Nuclear Reactor Physics Lecture Note (10)
-Neutron Spectrum(1): Neutron Slowing Down in Infinite Medium-

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9. Neutron spectrum

Group constants in each region in a reactor

ex. $$\Sigma_{t,g} = \frac{\int_{E_g}^{E_{g-1}} dE\Sigma_t(E)\phi(E)}{\int_{E_g}^{E_{g-1}} dE\,\phi(E)}$$

9.1 Neutron slowing down in thermal reactor

(1)Neutron spectrum in nuclear reactor

Neutron flux in nuclear reactors depends greatly on the energy.

$$\begin{cases} \text{depends on reactor type(Fast reactor, Thermal reator)} \\ \text{depends on the position in the reactor(fuel, moderator, etc)} \end{cases}$$

Thermal reactor:

$$\text{Fission neutrons} \xrightarrow{\text{Slowing down}} \text{Thermal neutrons} \Rightarrow \text{Fission}$$

The slowing down of fast neutrons is possible only by scattering of neutrons with nuclei.

In the following discussion, space depending is ignored. $\phi(E), \Sigma(E)$, etc

(2)Neutron slowing down equation

Slowing down of neutrons from uniformly distributed neutron source in infinite homogeneous medium

Number of collisions of neutron (energy E) with nuclei per unit time

＝Number of neutrons produced at energy E per unit time.

Since from simplification of neutron transport equation

$$[\Sigma_s(E) + \Sigma_a(E)]\phi(E) = \int_E^\infty dE'\Sigma_s(E' \to E)\phi(E') + S(E) \qquad \cdots (1)$$

If the scattering is elastic and isotropic in center of mass coordinate frame,

$$\Sigma_s(E' \to E) = \begin{cases} \dfrac{\Sigma_s(E')}{(1-\alpha)E'} & (\alpha E' \le E \le E') \\ 0 & \text{otherwise} \end{cases} \qquad \cdots (2)$$

where, $\alpha = \left(\dfrac{A-1}{A+1}\right)^2$ A : Mass number of nucleus in the medium

(3)Neutron slowing down in hydrogen

Slowing down of neutron from a source (neutron energy : E_0, number of emitted neutrons per unit time : S_0) in hydrogen. (no neutron absorption)

The neutron slowing down equation

$$\Sigma_s(E)\phi(E) = \int_E^{E_0} dE' \frac{\Sigma_s(E')\phi(E')}{E'} + S_0\delta(E - E_0) \qquad \cdots (3)$$

The solution

$$\phi(E) = \frac{S_0}{\Sigma_s(E)E} + \frac{S_0}{\Sigma_s(E)}\delta(E - E_0) \qquad \cdots (4)$$

If Σ_s is almost constant (often true)

$$\phi(E) \propto \frac{1}{E} \qquad \cdots (5)$$

(4)The neutron lethargy

In the discussion of neutron slowing down neutron lethargy u is often used instead of energy E.

Definition : $u \equiv \ln\dfrac{E_0}{E}$ $E_0 = 10\text{MeV}$ usually $\cdots (6)$

Reason : slowing down by scattering ratio of energy change is constant. So logarithm is useful.

Utilization of lethargy (1): Average gain of lethargy by a scattering

$$\xi = 1 + \frac{\alpha}{1-\alpha}\ln\alpha = 1 - \frac{(A-1)^2}{2A}\ln\left(\frac{A+1}{A-1}\right) \qquad \cdots (7)$$

It is constant. (independent of energy)

It depends on mass number of scattering nucleus only.

If ξ is large, it is possible to slow down fast neutrons to thermal region without a lot of scatterings with medium nuclei.

Indices of moderator:

Moderating power $(\equiv \xi\Sigma_s)$ The larger is better.

- Large scattering cross section
- Small number of scatterings for slowing down

Moderation ratio $(\equiv \xi\frac{\Sigma_s}{\Sigma_a})$ The larger is better.

- In addition to the above, possibility of absorption at collisions is small

Utilization of lethargy(2): Neutron flux per unit lethargy

Neutron flux per unit lethargy is usually used in the discussion of neutron slowing down instead of neutron flux per unit energy. It is used in the figure of neutron spectrum too.