5th Report for Topics in Mathematical Optimization

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1. Let $\mu \leq \gamma_0 \leq L$ such that $\gamma_0 > 0$ and $\mu < L$. Also let $\boldsymbol{x}_0 = \boldsymbol{v}_0 \in \mathbb{R}^n$. Defining

$$\begin{aligned}
\gamma_{k+1} &:= L\alpha_k^2 = (1-\alpha_k)\gamma_k + \alpha_k\mu \\
\boldsymbol{y}_k &= \frac{\alpha_k\gamma_k\boldsymbol{v}_k + \gamma_{k+1}\boldsymbol{x}_k}{\gamma_k + \alpha_k\mu} \\
\boldsymbol{x}_{k+1} &= \boldsymbol{y}_k - \frac{1}{L}f'(\boldsymbol{y}_k) \\
\boldsymbol{v}_{k+1} &= \frac{(1-\alpha_k)\gamma_k\boldsymbol{v}_k + \alpha_k\mu\boldsymbol{y}_k - \alpha_k f'(\boldsymbol{y}_k)}{\gamma_{k+1}},
\end{aligned}$$

where $\alpha_k \in (0,1)$ is the root of the equation $L\alpha_k^2 - (1-\alpha_k)\gamma_k - \alpha_k\mu = 0$, show the following expressions:

   (a) $\boldsymbol{v}_{k+1} = \boldsymbol{x}_k + \frac{1}{\alpha_k}(\boldsymbol{x}_{k+1} - \boldsymbol{x}_k)$.

   (b) $\boldsymbol{y}_{k+1} = \boldsymbol{x}_{k+1} + \beta_k(\boldsymbol{x}_{k+1} - \boldsymbol{x}_k)$ for $\beta_k = \frac{\alpha_{k+1}\gamma_{k+1}(1-\alpha_k)}{\alpha_k(\gamma_{k+1}+\alpha_{k+1}\mu)}$.

   (c) $\beta_k = \frac{\alpha_k(1-\alpha_k)}{\alpha_k^2+\alpha_{k+1}}$.

   (d) $\alpha_{k+1}^2 = (1-\alpha_{k+1})\alpha_k^2 + \frac{\mu}{L}\alpha_{k+1}$.

2. Prove Lemma 10.2.