2nd Report for Topics in Mathematical Optimization

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1. Consider a sequence $\{\beta_k\}_{k=0}^{\infty}$ which converges to zero.

   The sequence is said to converge *Q-linearly* if there exists a scalar $\rho \in (0,1)$ such that

   $$\left|\frac{\beta_{k+1}}{\beta_k}\right| \leq \rho,$$

   for all k sufficiently large. *Q-superlinear* convergence occurs when we have

   $$\lim_{k\to\infty} \frac{\beta_{k+1}}{\beta_k} = 0,$$

   while the convergence is *Q-quadratic* if there is a constant C such that

   $$\frac{|\beta_{k+1}|}{\beta_k^2} \leq C$$

   for all k sufficiently large. *Q-superquadratic* convergence is indicated by

   $$\lim_{k\to\infty} \frac{\beta_{k+1}}{\beta_k^2} = 0.$$

   (a) Show that the following implications are valid: If a sequence $\{\beta_k\}_{k=0}^{\infty}$ is Q-superquadratic converging, then it is Q-quadratic converging. If a sequence $\{\beta_k\}_{k=0}^{\infty}$ is Q-quadratic converging, then it is Q-superlinear converging. If a sequence $\{\beta_k\}_{k=0}^{\infty}$ is Q-superlinear converging, then it is Q-linear converging.

   (b) Give a example of a sequence which is Q-linear but not Q-superlinear converging.

   A zero converging sequence $\{\beta_k\}_{k=0}^{\infty}$ is said to converge *R-linearly* if it is dominated by a Q-linearly converging sequence. That is, if there is a Q-linearly converging sequence $\{\hat{\beta}_k\}_{k=0}^{\infty}$ such that $0 \leq |\beta_k| \leq \hat{\beta}_k$.

   (c) Give a sequence which is R-linearly converging but not Q-linearly converging.

2. In the Section 5.5, show that $\mathcal{L}_k = \{\boldsymbol{\delta}_0, \boldsymbol{\delta}_1, \ldots, \boldsymbol{\delta}_{k-1}\}$.

3. Prove Lemma 6.3.

4. Prove Theorem 6.5.