Assignment for Fundamentals of Mathematical and Computing Sciences: Applied Mathematical Sciences

Naoto Miyoshi

Solve the following problems and submit it to the report box (located on 3rd floor between East and West wings of W8 building) or by an e-mail to miyoshi@is.titech.ac.jp.

Due Date: August 1, 2013.

1. Let $\mathcal{X}$ denote an uncountable set and let $\mathcal{F}_x$, $x \in \mathcal{X}$, be a collection of σ-fields on a given sample space Ω. Show that $\mathcal{F} = \bigcap_{x\in\mathcal{X}} \mathcal{F}_x$ is a σ-field on Ω.

In the following, let $(\Omega, \mathcal{F}, \mathsf{P})$ denote a probability space.

2. Let $X_1, X_2, \dots$ denote a sequence of independent and identically distributed random variables such that $\mathsf{P}(X_1 = 0) = \mathsf{P}(X_1 = 2) = 1/2$. Compute the expectation $\mathsf{E}X = \int_\Omega X(\omega)\,\mathsf{P}(\mathrm{d}\omega)$ and variance $\mathsf{Var}X = \int_\Omega \bigl(X(\omega) - \mathsf{E}X\bigr)^2\,\mathsf{P}(\mathrm{d}\omega)$ of $X = \sum_{i=1}^\infty X_i/3^i$.

3. Let $X_1, X_2, \dots, X, Y \colon \Omega \to \mathbb{R}$ denote random variables such that $|X_n| \le Y$ a.s. for all n, $Y \in L^p$ for some $p \in (0, \infty)$ and $X_n \to X$ a.s. as $n \to \infty$. Show $X \in L^p$ and $X_n \xrightarrow{L^p} X$ as $n \to \infty$.