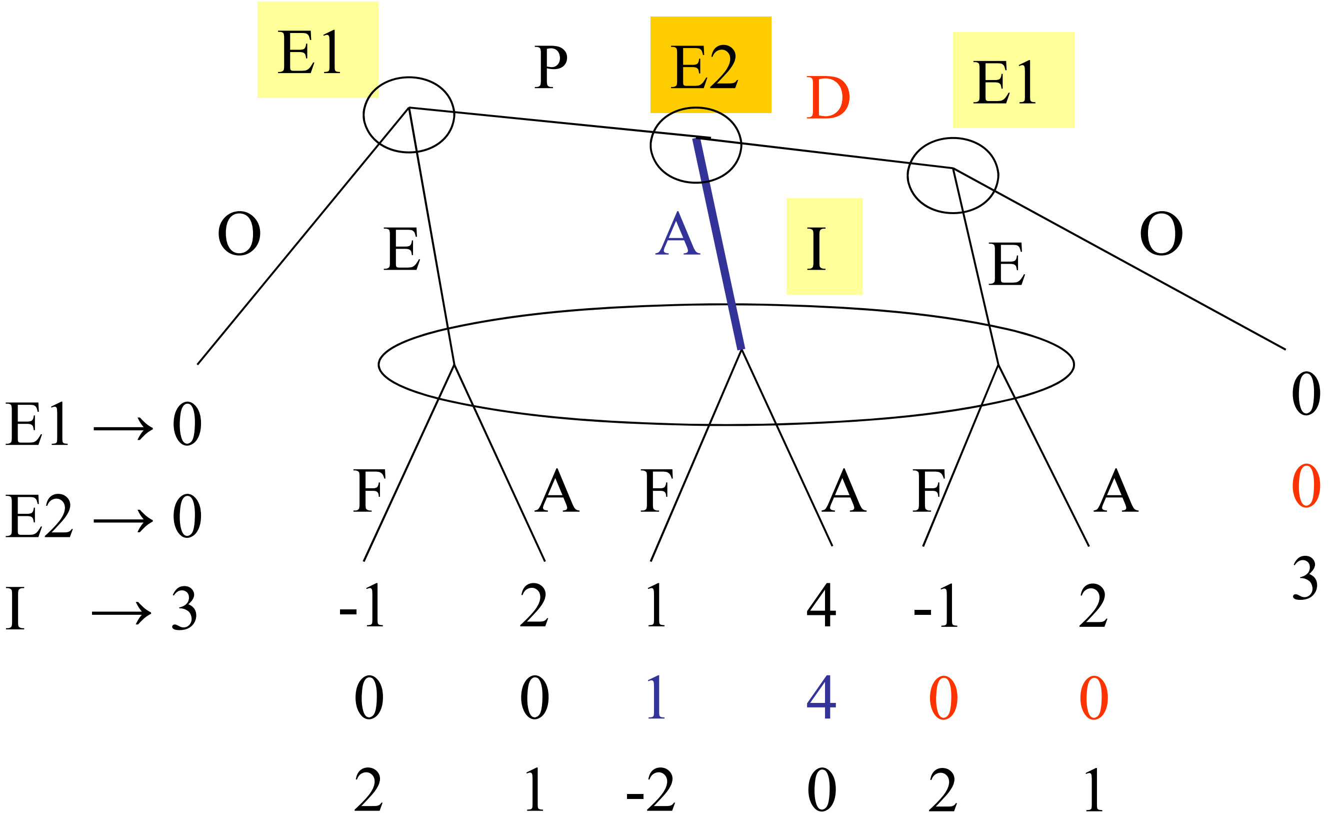
$-1 < 0, \ -1 < 1$

WPBE → ((I_1, A), $\mu = (1,0)$)

WPBE in Ex.9.C.2

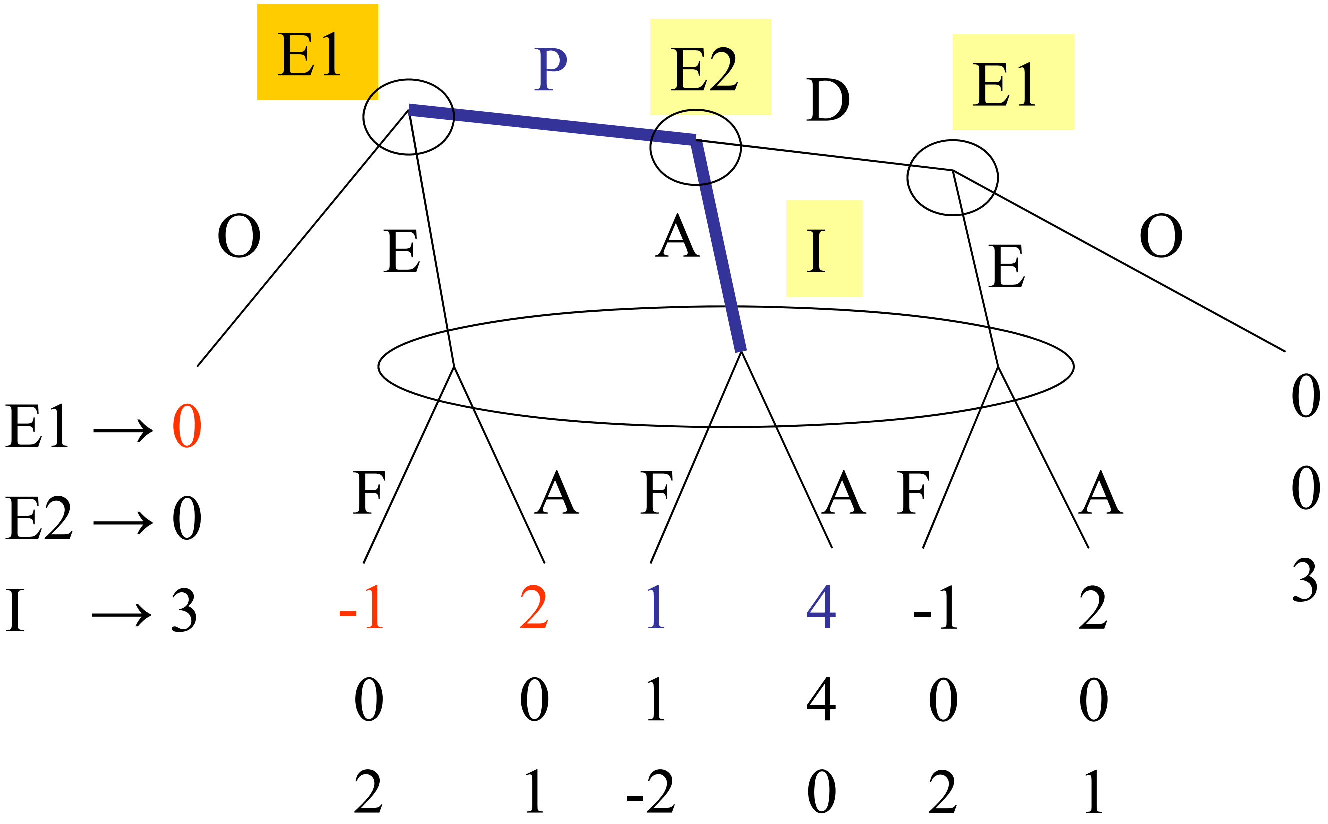


WPBE in Ex.9.C.2



E2 plays “A” since 1, 4 > 0

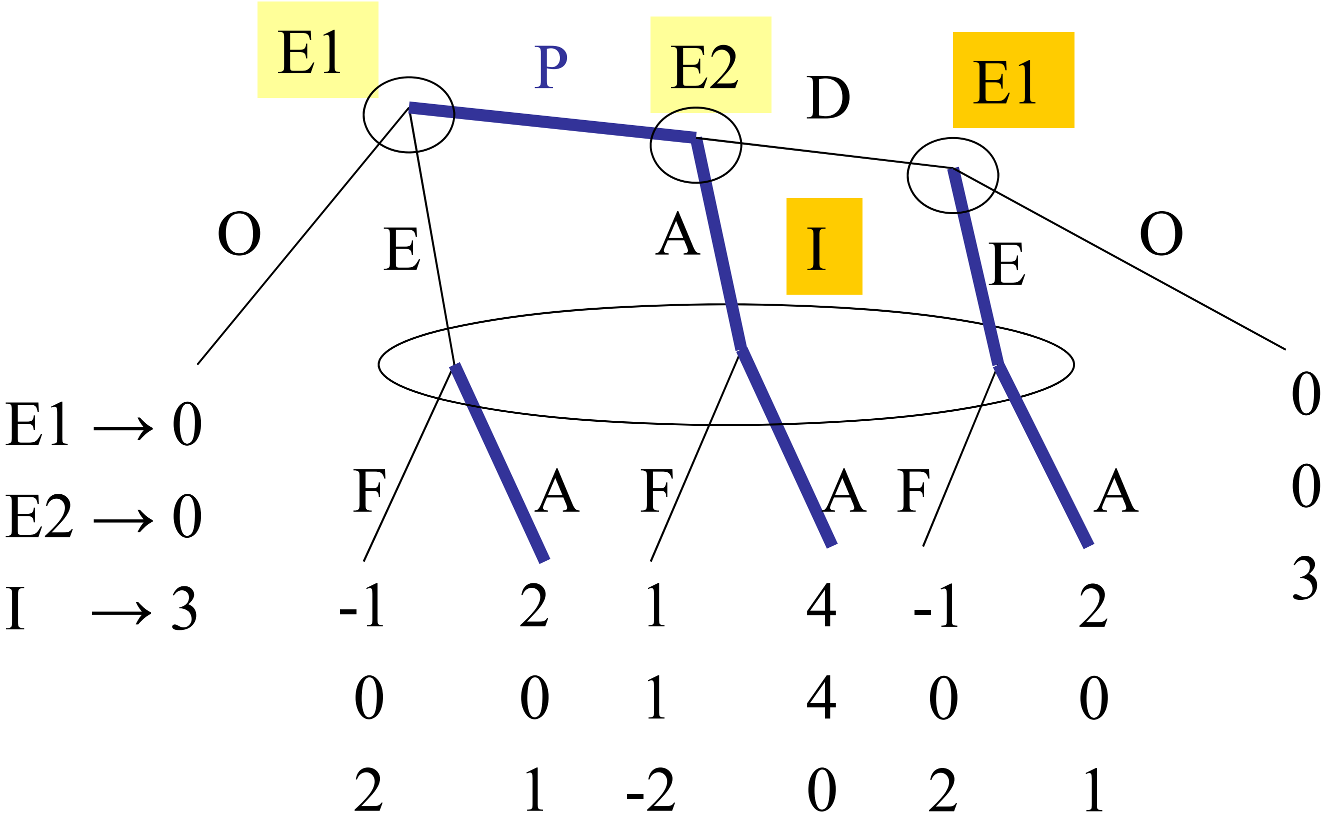
WPBE in Ex.9.C.2



E1 plays “P” since 4 > 2, 1 > -1 → P > E

4, 1 > 0 → P > O

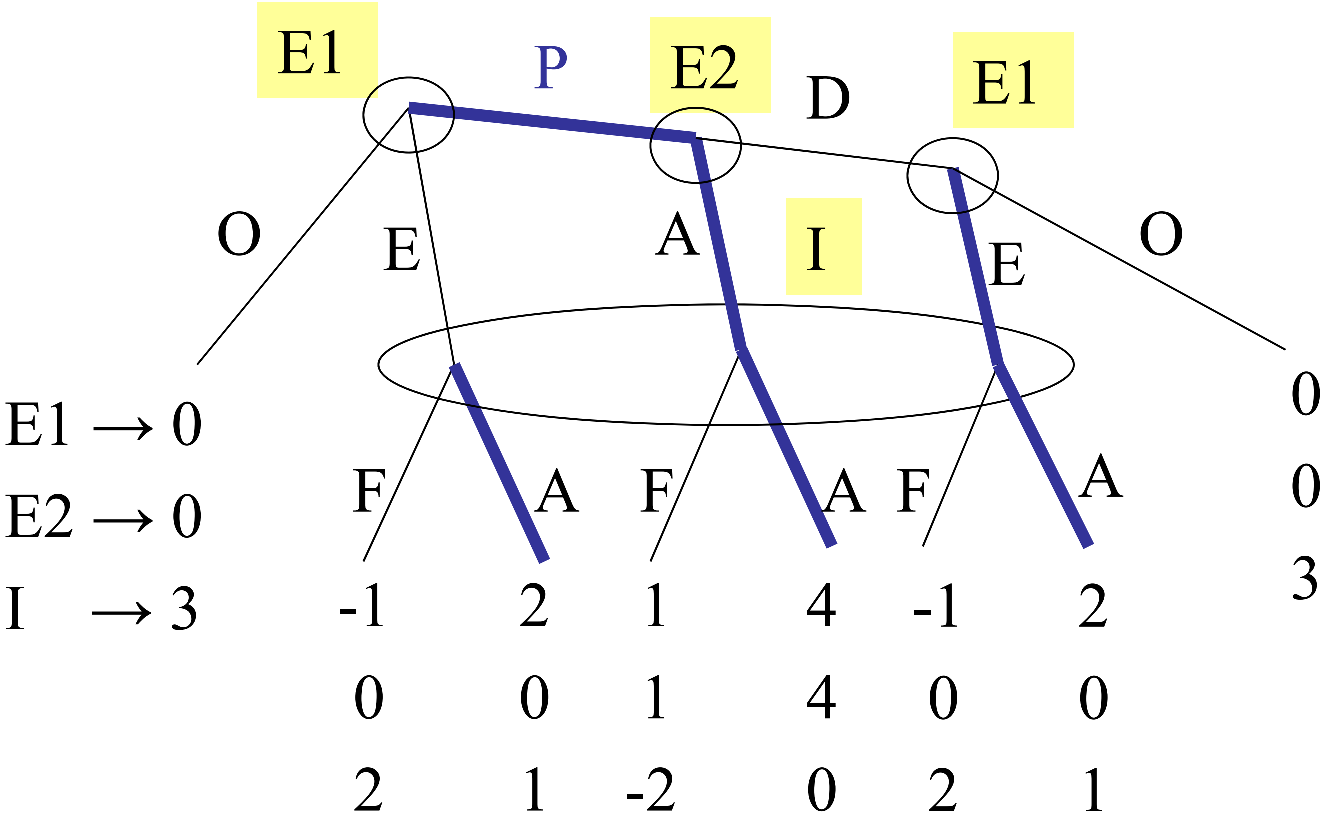
WPBE in Ex.9.C.2



I's belief (0, 1, 0) → I plays "A" since 0 > -2

Then E1 plays "E" since 2 > 0.

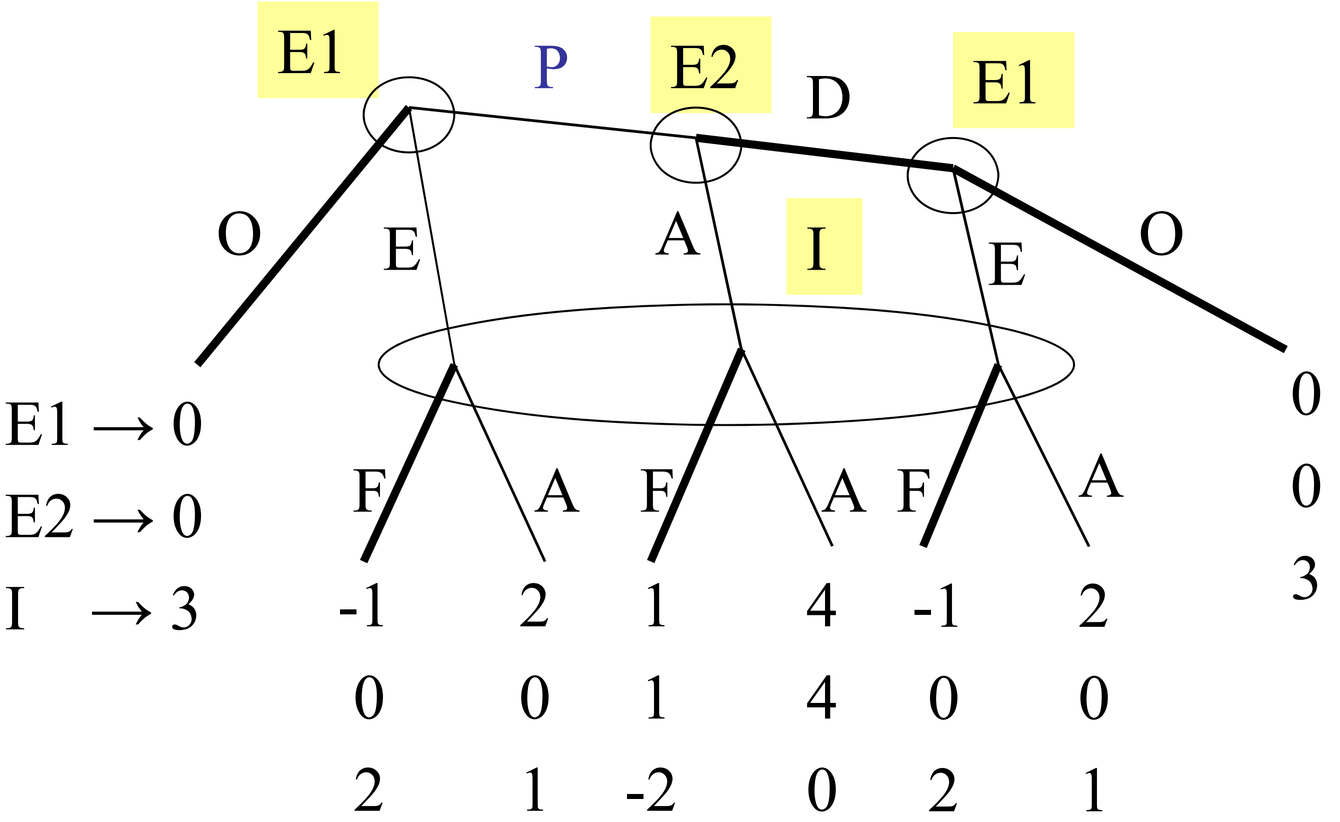
WPBE in Ex.9.C.2



WPBE : ((P, E), (A), (A), (0, 1, 0))

Note: ((O, O), (D), (F)) Nash eq. (SPNE)

WPBE in Ex.9.C.2



((O, O), (D), (F)) Nash eq. (SPNE)

Assignments

Problem Set 9 (due July 11)

Exercises (pp.301-305)

9.C.1

Reading Assignment:

Text, Chapter 9, pp.287-291