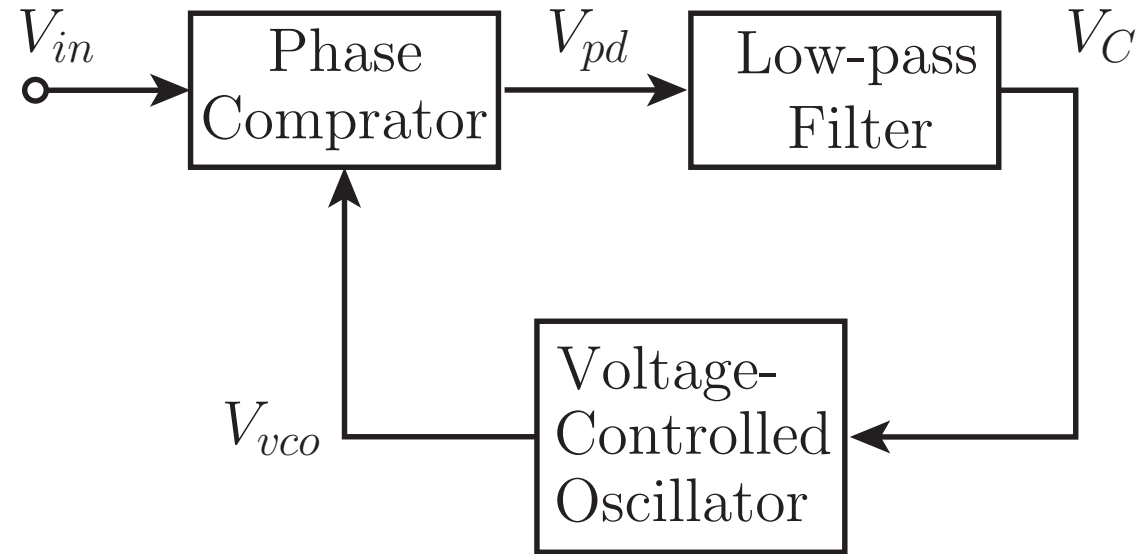


Phase Locked Loop(PLL)の構成

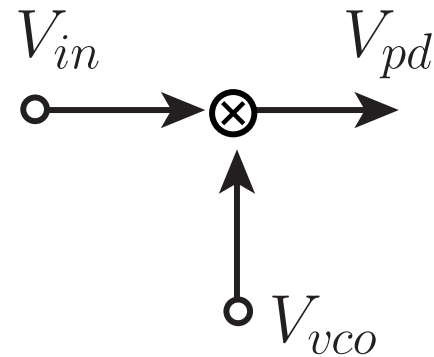
PLLの動作の概要



$$V_{in}(t) = A_{in} \sin(2\pi f_{in} t + \theta_{in})$$

$$V_{vco}(t) = A_{vco} \sin(2\pi f_{vco} t + \theta_{vco} + \frac{\pi}{2})$$

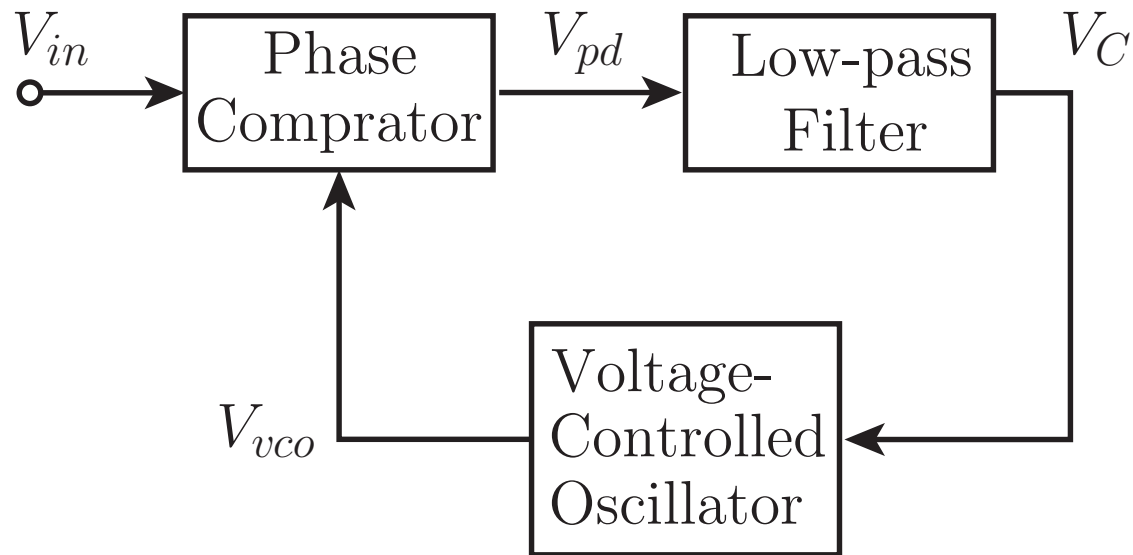
位相比較回路→アナログ乗算回路



$$V_{pd}(t) = K_{mul} V_{in}(t) V_{vco}(t)$$

$$\begin{aligned} &= \frac{K_{mul} A_{in} A_{vco}}{2} \left[\cos \left\{ 2\pi(f_{in} - f_{vco})t + (\theta_{in} - \theta_{vco}) - \frac{\pi}{2} \right\} \right. \\ &\quad \left. - \cos \left\{ 2\pi(f_{in} + f_{vco})t + (\theta_{in} + \theta_{vco}) + \frac{\pi}{2} \right\} \right] \\ &= \frac{K_{mul} A_{in} A_{vco}}{2} [\sin(\theta_{in} - \theta_{vco}) + \sin\{4\pi f_{in}t + (\theta_{in} + \theta_{vco})\}] \end{aligned}$$

$\because f_{in} = f_{vco}$ と仮定



$$V_C = \frac{K_{lpf} K_{mul} A_{in} A_{vco}}{2} \sin(\theta_{in} - \theta_{vco})$$

$$\theta_{in} - \theta_{vco} = \sin^{-1} \left(\frac{2V_C}{K_{lpf} K_{mul} A_{in} A_{vco}} \right)$$

$$K_{lpf} K_{mul} A_{in} A_{vco} \rightarrow \infty$$

$$\boxed{\theta_{in} - \theta_{vco} \rightarrow 0}$$

$f_{in} \neq f_{vco}$ と仮定

$$f_{vco} = K_{vco} V_C(t) + f_{free} \quad K_{vco} > 0$$

$$V_C(t) = \frac{K_{lpf} K_{mul} A_{in} A_{vco}}{2} \sin\{2\pi(f_{in} - f_{vco})t + (\theta_{in} - \theta_{vco})\}$$

$f_{in} > f_{vco}$

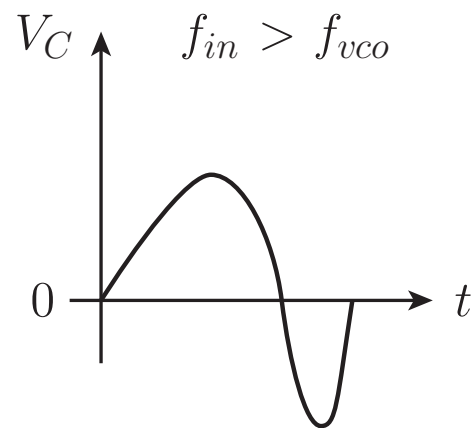
$$V_C(t) > 0 \quad f_{vco} \rightarrow \text{大} \quad |f_{in} - f_{vco}| \rightarrow \text{小} \quad \boxed{\text{長い}}$$

$$V_C(t) < 0 \quad f_{vco} \rightarrow \text{小} \quad |f_{in} - f_{vco}| \rightarrow \text{大} \quad \boxed{\text{短い}}$$

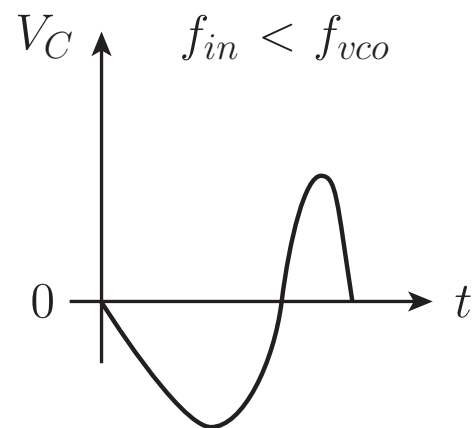
$f_{in} < f_{vco}$

$$V_C(t) > 0 \quad f_{vco} \rightarrow \text{大} \quad |f_{in} - f_{vco}| \rightarrow \text{大} \quad \boxed{\text{短い}}$$

$$V_C(t) < 0 \quad f_{vco} \rightarrow \text{小} \quad |f_{in} - f_{vco}| \rightarrow \text{小} \quad \boxed{\text{長い}}$$



(a)



(b)

平均值

$$V_C(t) > 0$$

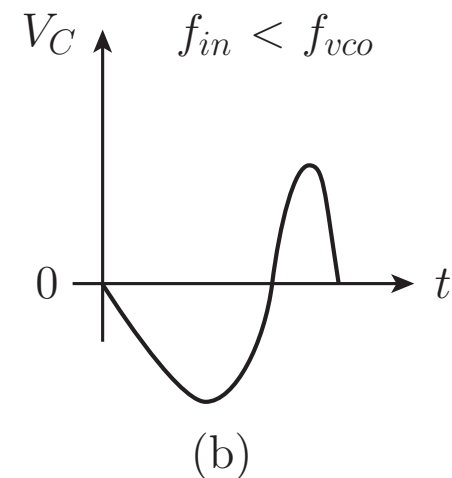
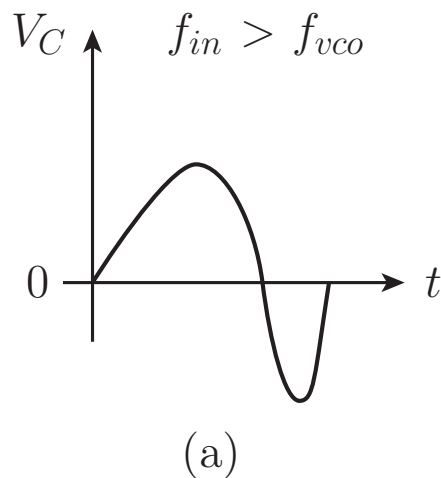
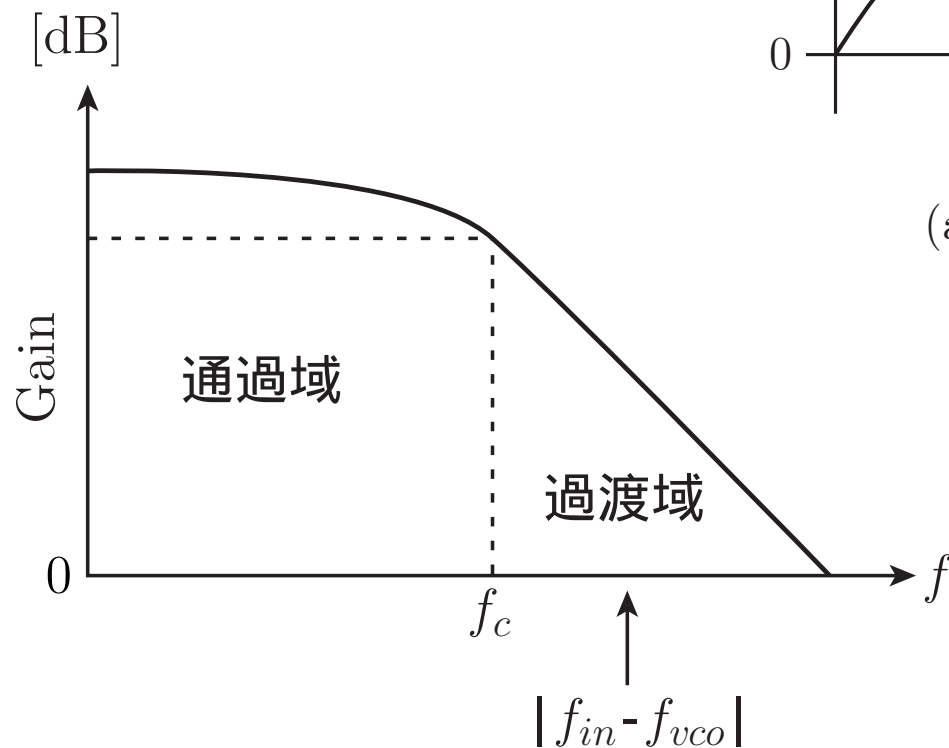
$$V_C(t) < 0$$

$$f_{vco} \rightarrow \text{大}$$

$$f_{vco} \rightarrow \text{小}$$

$$f_{vco} \rightarrow f_{in}$$

低域通過フィルタ特性



利得増加 $\rightarrow V_C$: 大

利得減少 $\rightarrow V_C$: 小

$$|f_{in} - f_{vco}| \rightarrow 0$$

PLLの線形解析手法

$$K_{lpf} K_{mul} A_{in} A_{vco} \rightarrow \infty?$$

$$\theta_{in} - \theta_{vco} \neq 0$$

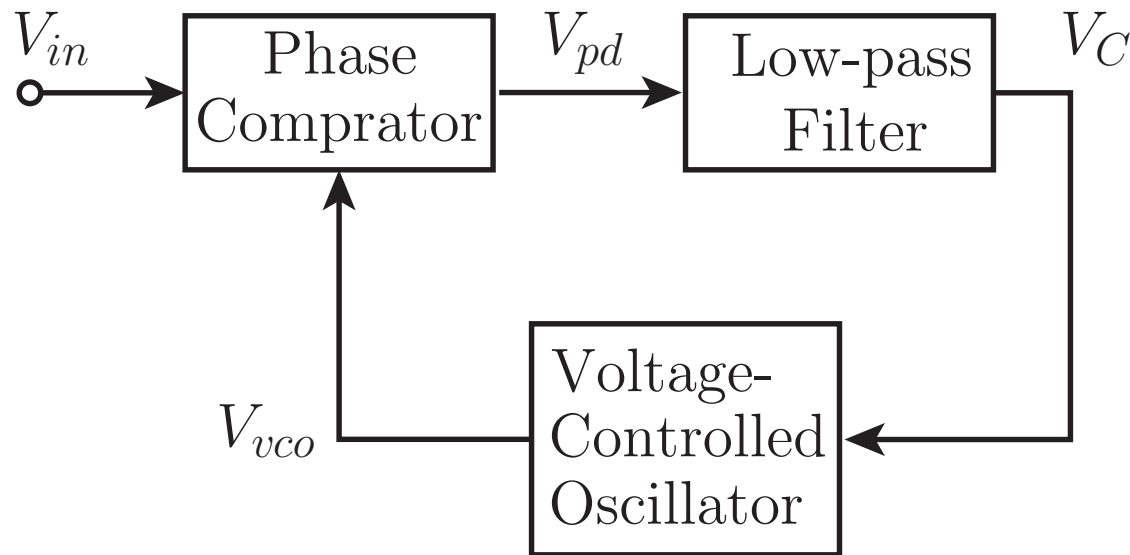
「同期」の定義

$$\frac{d\theta_{in}}{dt} = \frac{d\theta_{vco}}{dt}$$

「瞬時周波数」の定義

$$f_{inst} = \frac{1}{2\pi} \cdot \frac{d\theta}{dt}$$

「同期」とは $f_{in} = f_{vco}$



$$V_{pd}'(t) = \frac{K_{mul} A_{in} A_{vco}}{2} \sin(\theta_{in} - \theta_{vco})$$

$$\approx \frac{K_{mul} A_{in} A_{vco}}{2} (\theta_{in} - \theta_{vco})$$

ラプラス変換

$$V_{pd}(s) = K_{pd} \{ \Theta_{in}(s) - \Theta_{vco}(s) \}$$

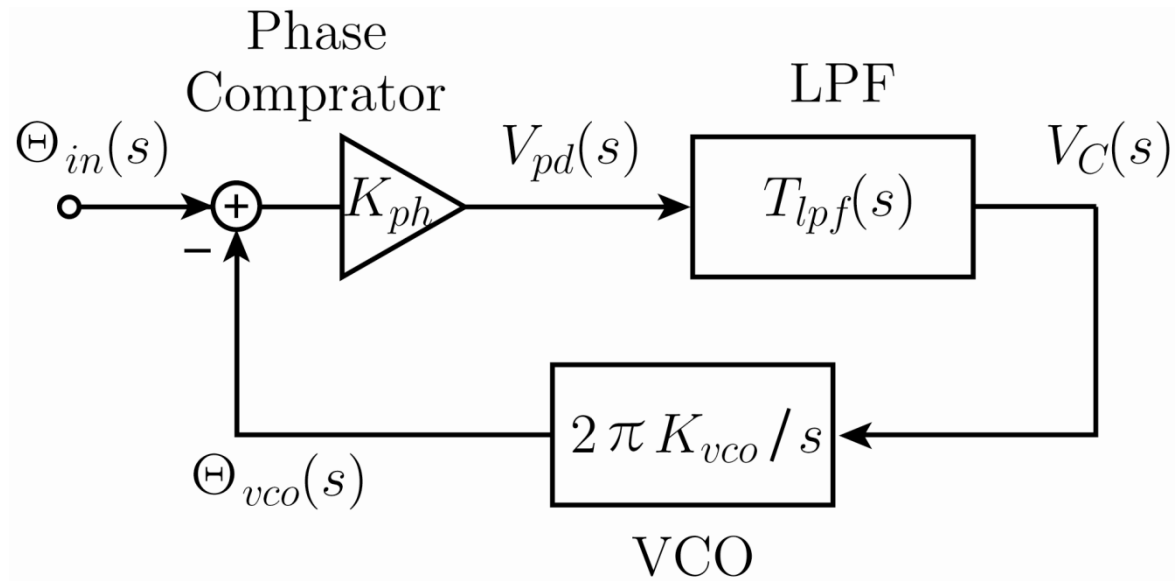
$$V_C(s) = T_{lpf}(s) V_{pd}(s)$$

$$f_{vco} = K_{vco} V_C(t) + f_{free}$$

$$f_{inst} = \frac{1}{2\pi} \cdot \frac{d\theta}{dt}$$

$$\begin{aligned}\theta_{vco} &= 2\pi \int (f_{vco} - f_{free}) dt + \text{Const.} \\ &= 2\pi \int K_{vco} V_C(t) dt + \text{Const.}\end{aligned}$$

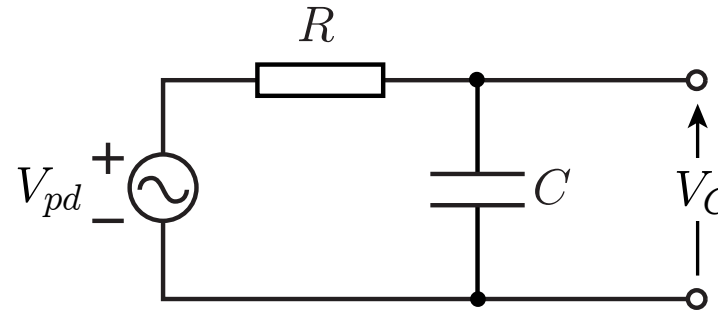
$$\Theta_{vco}(s) = \frac{2\pi K_{vco} V_C(s)}{s}$$



$$\Theta_{vco}(s) = \frac{2\pi K_{vco} K_{pd} T_{lpf}(s)}{s + 2\pi K_{vco} K_{pd} T_{lpf}(s)} \Theta_{in}(s)$$

$$V_C(s) = \frac{s K_{pd} T_{lpf}(s)}{s + 2\pi K_{vco} K_{pd} T_{lpf}(s)} \Theta_{in}(s)$$

$$T_{lpf}(s) = \frac{K_{lpf}a_0}{a_0 + a_1s}$$



$$\Theta_{vco}(s) = \frac{2\pi K_{vco} K_{pd} K_{lpf} a_0}{a_1 s^2 + a_0 s + 2\pi K_{vco} K_{pd} K_{lpf} a_0} \Theta_{in}(s)$$

直流利得が1倍 $\longrightarrow \theta_{in}$ の初期値からの偏差
 $= \theta_{vco}$ の初期値からの偏差

$$V_C(s) = \frac{s K_{pd} K_{lpf} a_0}{a_1 s^2 + a_0 s + 2\pi K_{vco} K_{pd} K_{lpf} a_0} \Theta_{in}(s)$$

直流利得が0倍 $\longrightarrow \theta_{in}$ の初期値からの偏差は
 V_C へ影響しない

$$\Theta_{vco}(s) = \frac{2\pi K_{vco} K_{pd} K_{lpf} a_0}{a_1 s^2 + a_0 s + 2\pi K_{vco} K_{pd} K_{lpf} a_0} \Theta_{in}(s)$$

$$\Theta_{vco}(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \Theta_{in}(s)$$

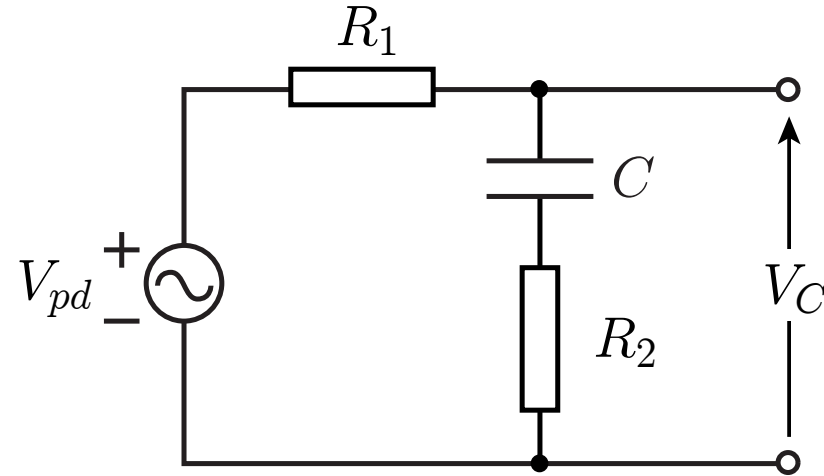
$$\omega_0 = \sqrt{\frac{2\pi K_{loop} a_0}{a_1}}$$

$$Q = \sqrt{\frac{2\pi K_{loop} a_1}{a_0}}$$

$$K_{loop} = K_{vco} K_{pd} K_{lpf} \gg 1 \quad a_1 \gg a_0$$

Q は大

$$T_{lpf}(s) = \frac{K_{lpf}(a_0 + a_2 s)}{a_0 + a_1 s}$$



$$\Theta_{vco}(s) = \frac{2\pi K_{loop}(a_0 + a_2 s)}{a_1 s^2 + (a_0 + 2\pi K_{loop} a_2)s + 2\pi K_{loop} a_0} \Theta_{in}(s)$$

$$V_C(s) = \frac{s K_{pd} K_{lpf}(a_0 + a_2 s)}{a_1 s^2 + (a_0 + 2\pi K_{loop} a_2)s + 2\pi K_{loop} a_0} \Theta_{in}(s)$$

$$\omega_0 = \sqrt{\frac{2\pi K_{loop} a_0}{a_1}}$$

$$Q = \sqrt{\frac{2\pi K_{loop} a_1 / a_0}{1 + K_{loop} a_2 / a_0}}$$

ロックレンジ

発振周波数の範囲＝制御電圧の可変範囲

$$V_C(t) = \frac{K_{lpf} K_{mul} A_{in} A_{vco}}{2} \sin \{ 2\pi (f_{in} - f_{vco}) t + (\theta_{in} - \theta_{vco}) \}$$

$$V_{Cmax} = \frac{K_{lpf} K_{mul} A_{in} A_{vco}}{2}$$

$$f_{vco} = K_{vco} V_C(t) + f_{free}$$

$$f_{lock} = K_{vco} V_{Cmax} \leq \frac{K_{vco} K_{lpf} K_{mul} A_{in} A_{vco}}{2} = K_{loop}$$

キャプチャレンジ

近似式

$$f_{capture} = \sqrt{\frac{2a_2}{a_1} - \left(\frac{a_2}{a_1}\right)^2} f_{lock} \quad (a_2 < a_1)$$

引き込み時間

近似式

$$t_{pull-in} = \frac{Q\{2\pi(f_{in} - f_{vco})\}^2}{\omega_0^3}$$

ω_0 が大

θ_{in} の高い周波数成分を含む揺らぎが θ_{vco} へ伝達

(位相雑音)

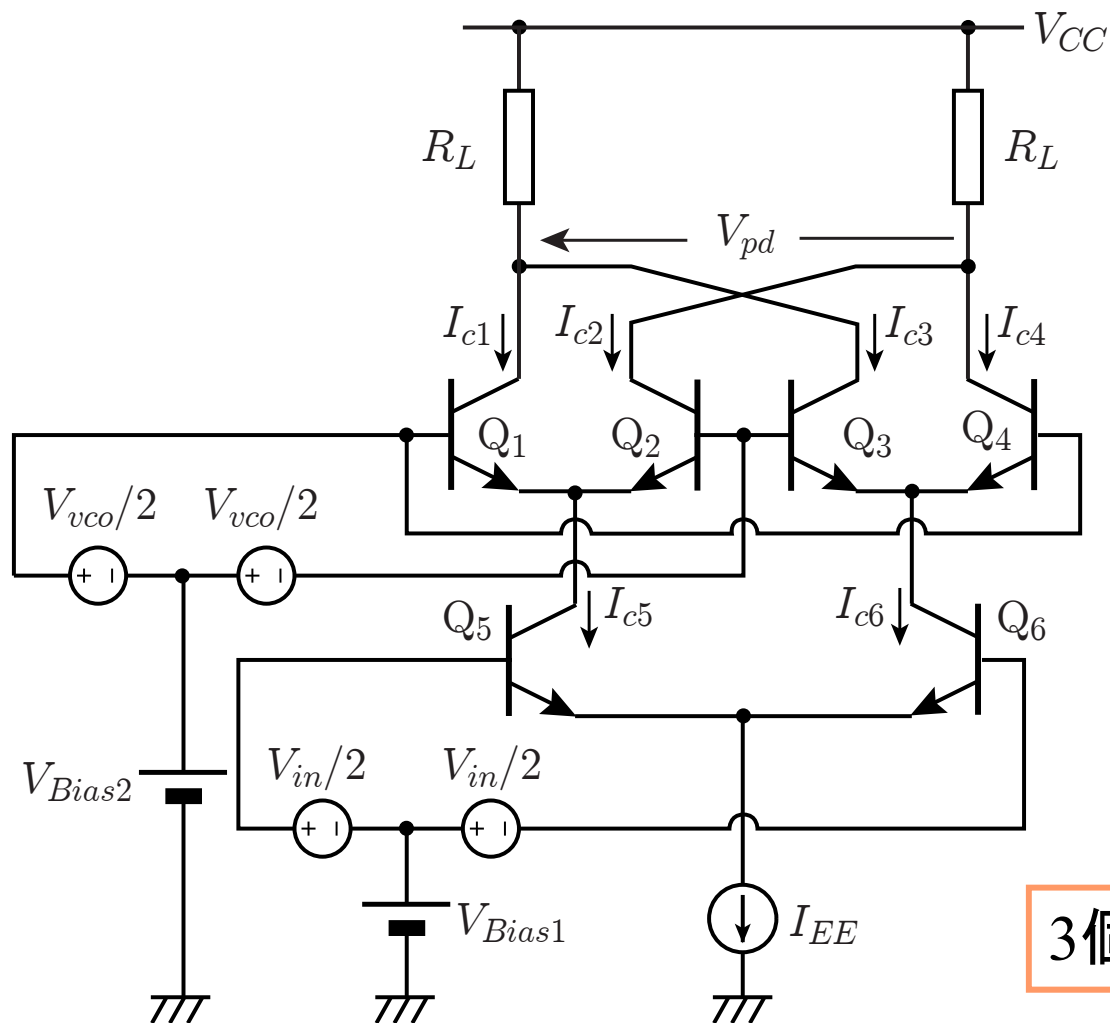
ω_0 が小

引き込み時間が増加

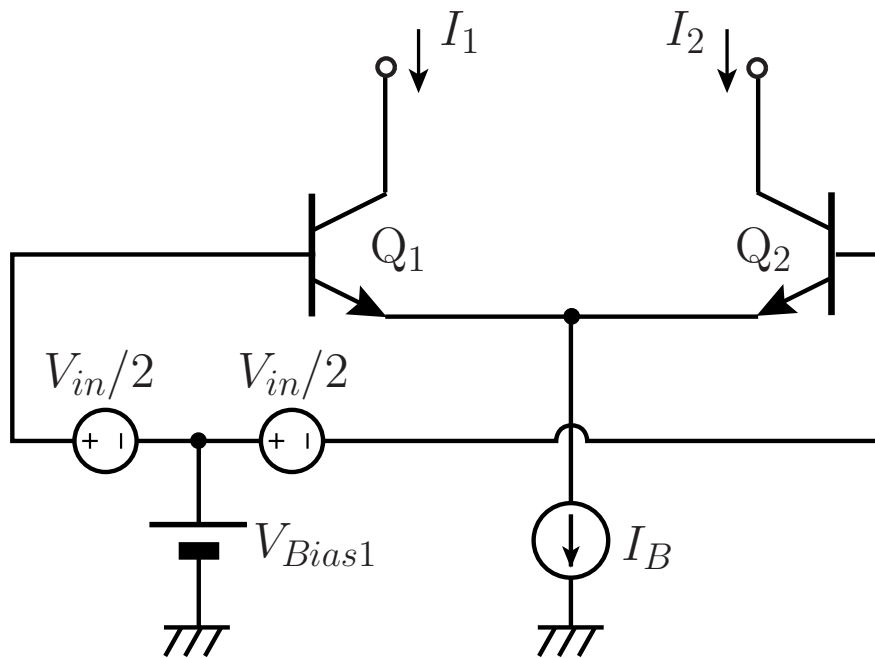
PLLの構成要素

位相比較回路

アナログ乗算回路



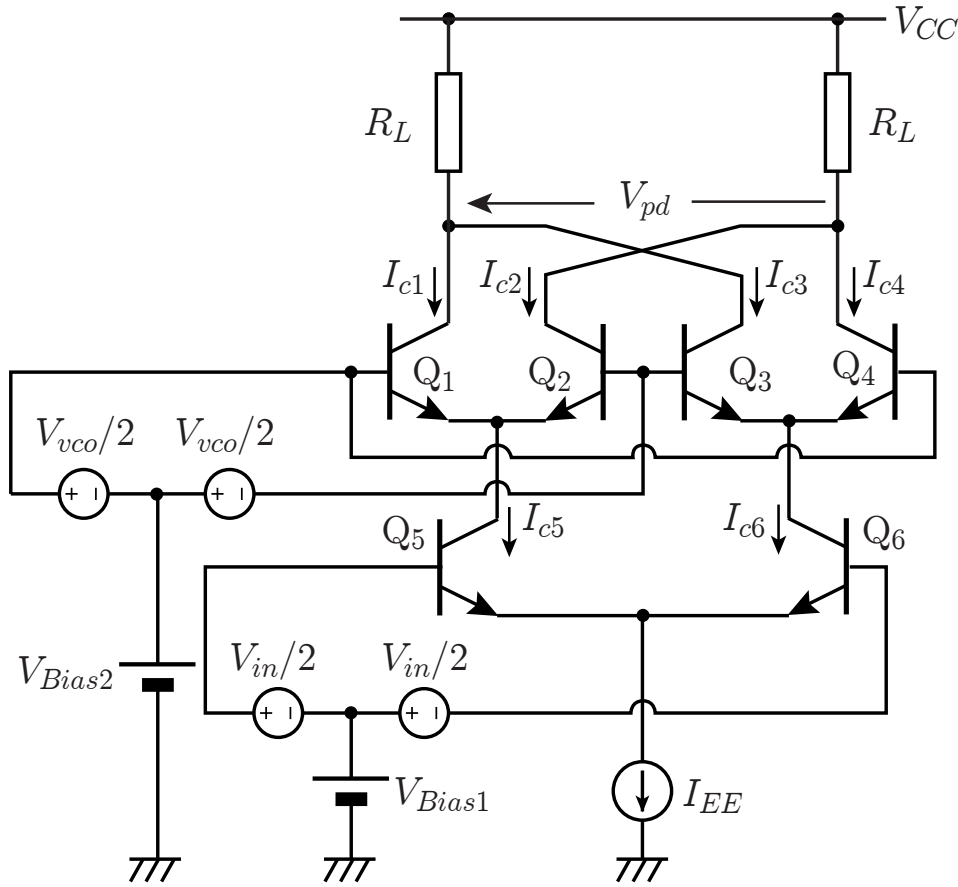
3個の差動増幅回路



$$I_1 = \frac{I_B}{2} + \frac{1}{2r_e} V_{in}$$

$$I_2 = \frac{I_B}{2} - \frac{1}{2r_e} V_{in}$$

$$r_e = \frac{kT}{q(I_B/2)} = 2 \frac{kT}{qI_B}$$



$$I_{c5} = \frac{I_{EE}}{2} + \frac{1}{2r_{e1}} V_{in}$$

$$I_{c6} = \frac{I_{EE}}{2} - \frac{1}{2r_{e1}} V_{in}$$

$$r_{e1} = 2 \frac{kT}{qI_{EE}}$$

$$I_{c1} = \frac{I_{c5}}{2} + \frac{1}{2r_{e2}} V_{vco}$$

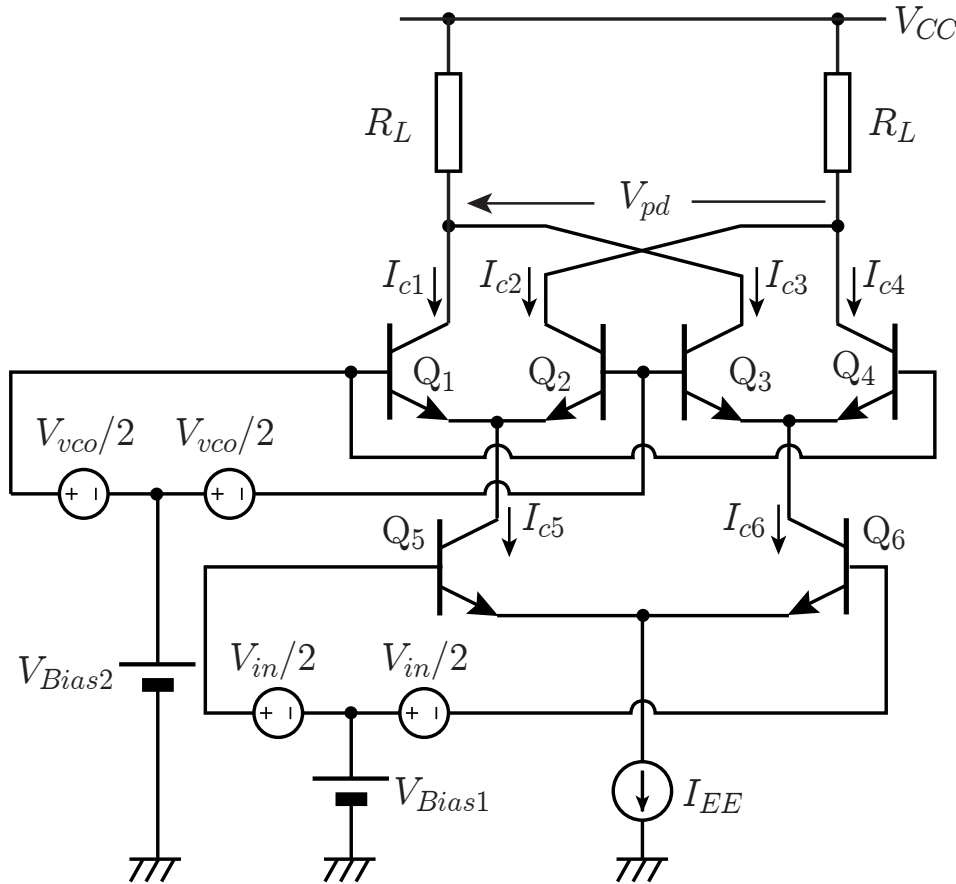
$$I_{c2} = \frac{I_{c5}}{2} - \frac{1}{2r_{e2}} V_{vco}$$

$$r_{e2} = 2 \frac{kT}{qI_{c5}}$$

$$I_{c3} = \frac{I_{c6}}{2} - \frac{1}{2r_{e3}} V_{vco}$$

$$I_{c4} = \frac{I_{c6}}{2} + \frac{1}{2r_{e3}} V_{vco}$$

$$r_{e3} = 2 \frac{kT}{qI_{c6}}$$



$$V_{pd} = -R_L (I_{c1} - I_{c2} + I_{c3} - I_{c4})$$

$$I_{c1} - I_{c2} = \frac{1}{r_{e2}} V_{vco}$$

$$I_{c3} - I_{c4} = -\frac{1}{r_{e3}} V_{vco}$$

$$I_{c5} - I_{c6} = \frac{1}{r_{e1}} V_{in}$$

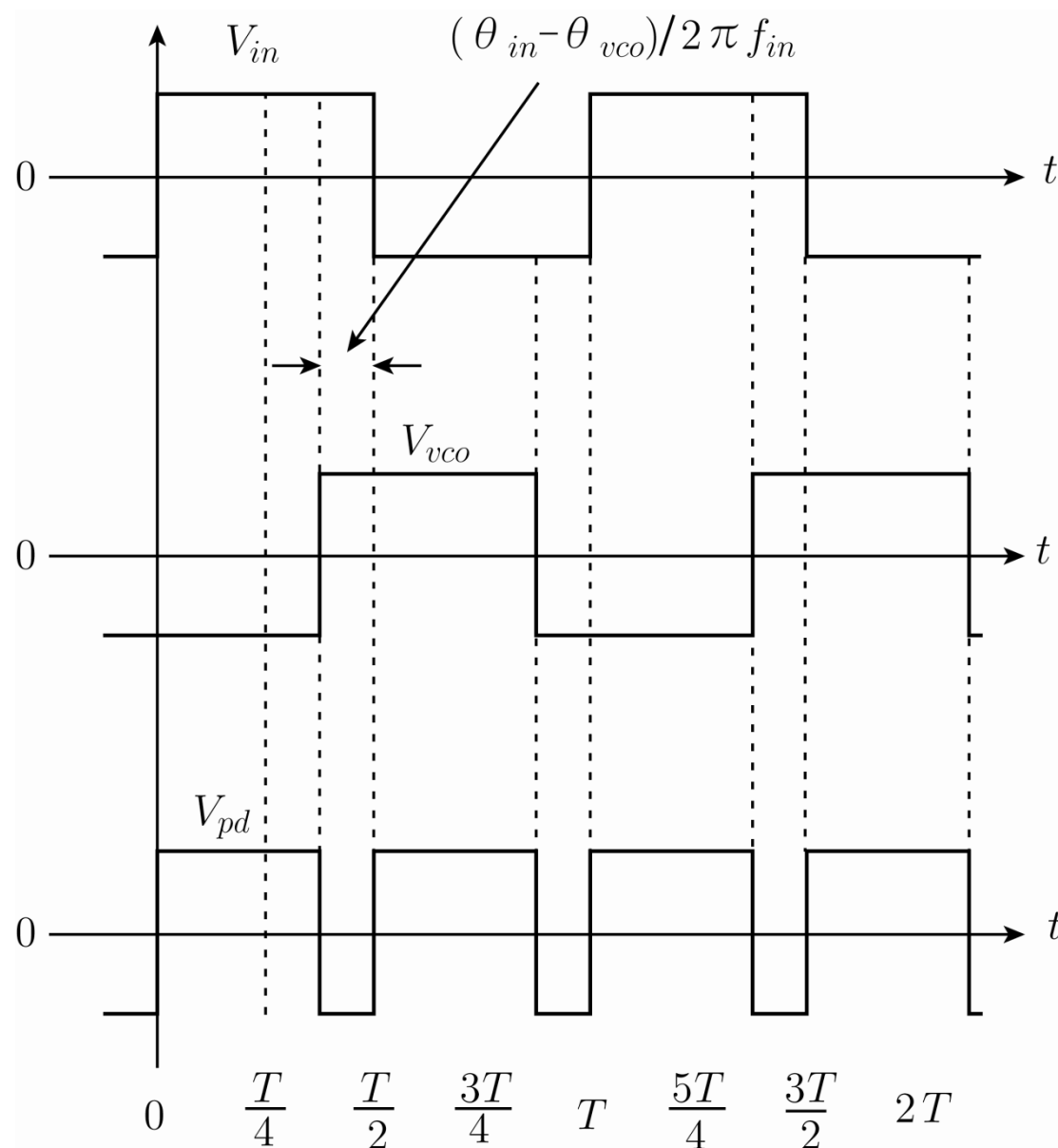
$$V_{pd} = -R_L \left(\frac{V_{vco}}{r_{e2}} - \frac{V_{vco}}{r_{e3}} \right) = -R_L V_{vco} \left(\frac{qI_{c5}}{2kT} - \frac{qI_{c6}}{2kT} \right)$$

$$= -R_L V_{vco} \frac{qV_{in}}{2kTr_{e1}} = -R_L \frac{qI_{EE}}{(2kT)^2} V_{vco} V_{in}$$

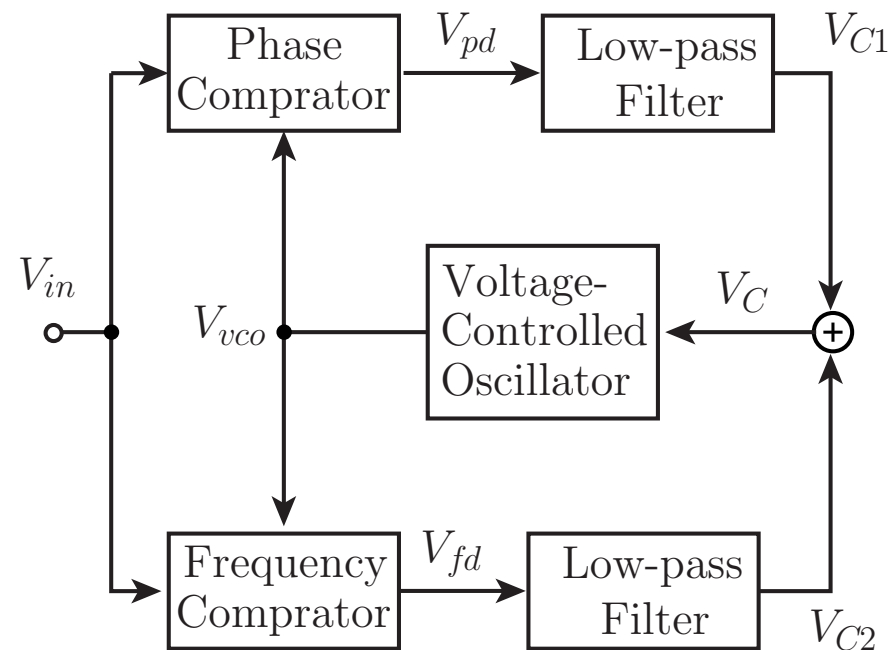
Exclusive OR回路

$x \oplus y \rightarrow$
 $00 \rightarrow 0$
 $01 \rightarrow 1$
 $10 \rightarrow 1$
 $11 \rightarrow 0$

$0 \rightarrow +$ (プラス)
 $1 \rightarrow -$ (マイナス)



チャージポンプ型位相比較回路



位相比較+周波数比較=高速な同期を実現

周波数・位相差比較回路

方形波 V_{in} と V_{vco} の立ち上がり反応

$V_{in}:Hレベル \rightarrow Up=1$

→

$V_{vco}:Hレベル \rightarrow Up=0$

$Down=0$

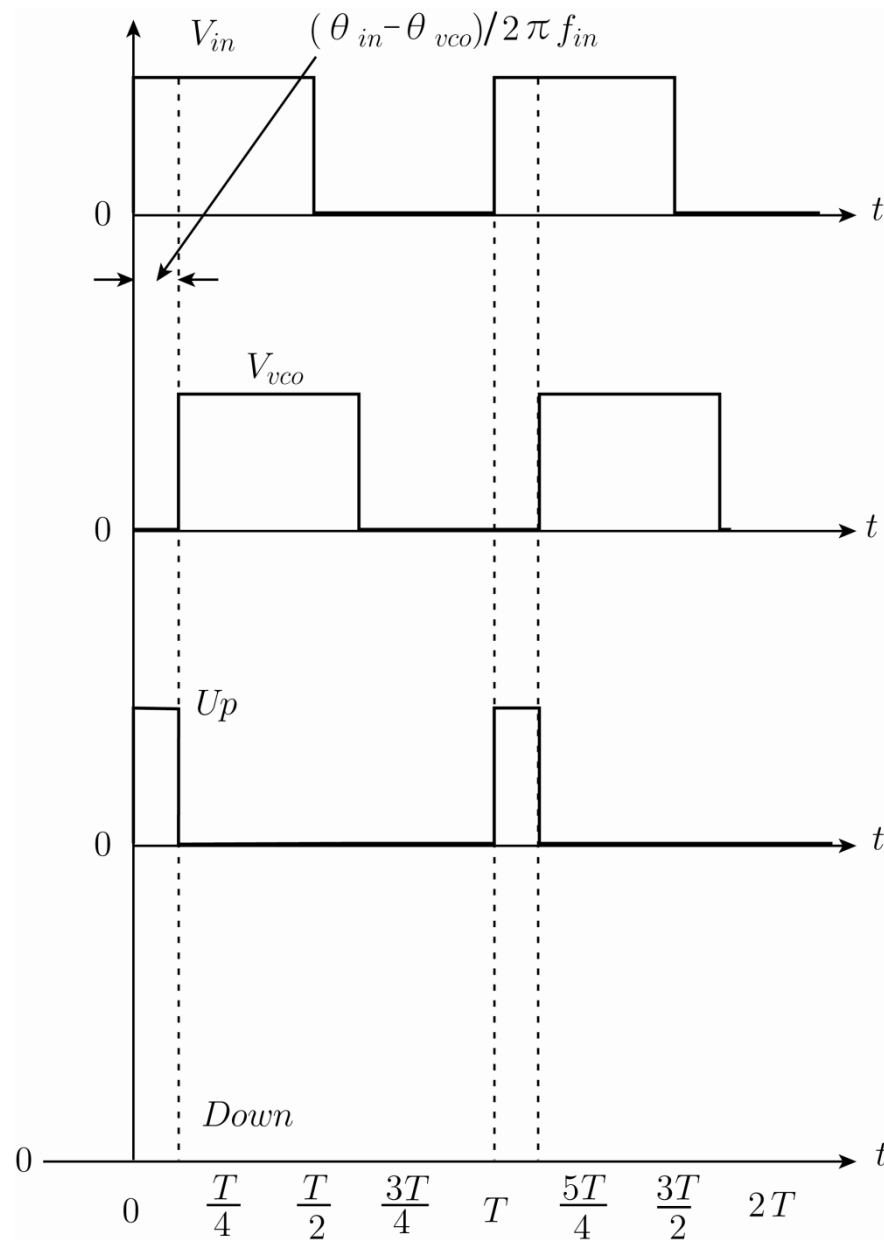
$V_{vco}:Hレベル \rightarrow Down=1$

→

$V_{in}:Hレベル \rightarrow Down=0$

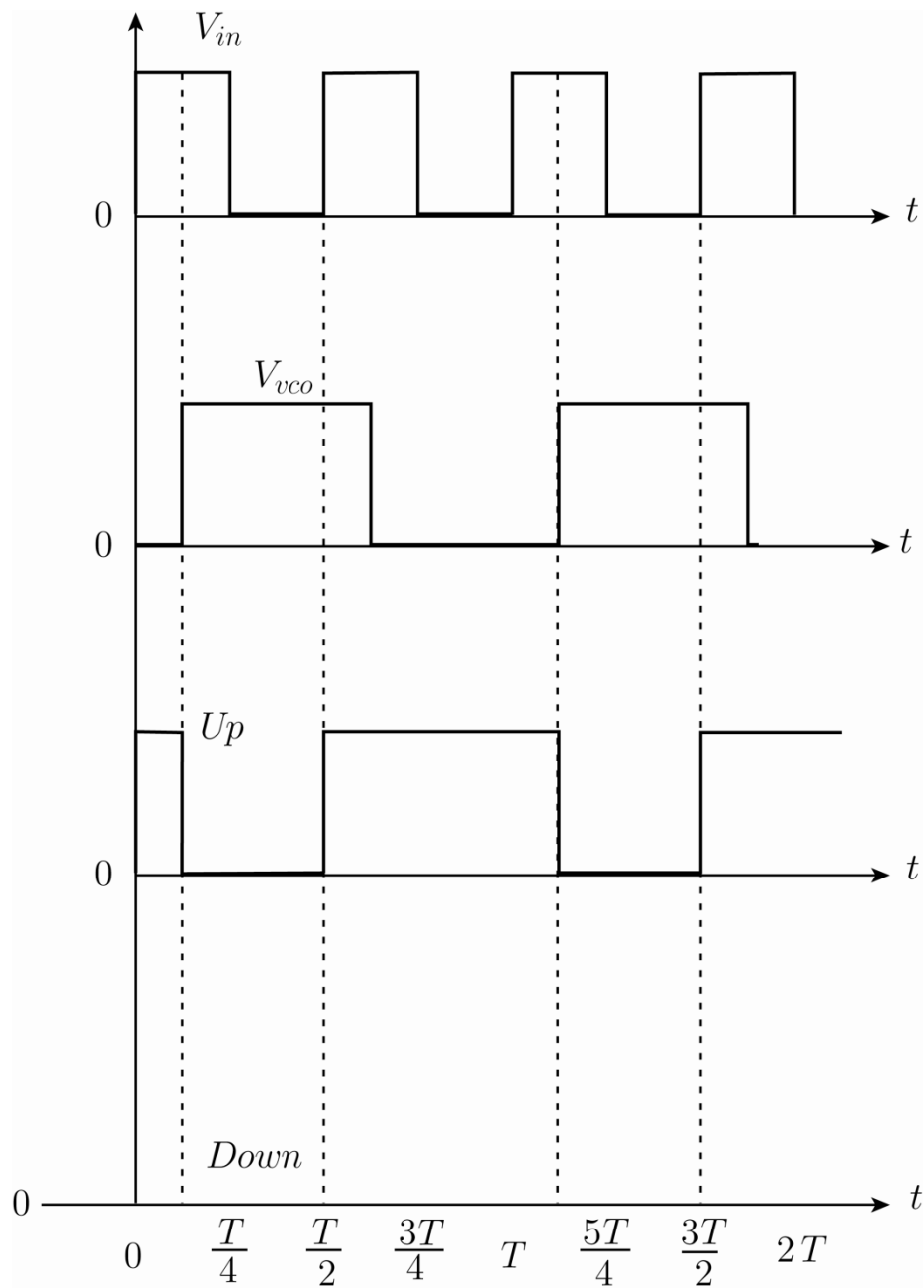
$Up=0$

周波数が等しく， V_{in} が V_{vco} よりも進んでいる場合



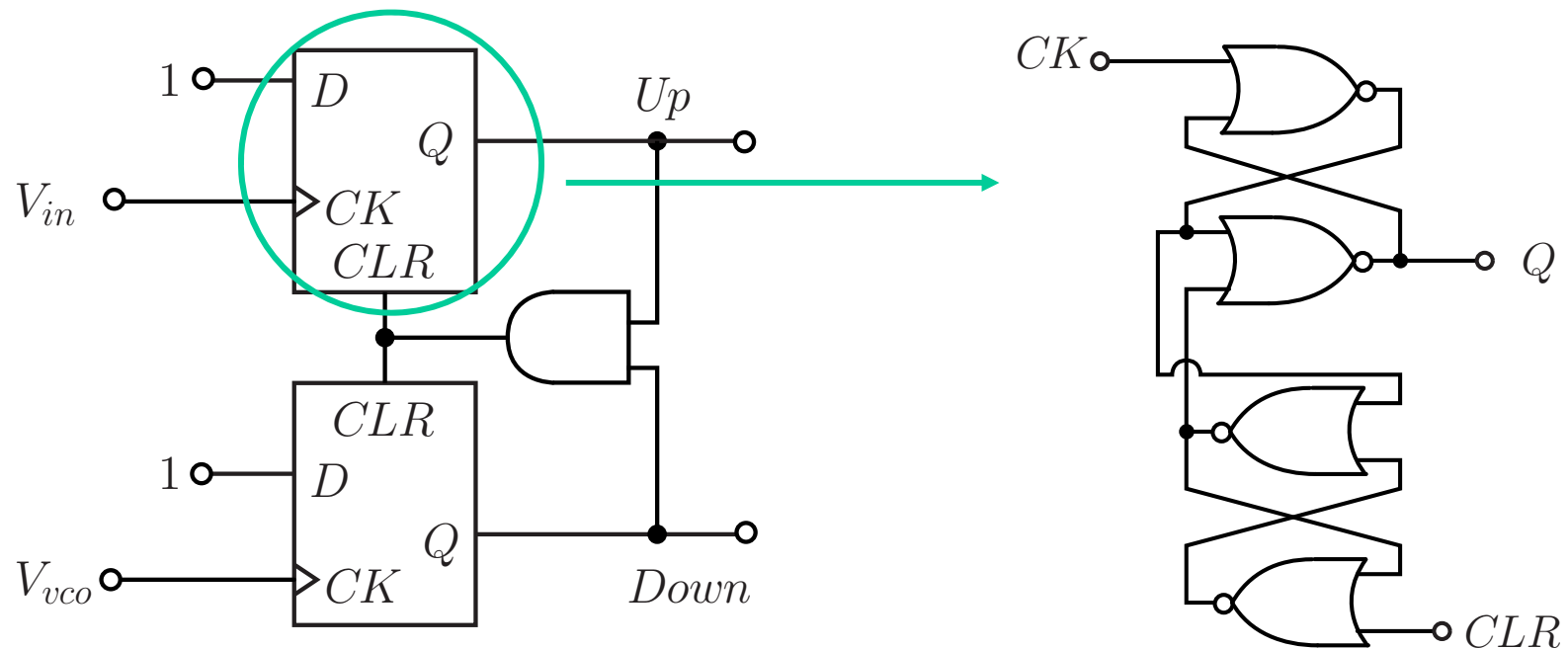
位相差の検出

周波数が異なる場合



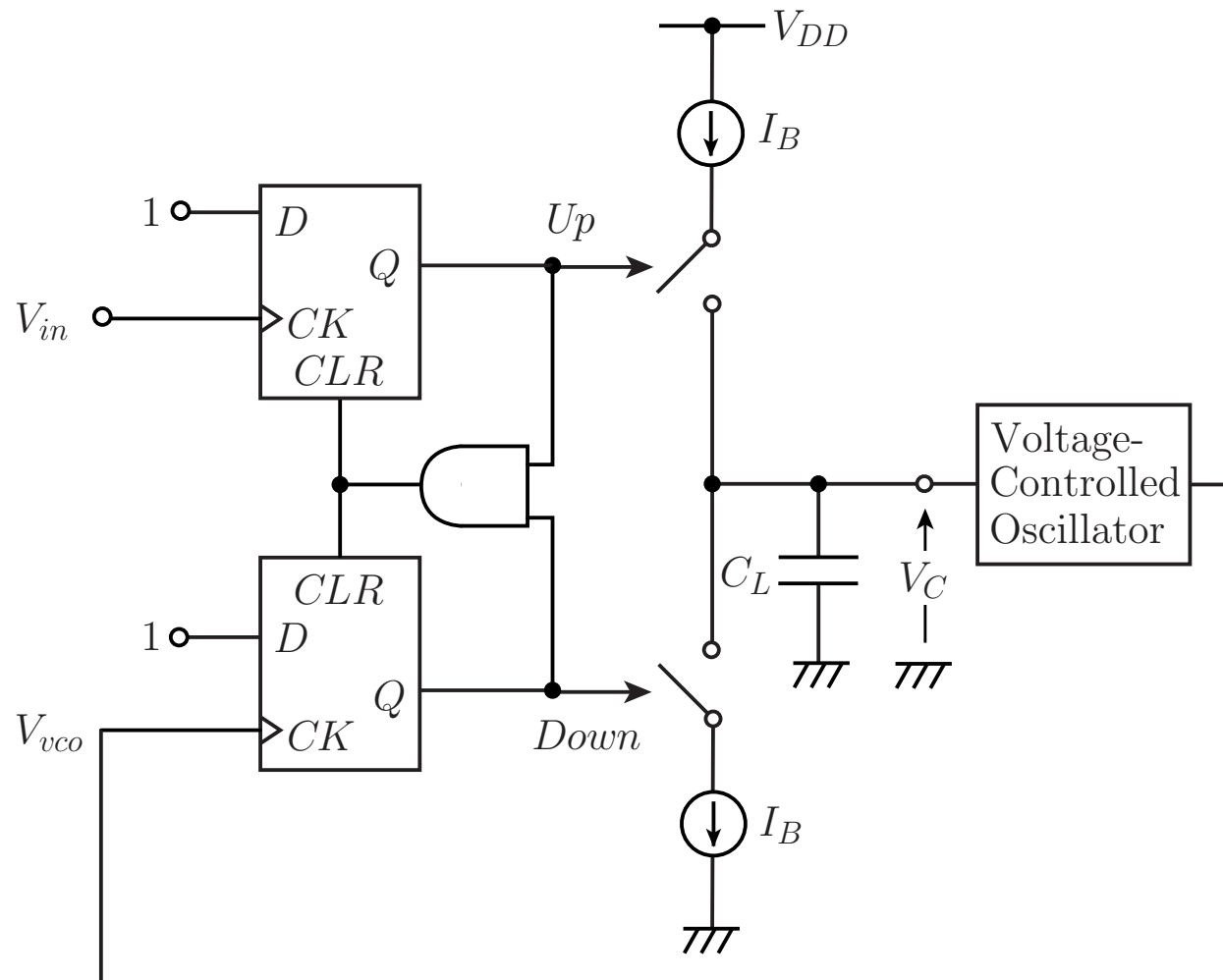
周波数差の検出

周波数・位相差比較回路の構成



$Up = 1, Down = 1$ という状態が存在

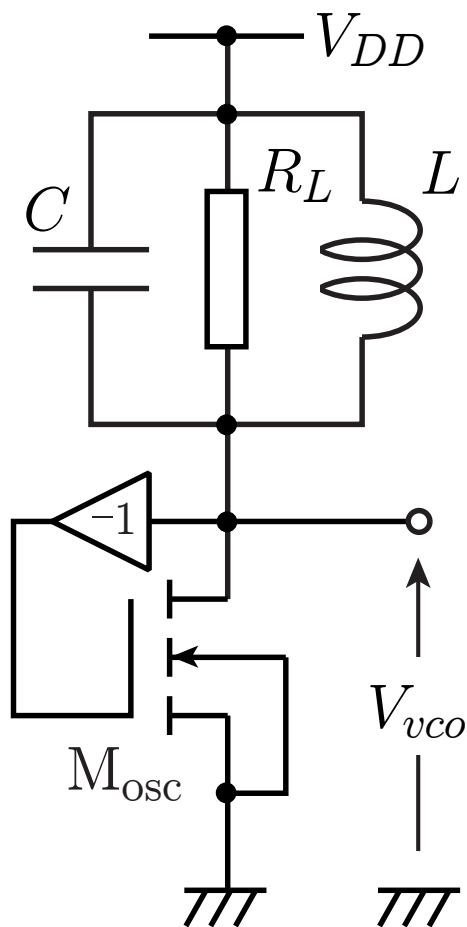
チャージポンプ型PLLの構成



$Up=1, Down=1$ という状態には不感

電圧制御発振回路

LC発振回路



1GHz以上→インダクタの使用が可能

$$f_{res} = \frac{1}{2\pi\sqrt{LC}} \text{ のとき}$$

負荷インピーダンス $Z_{LC} = R_L$

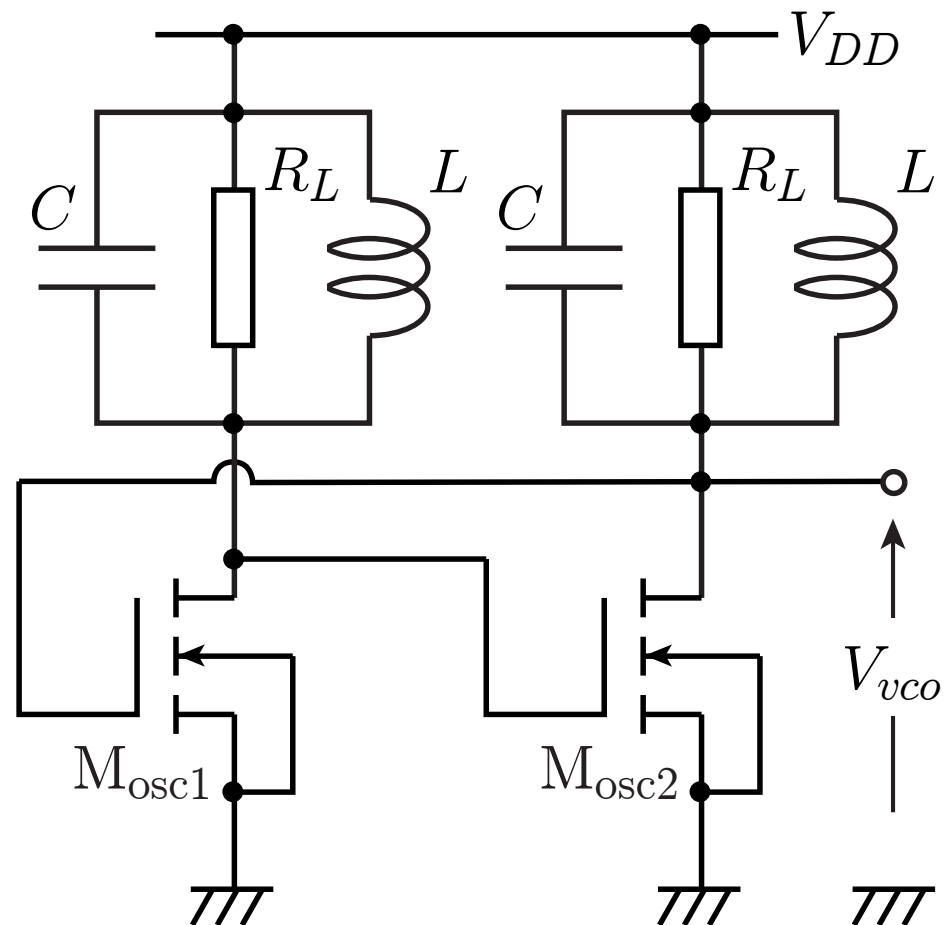
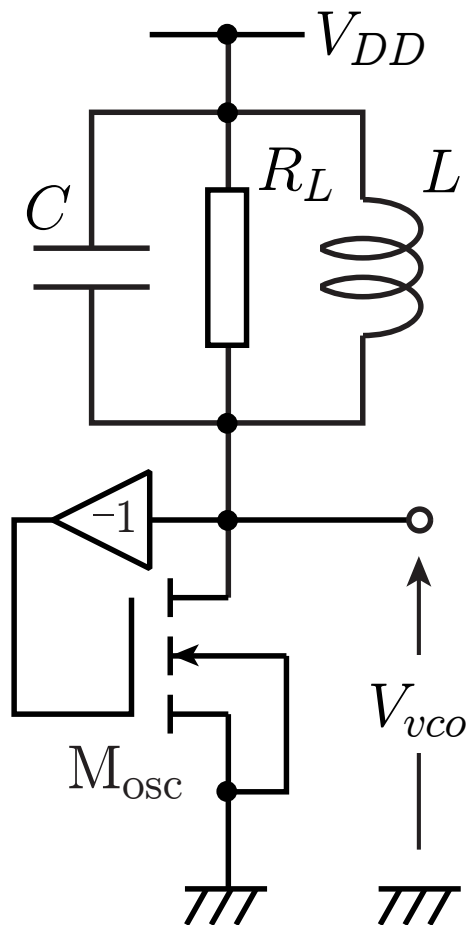
$M_{osc} + R_L \rightarrow \text{Inverter}$

+

-1倍のInverter

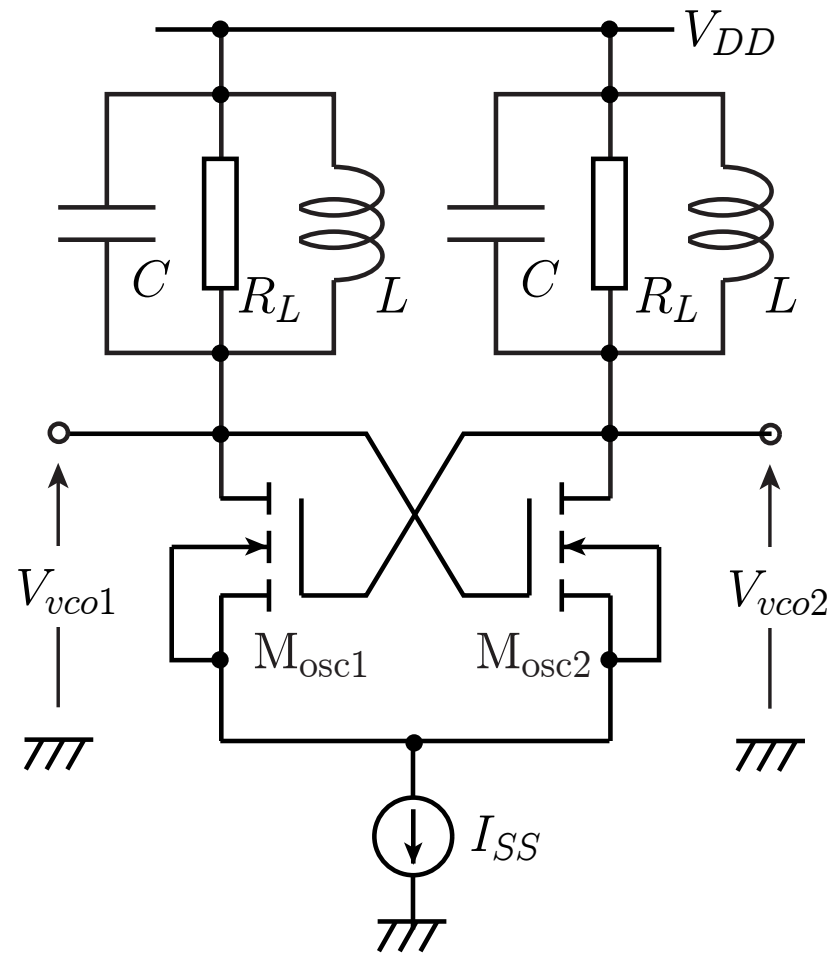
||

正帰還(発振)



消費電力を定めるのが困難

非飽和領域動作の可能性

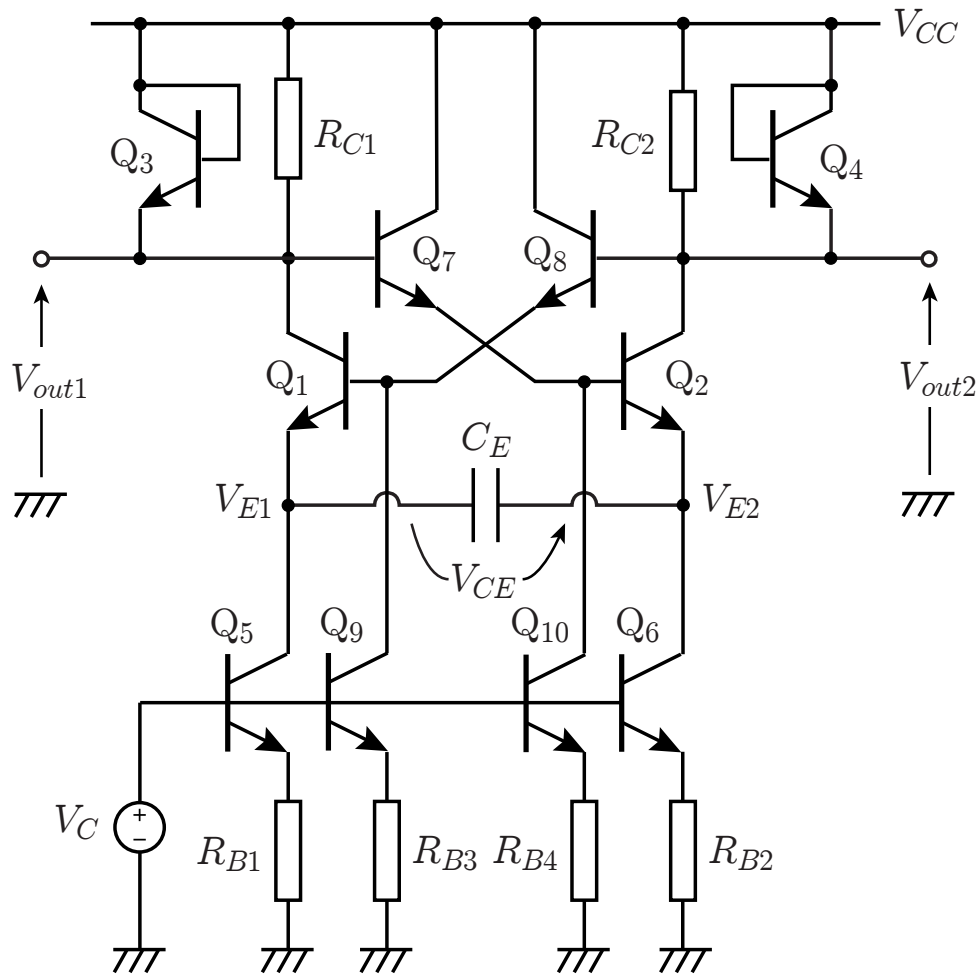


周波数条件: $f_{res} = \frac{1}{2\pi\sqrt{LC}}$

電力条件: $(g_m R_L)^2 \geq 1$

接合容量(可変容量)の使用

電圧制御マルチバイブレータ



Q₁.オン, Q₂.オフ

$$V_{E1} = V_{CC} - 2V_{BE}$$

$$V_{CE} = V_{BE} \rightarrow -V_{BE}$$

$$V_{E2} = V_{CC} - V_{BE} \rightarrow V_{CC} - 3V_{BE}$$



Q₂.オン → Q₁.オフ

$$V_{E2} = V_{CC} - 3V_{BE} \rightarrow V_{CC} - 2V_{BE}$$

$$V_{CE} = -V_{BE}$$

$$V_{E1} = V_{CC} - 2V_{BE} \rightarrow V_{CC} - V_{BE}$$

1 周期: $V_{CE}=V_{BE} \rightarrow -V_{BE} \rightarrow V_{BE}$

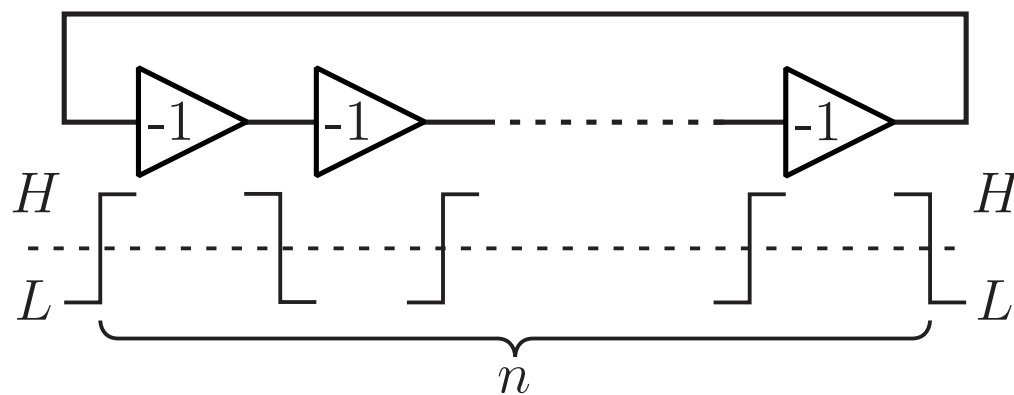
充電電流: $I_E = \frac{V_C - V_{BE}}{R_B}$

周期 T : $4V_{BE} = \frac{I_E}{C_E} T$

$$T = \frac{4V_{BE}C_E}{I_E} = \frac{4C_ER_BV_{BE}}{V_C - V_{BE}}$$

リングオシレータ

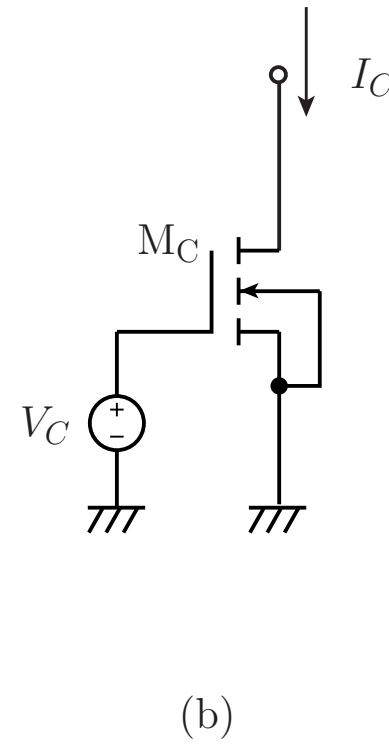
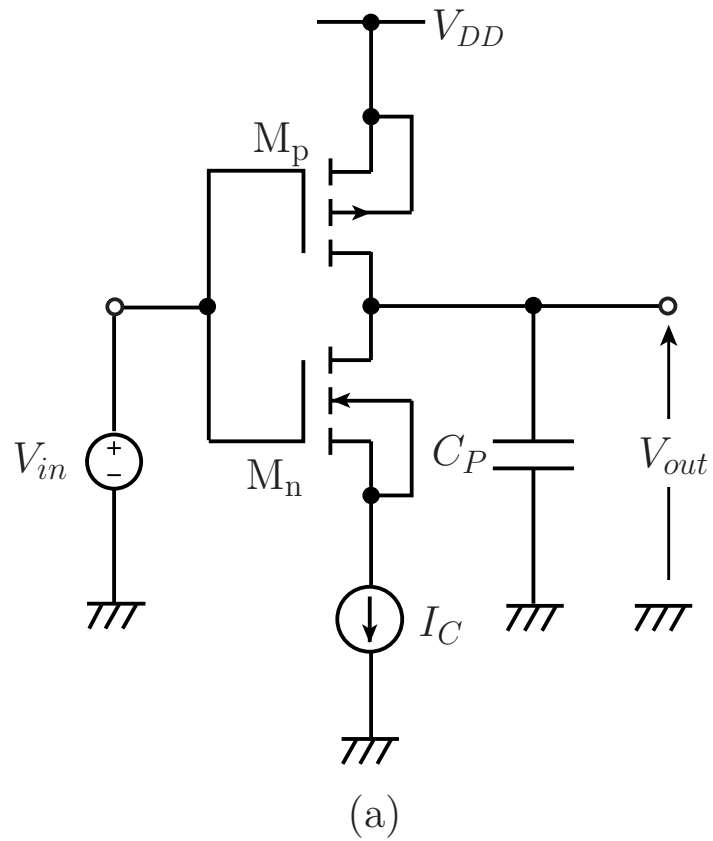
n は奇数



$$t_{delay} = nt_{inv}$$

$$\text{周期 } T = 2t_{delay}$$

インバータの構成



I_C により C_P への充電時間を調整

PLLの応用

復調回路

周波数シンセサイザ

クロック信号の再生

その他

振幅変調信号とその復調

$$\text{搬送波} : V_{carrier}(t) = A_c \cos(2\pi f_{carrier} t + \theta_{carrier})$$

振幅変調

$$A_c \rightarrow K_{AM} S_{org}(t)$$

$$\text{信号} : S_{org}(t) = V_{signal} \cos(2\pi f_{signal} t)$$

変調波 :

$$V_{AM}(t) = K_{AM} V_{signal} \cos(2\pi f_{signal} t) \cos(2\pi f_{carrier} t + \theta_{carrier})$$

$$\begin{aligned}
 V_{AM}(t) &= K_{AM} V_{signal} \cos(2\pi f_{signal} t) \cos(2\pi f_{carrier} t + \theta_{carrier}) \\
 &= \frac{1}{2} K_{AM} V_{signal} [\cos\{2\pi(f_{carrier} - f_{signal})t - \theta_{carrier}\} \\
 &\quad - \cos\{2\pi(f_{carrier} + f_{signal})t + \theta_{carrier}\}]
 \end{aligned}$$

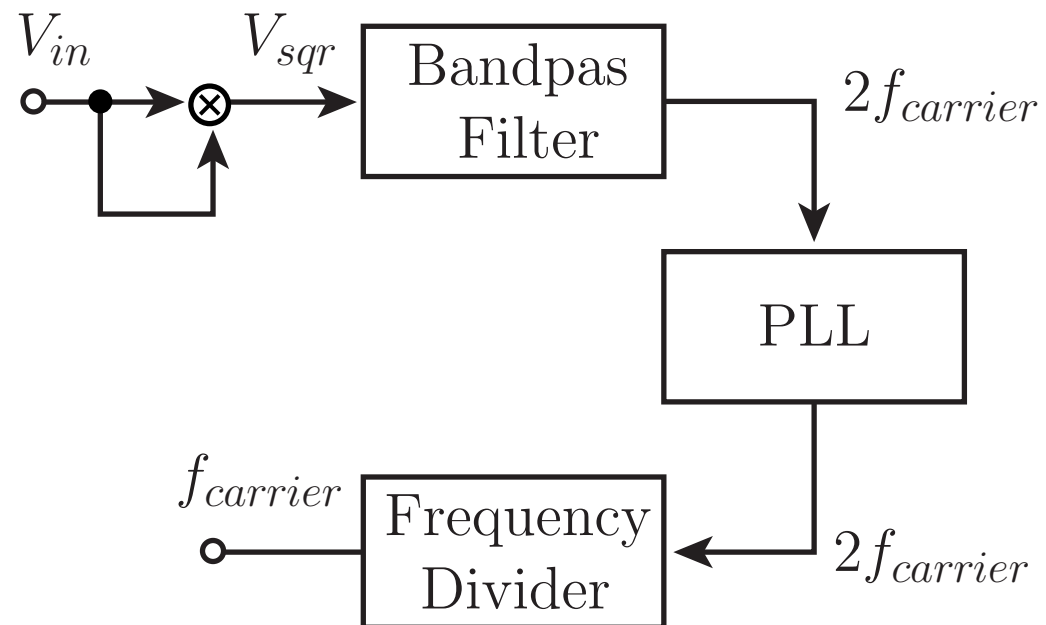
復調

$$f_{carrier} - f_{signal}, \quad f_{carrier} + f_{signal} \times f_{carrier}$$

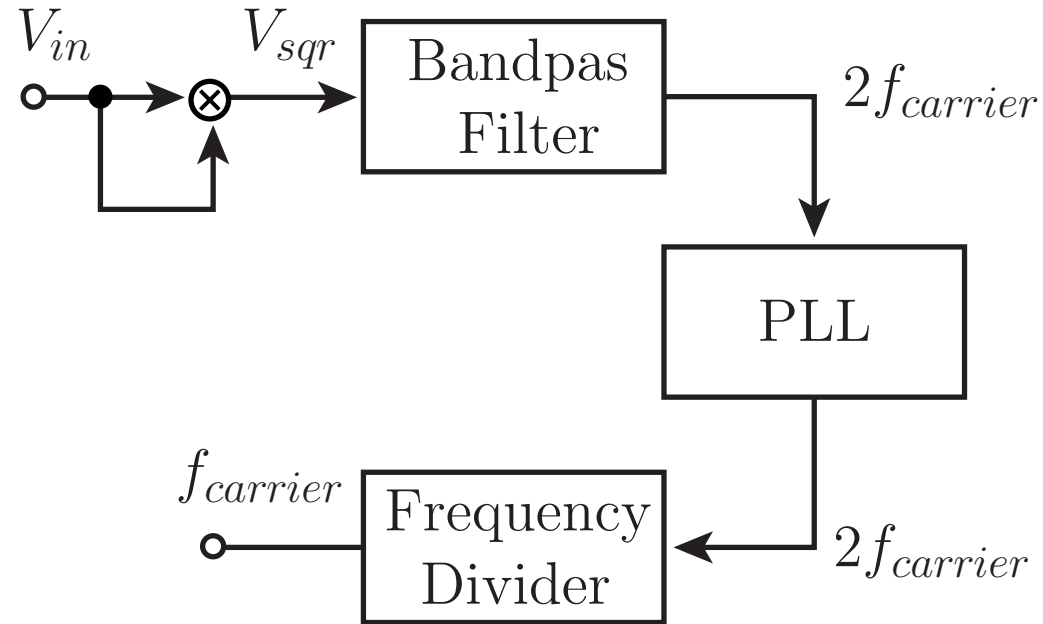
$$\rightarrow f_{signal}, \quad 2f_{carrier} - f_{signal}, \quad 2f_{carrier} + f_{signal}$$

$f_{carrier}$ が未知の場合は？

2乗ループ回路

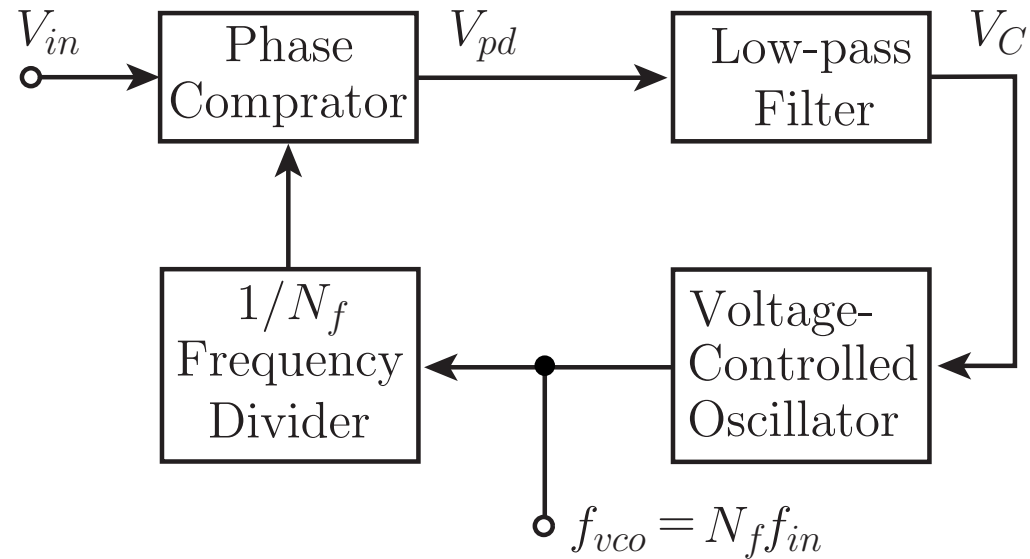


$$V_{sqr}(t) = V_{AM}^2(t)$$



$$\begin{aligned}
 V_{sqr}(t) = & K_{sqr} K_{AM}^2 V_{signal}^2 [\cos\{4\pi(f_{carrier} + f_{signal})t + \theta_{carrier}\} \\
 & + 2\cos(4\pi f_{carrier}t + \theta_{carrier}) \\
 & + 2\cos\{4\pi(f_{carrier} - f_{signal})t + \theta_{carrier}\} \\
 & + 2\cos(4\pi f_{signal}t + \theta_{carrier}) \\
 & + 2\cos\theta_{carrier}] / 8
 \end{aligned}$$

周波数シンセサイザの構成



f_{in} : 基準周波数

$f_{vco} = N_f f_{in}$: 出力周波数

N_f により可変