スイッチトキャパシタフィルタの構成

双一次s-z変換

$$s \rightarrow \frac{2(1-z^{-1})}{T(1+z^{-1})}$$

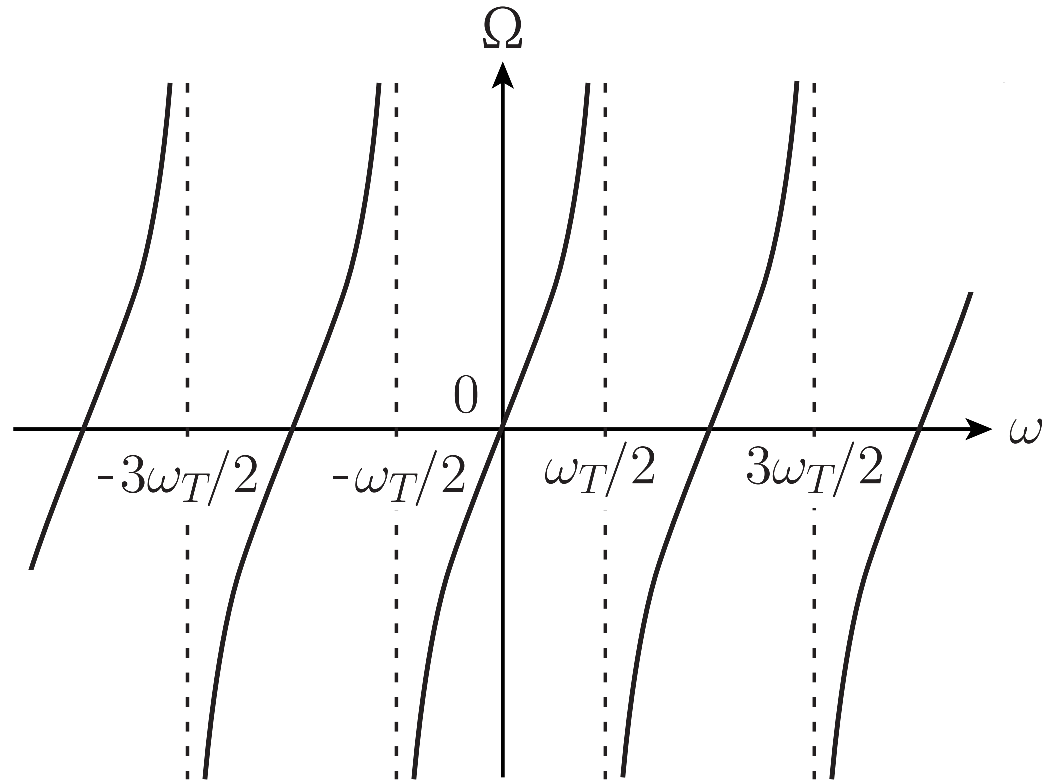
$s=j\Omega,\quad z=e^{j\omega T}$を代入

$$\frac{2(1-z^{-1})}{T(1+z^{-1})}=\frac{2(1-e^{-j\omega T})}{T(1+e^{-j\omega T})}=\frac{2(e^{j\omega T/2}-e^{-j\omega T/2})}{T(e^{j\omega T/2}+e^{-j\omega T/2})}$$

$$=\frac{2\{\cos(\omega T/2)+j\sin(\omega T/2)-\cos(\omega T/2)+j\sin(\omega T/2)\}}{T\{\cos(\omega T/2)+j\sin(\omega T/2)+\cos(\omega T/2)-j\sin(\omega T/2)\}}$$

$$=\frac{4j\sin(\omega T/2)}{2T\cos(\omega T/2)}\boxed{=j\frac{2}{T}\tan\frac{\omega T}{2}}$$

$$\boxed{s \to \frac{2(1-z^{-1})}{T(1+z^{-1})}} \longrightarrow \boxed{\Omega \to \frac{2}{T}\tan(\frac{\omega T}{2})}$$



$$\omega_T = \frac{2\pi}{T}$$

$\omega T << 1$のとき$\Omega = \frac{2}{T}\tan(\frac{\omega T}{2}) \approx \frac{2}{T} \times \frac{\omega T}{2} = \omega$

LDI変換(Lossless Discrete Integrator変換)

$$\boxed{s \to \frac{z^{1/2} - z^{-1/2}}{T}}$$

$s = j\Omega$,　$z = e^{j\omega T}$を代入

$$\frac{z^{1/2} + z^{-1/2}}{T} = \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{T}$$

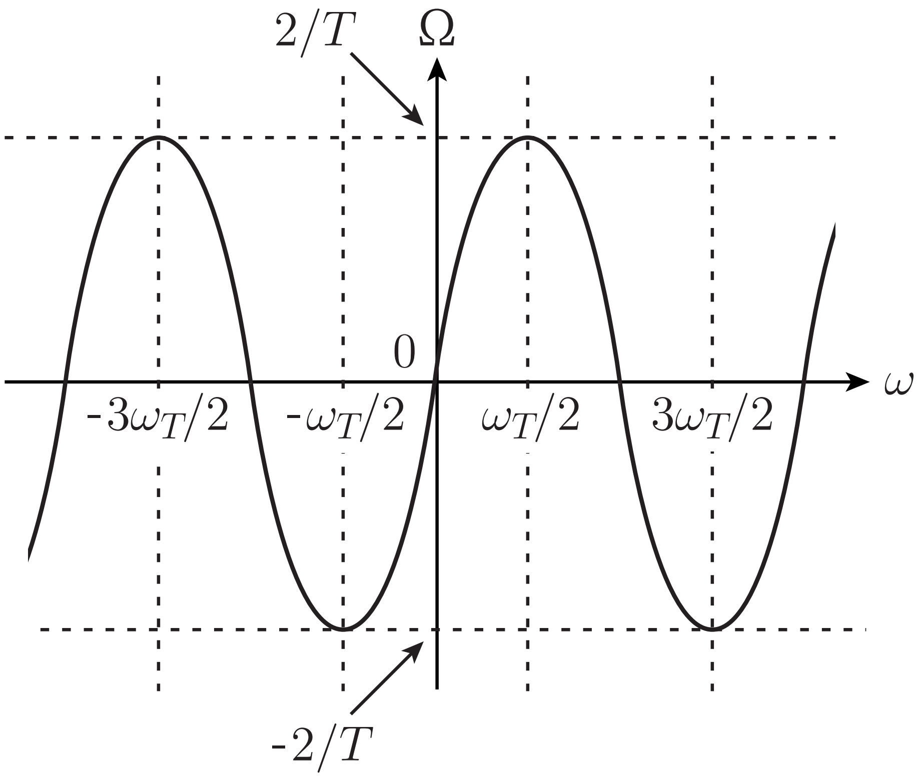
$$= \frac{\{\cos(\omega T/2) + j\sin(\omega T/2)\} - \{\cos(\omega T/2) - j\sin(\omega T/2)\}}{T}$$

$$\boxed{= j\frac{2}{T}\sin(\omega T/2)}$$

$$\Omega \to \frac{2}{T}\sin\left(\frac{\omega T}{2}\right)$$

$$\Omega \rightarrow \frac{2}{T}\sin\left(\frac{\omega T}{2}\right) \approx \omega$$

$(\because x \ll 1$のとき $\sin x \approx x)$



$$\omega_T = \frac{4\pi}{T}$$

Ωの一部がωへ写像

スイッチトキャパシタ2次区間回路

$$T_{second}(s)=\frac{N(s)}{s^2+\frac{\Omega_0}{Q}s+\Omega_0{}^2}$$

＋　双一次s-z変換

$$T_{SC2}(z)=T_{second}\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)=\frac{N\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)}{\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)^2+\frac{\Omega_0}{Q}\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)+\Omega_0{}^2}$$

$$=\frac{N\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)(1+z^{-1})^2}{\frac{4}{T^2}(1-z^{-1})^2+\frac{\Omega_0}{Q}(1-z^{-2})+\Omega_0{}^2(1+z^{-1})^2}$$

$$T_{SC2}(z)=\frac{N\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)(1+z^{-1})^2}{\frac{4}{T^2}(1-z^{-1})^2+\frac{\Omega_0}{Q}(1-z^{-2})+\Omega_0^{\,2}(1+z^{-1})^2}$$

低域通過型 $N(s)=K\Omega_0^{\,2}$ $\boxed{N_{SC2}(z)\propto(1+z^{-1})^2}$

帯域通過型 $N(s)=K\frac{\Omega_0}{Q}s\Rightarrow K\frac{\Omega_0}{Q}\cdot\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}$ $\boxed{N_{SC2}(z)\propto(1-z^{-2})}$

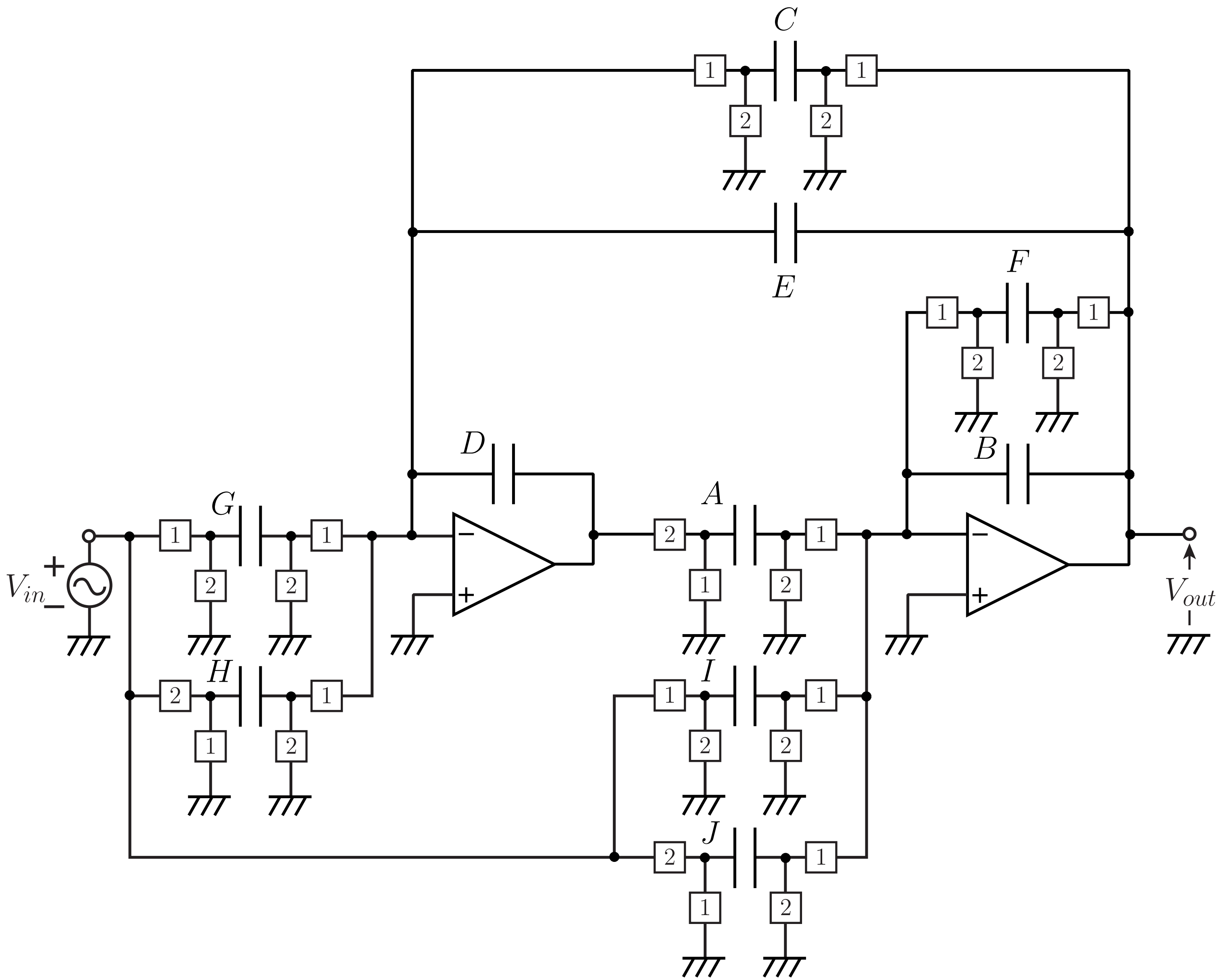
高域通過型 $N(s)=Ks^2\Rightarrow K\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)^2$ $\boxed{N_{SC2}(z)\propto(1-z^{-1})^2}$

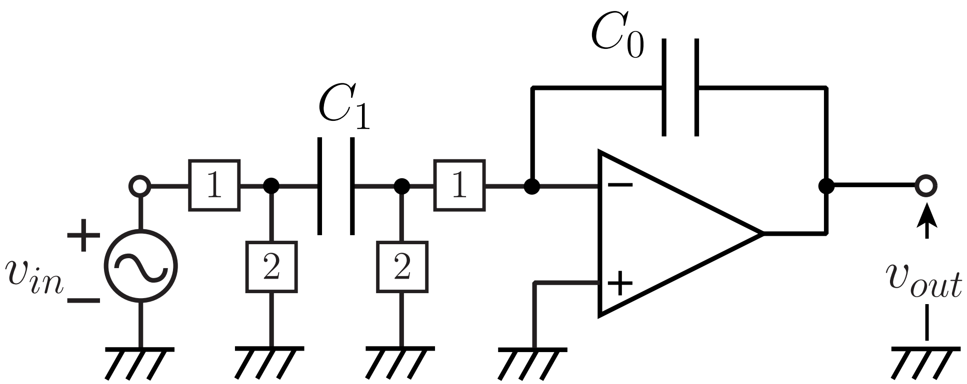
例：低域通過型伝達関数

$$T_{SC2}(z)=\frac{K\Omega_0{}^2}{\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)^2+\frac{\Omega_0}{Q}\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)+\Omega_0{}^2}$$

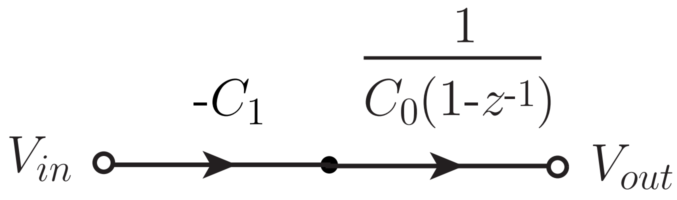
$$=\frac{\frac{K\Omega_0{}^2}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0{}^2}(1+z^{-1})^2}{1-2\frac{\frac{4}{T^2}-\Omega_0{}^2}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0{}^2}z^{-1}+\frac{\frac{4}{T^2}-\frac{2\Omega_0}{TQ}+\Omega_0{}^2}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0{}^2}z^{-2}}$$

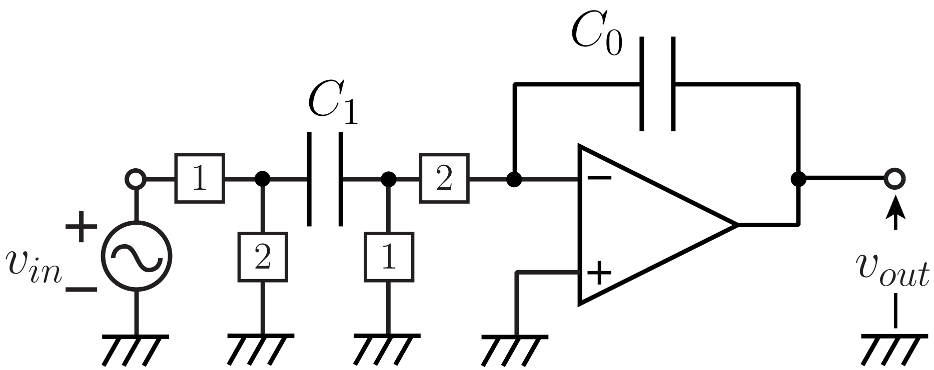
FleischerとLakerの回路



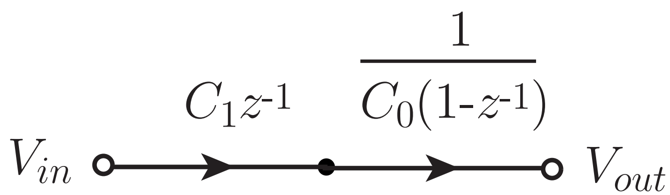


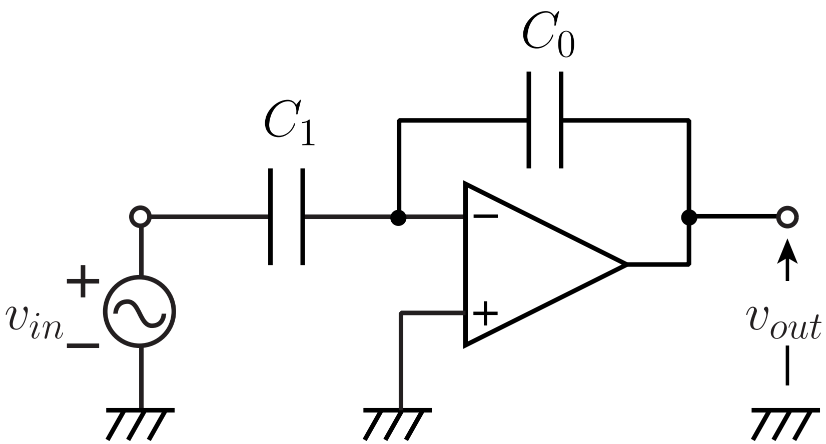
$$\frac{V_{out}(z)}{V_{in}(z)}=\frac{-C_1}{C_0(1-z^{-1})}=-C_1\times\frac{1}{C_0(1-z^{-1})}$$



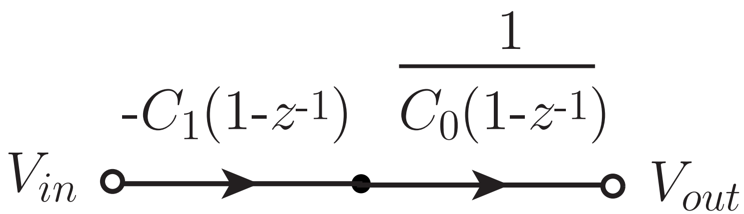


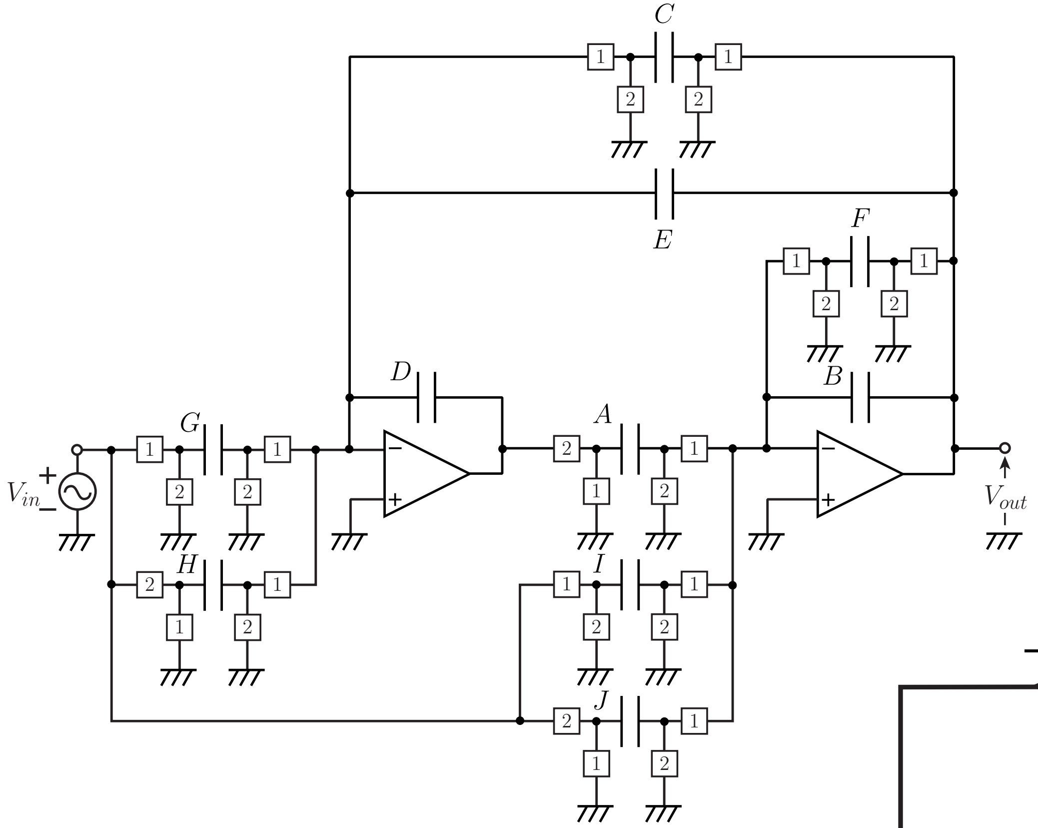
$$\frac{V_{out}(z)}{V_{in}(z)}=\frac{C_1z^{-1}}{C_0(1-z^{-1})}=C_1z^{-1}\times\frac{1}{C_0(1-z^{-1})}$$

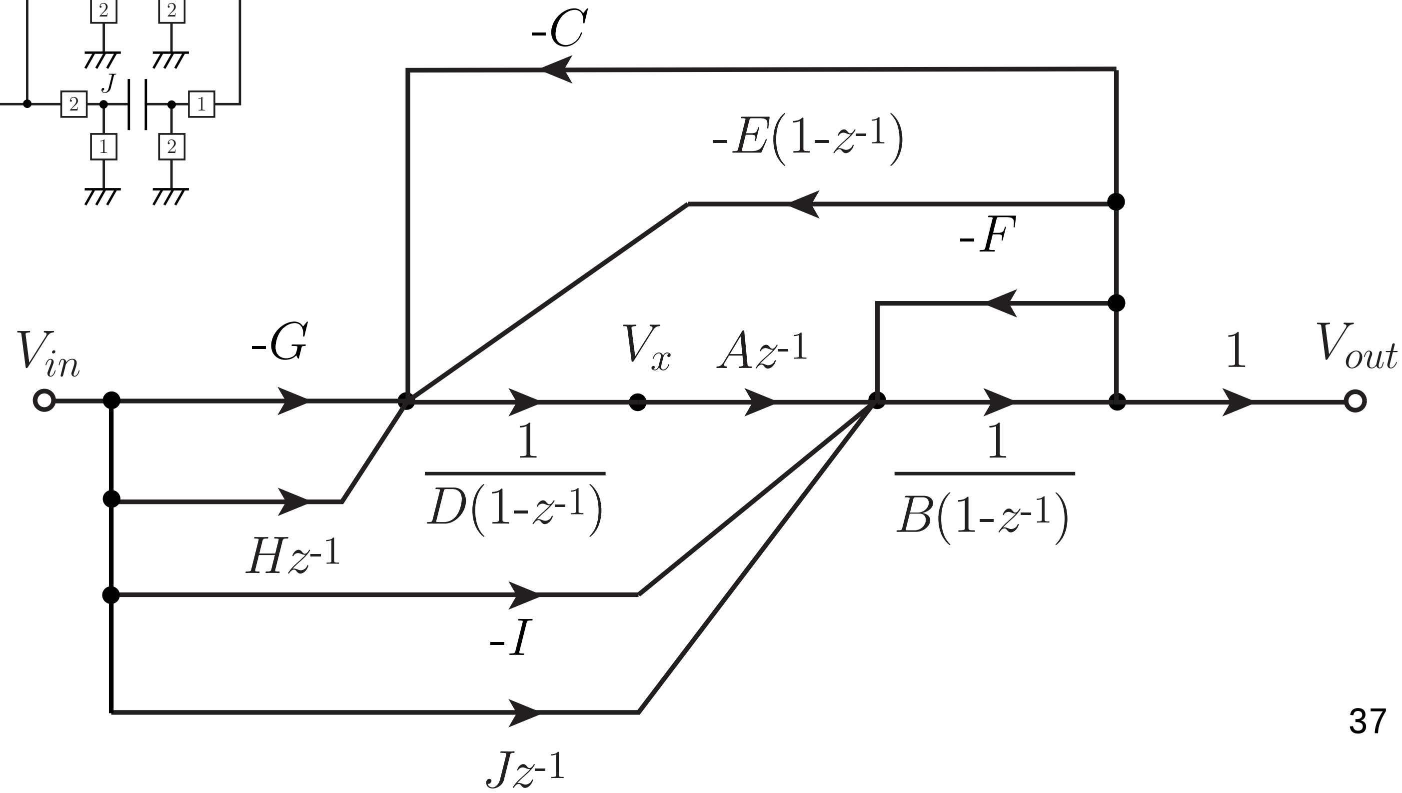


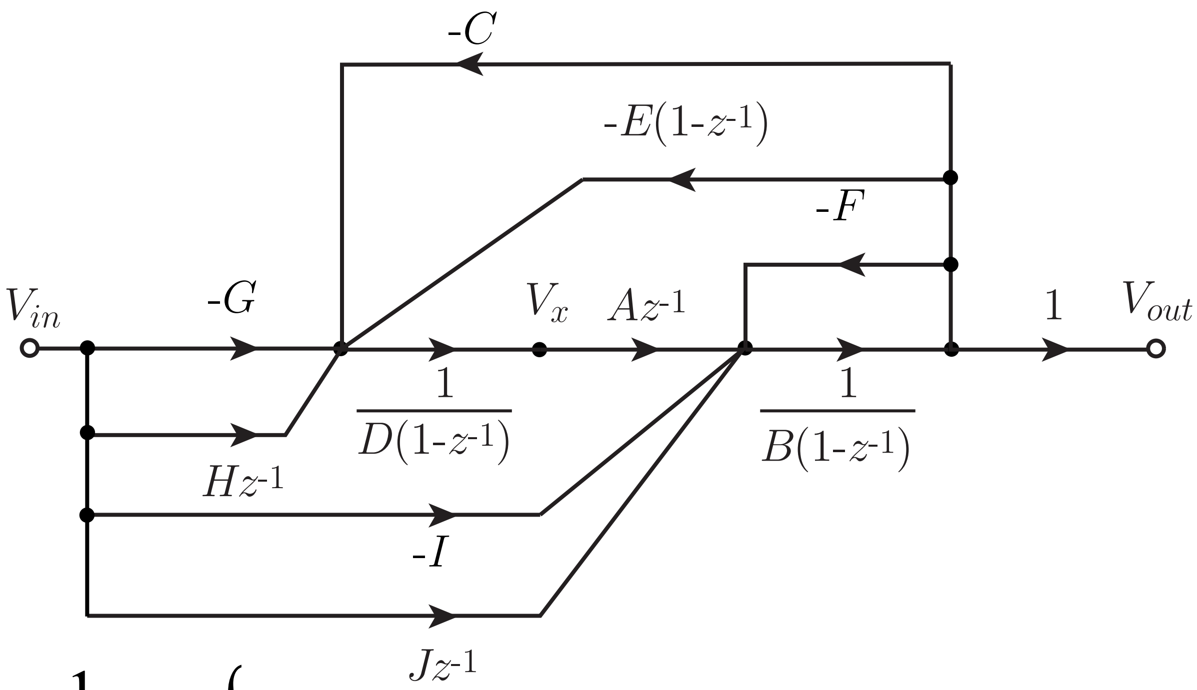


$$\boxed{\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1}{C_0} = -C_1(1-z^{-1}) \times \frac{1}{C_0(1-z^{-1})}}$$




C
E
F
B
D
A
G
H
I
J
V_{in}
V_{out}


-C
-E(1-z-1)
-F
V_{in}
-G
V_x
Az-1
1
V_{out}
1
D(1-z-1)
1
B(1-z-1)
Hz-1
-I
Jz-1

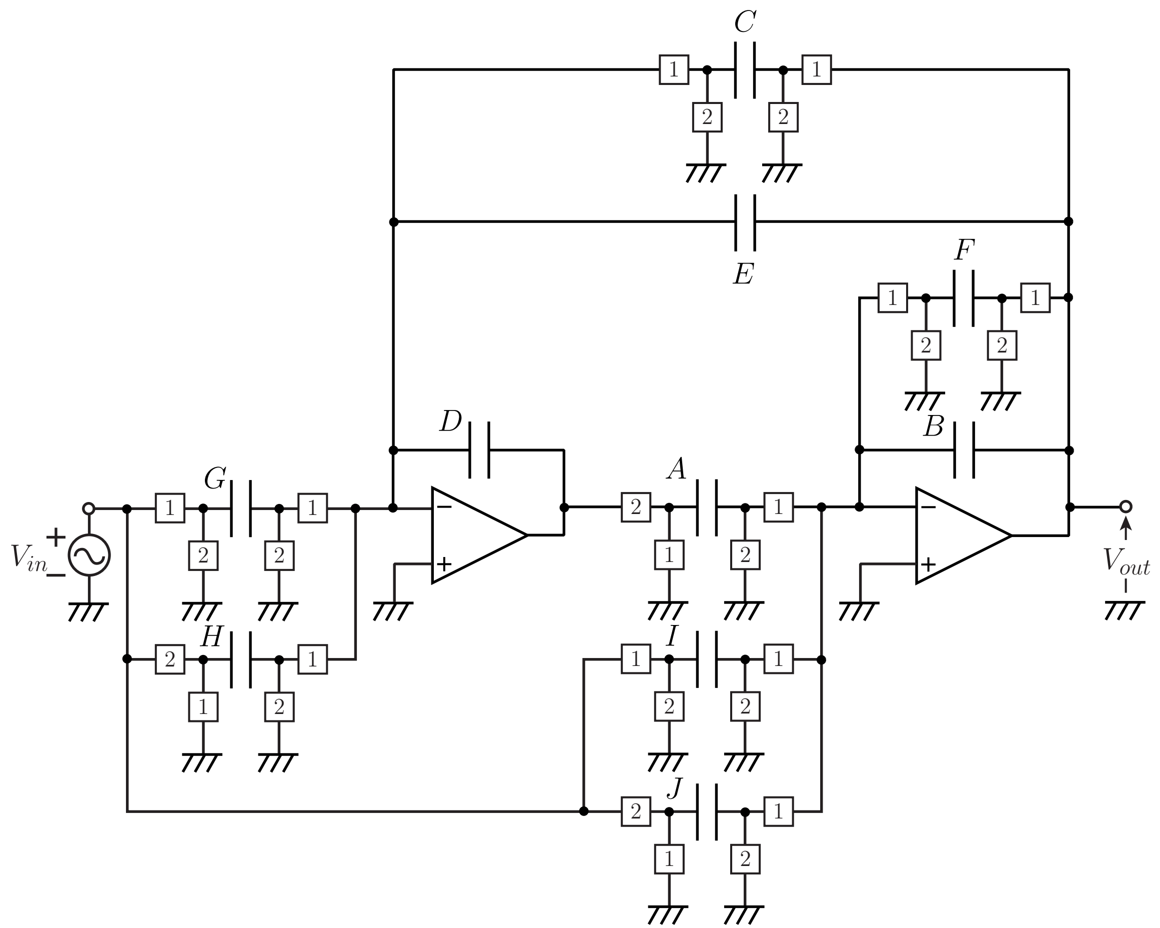


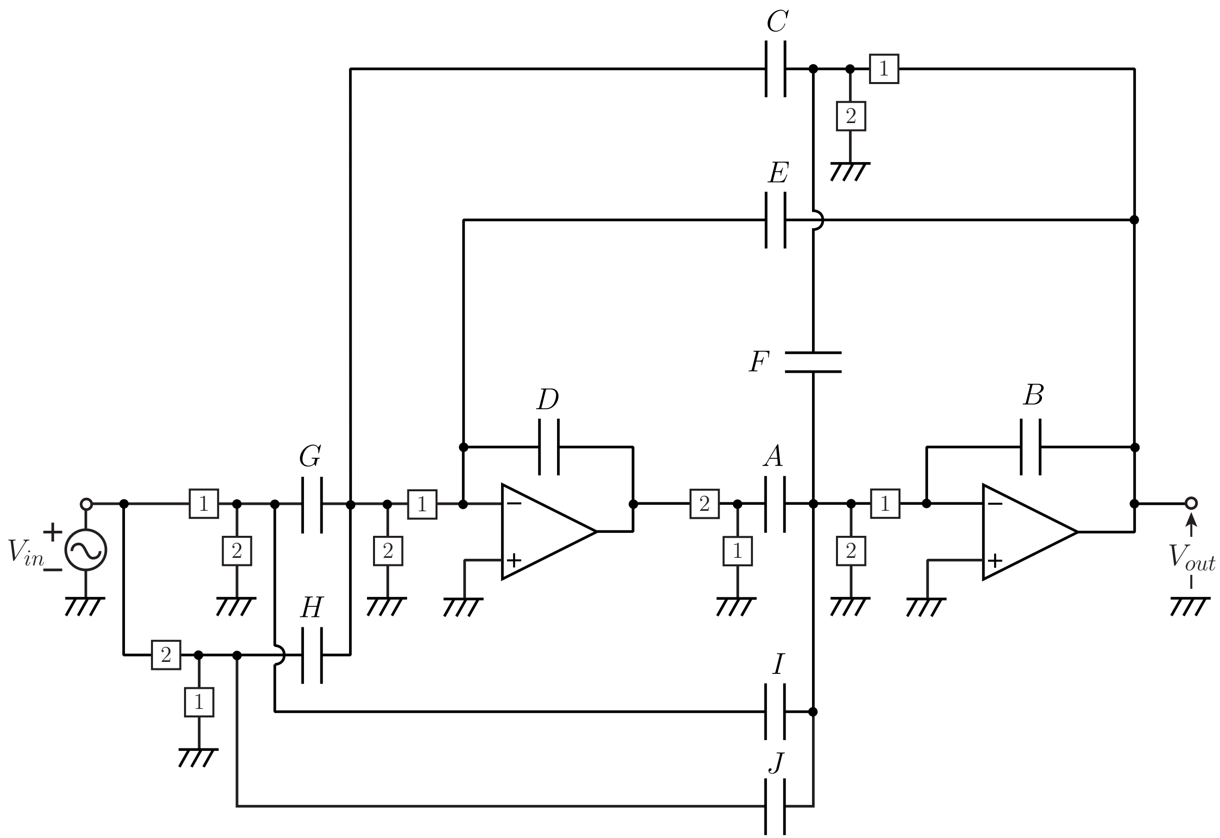
$$V_x = \frac{1}{D(1-z^{-1})}\left\{-GV_{in} + Hz^{-1}V_{in} - CV_{out} - E(1-z^{-1})V_{out}\right\}$$

$$V_{out} = \frac{1}{B(1-z^{-1})}\left\{Az^{-1}V_x - IV_{in} + Jz^{-1}V_{in} - FV_{out}\right\}$$

$$\boxed{T_{FL}(z) = -\frac{DI + (AG - DI - DJ)z^{-1} + (DJ - AH)z^{-2}}{D(B+F) + (AC + AE - DF - 2BD)z^{-1} + (BD - AE)z^{-2}}}$$

スイッチの共有




C
1
2
E
F
D
B
G
A
1
1
2
1
2
2
2
1
2
V_in
H
2
I
1
J
V_out

$$T_{FL}(z) = -\frac{DI + (AG - DI - DJ)z^{-1} + (DJ - AH)z^{-2}}{D(B+F) + (AC + AE - DF - 2BD)z^{-1} + (BD - AE)z^{-2}}$$

特性:容量比で決定 → 規格化 → $A = B = D = 1$

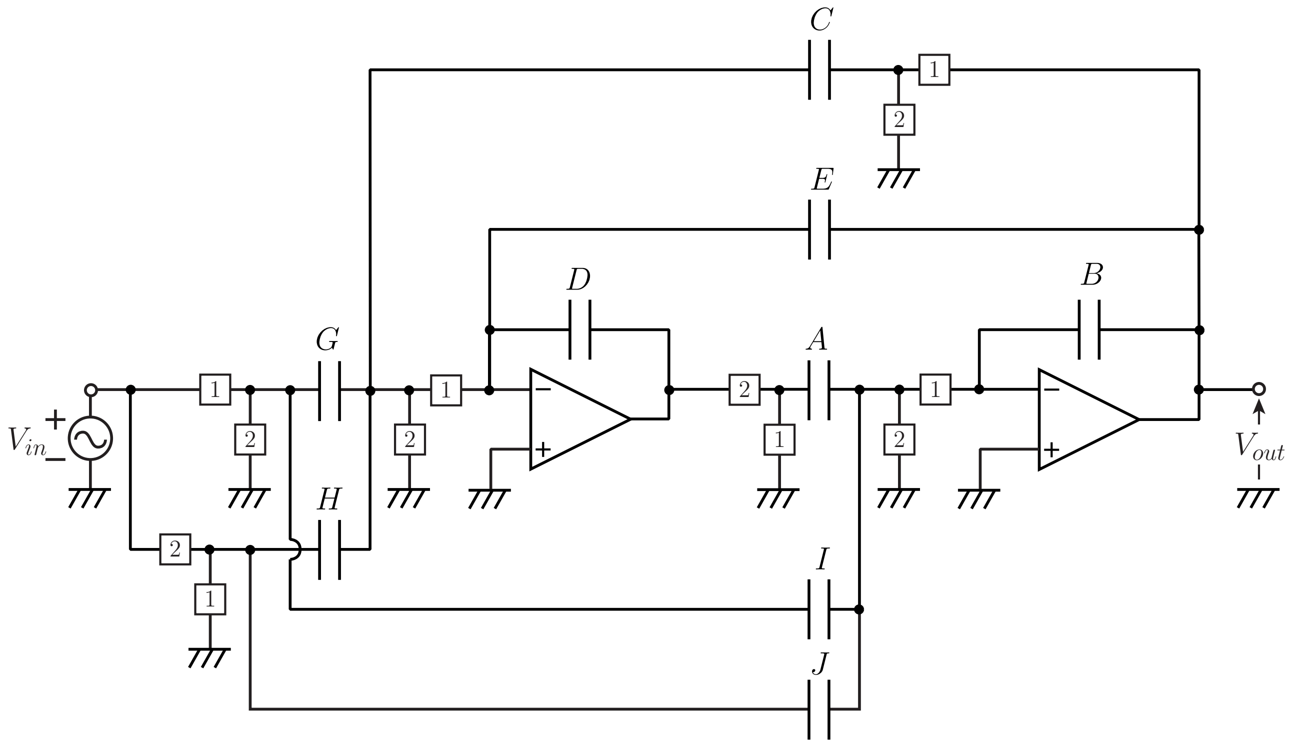
$$\boxed{T_{FL}(z) = \frac{I + (G - I - J)z^{-1} + (J - H)z^{-2}}{1 + F + (C + E - F - 2)z^{-1} + (1 - E)z^{-2}}}$$

安定性:分母多項式の根は単位円内

$$1 + F > 1 - E \qquad F = 0 \text{ or } E = 0$$

例： $F=0$

$$\boxed{T_{FL}(z)=\frac{I+(G-I-J)z^{-1}+(J-H)z^{-2}}{1+(C+E-2)z^{-1}+(1-E)z^{-2}}}$$



FleischerとLakerのE回路

$$T_{SC2}(z)=\frac{\dfrac{K\Omega_0{}^2}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}(1+z^{-1})^2}{1-2\dfrac{\dfrac{4}{T^2}-\Omega_0{}^2}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}z^{-1}+\dfrac{\dfrac{4}{T^2}-\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}z^{-2}}$$

$$T_{FL}(z)=\frac{I+(G-I-J)z^{-1}+(J-H)z^{-2}}{1+(C+E-2)z^{-1}+(1-E)z^{-2}}$$

$$\boxed{E=\frac{\dfrac{4\Omega_0}{TQ}}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}}$$

$$\boxed{C=\frac{4\Omega_0{}^2}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}}$$

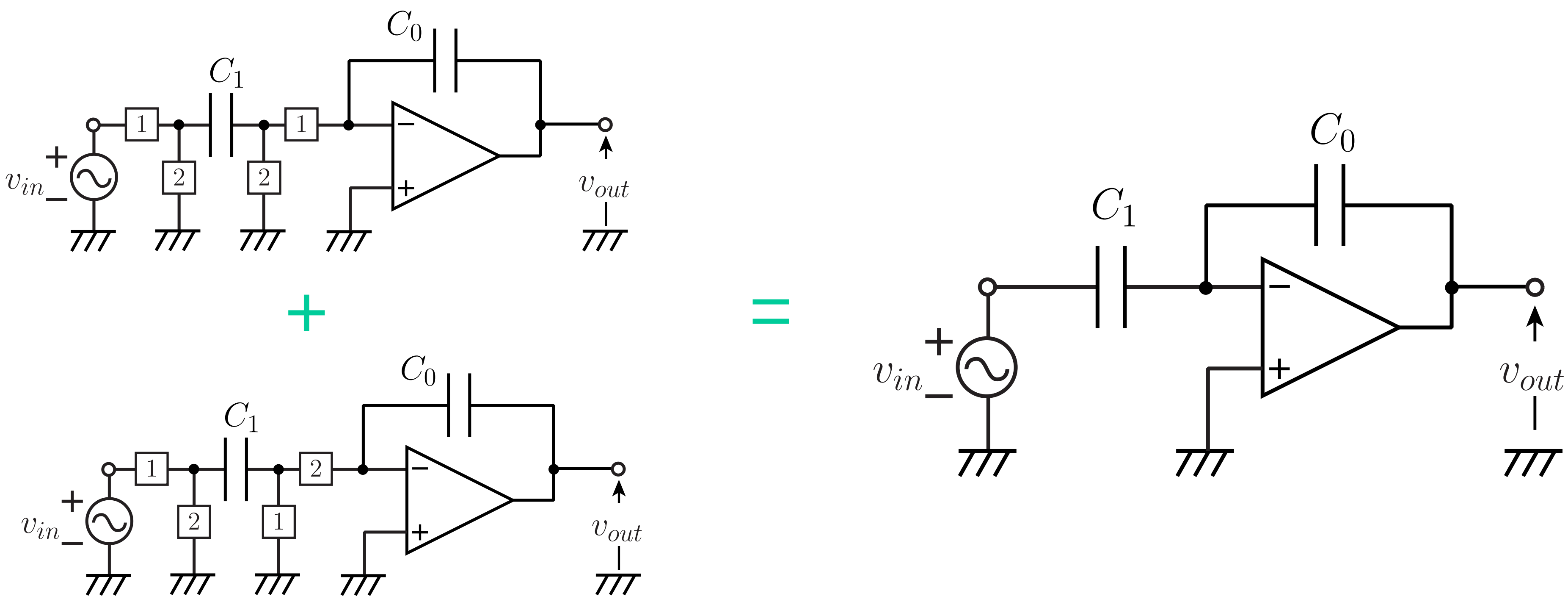
$$T_{FL}(z) = \frac{I + (G - I - J)z^{-1} + (J - H)z^{-2}}{1 + (C + E - 2)z^{-1} + (1 - E)z^{-2}}$$

$$T_{SC2}(z) = \frac{\dfrac{K\Omega_0{}^2}{\dfrac{4}{T^2} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2}(1 + 2z^{-1} + z^{-2})}{1 - 2\dfrac{\dfrac{4}{T^2} - \Omega_0{}^2}{\dfrac{4}{T^2} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2}z^{-1} + \dfrac{\dfrac{4}{T^2} - \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2}{\dfrac{4}{T^2} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2}z^{-2}}$$

$$I = \frac{K\Omega_0{}^2}{\dfrac{4}{T^2} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2} \quad J = \frac{K\Omega_0{}^2}{\dfrac{4}{T} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2} \quad H = 0 \quad G = \frac{4K\Omega_0{}^2}{\dfrac{4}{T^2} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2}$$

$$A = B = D = 1 \quad C = \frac{4{\Omega_0}^2}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + {\Omega_0}^2} \quad E = \frac{\frac{4\Omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + {\Omega_0}^2}$$

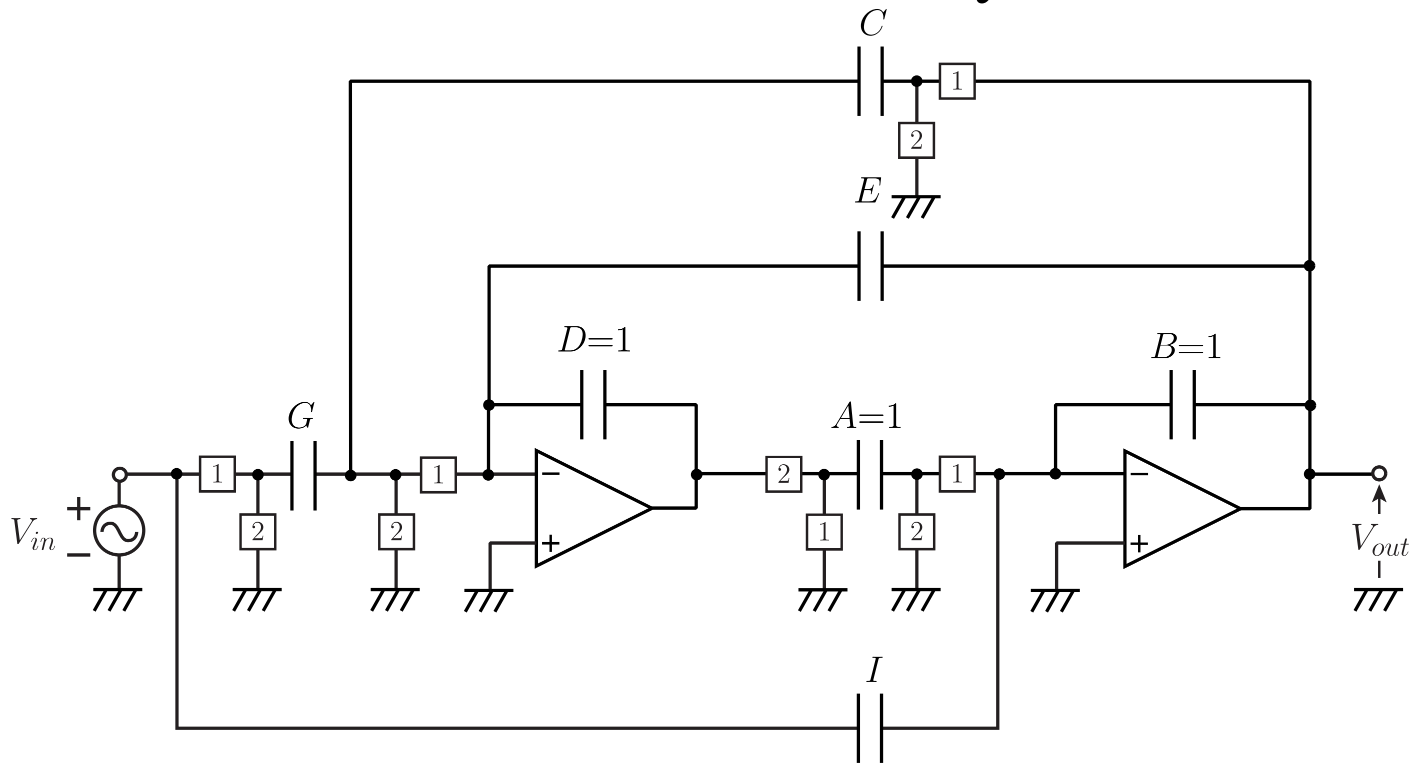
$$I = J = \frac{K{\Omega_0}^2}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + {\Omega_0}^2} \quad G = \frac{4K{\Omega_0}^2}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + {\Omega_0}^2} \quad H = 0$$



$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1}{C_0(1-z^{-1})} + \frac{C_1 z^{-1}}{C_0(1-z^{-1})} = \frac{-C_1(1-z^{-1})}{C_0(1-z^{-1})} = \frac{-C_1}{C_0}$$

$$A = B = D = 1 \qquad C = \frac{4{\Omega_0}^2}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + {\Omega_0}^2} \qquad E = \frac{\frac{4\Omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + {\Omega_0}^2}$$

$$I = J = \frac{K{\Omega_0}^2}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + {\Omega_0}^2} \qquad G = \frac{4K{\Omega_0}^2}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + {\Omega_0}^2} \qquad H = 0$$



帯域通過型関数:

$$T_{SC2}(z)=T_{second}\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)=\frac{N\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)}{\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)^2+\frac{\omega_0}{Q}\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)+\omega_0{}^2}$$

$$=\frac{\dfrac{\frac{2K\Omega_0}{TQ}}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0{}^2}(1-z^{-2})}{1-2\dfrac{\frac{4}{T^2}-\Omega_0{}^2}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0{}^2}z^{-1}+\dfrac{\frac{4}{T^2}-\frac{2\Omega_0}{TQ}+\Omega_0{}^2}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0{}^2}z^{-2}}$$

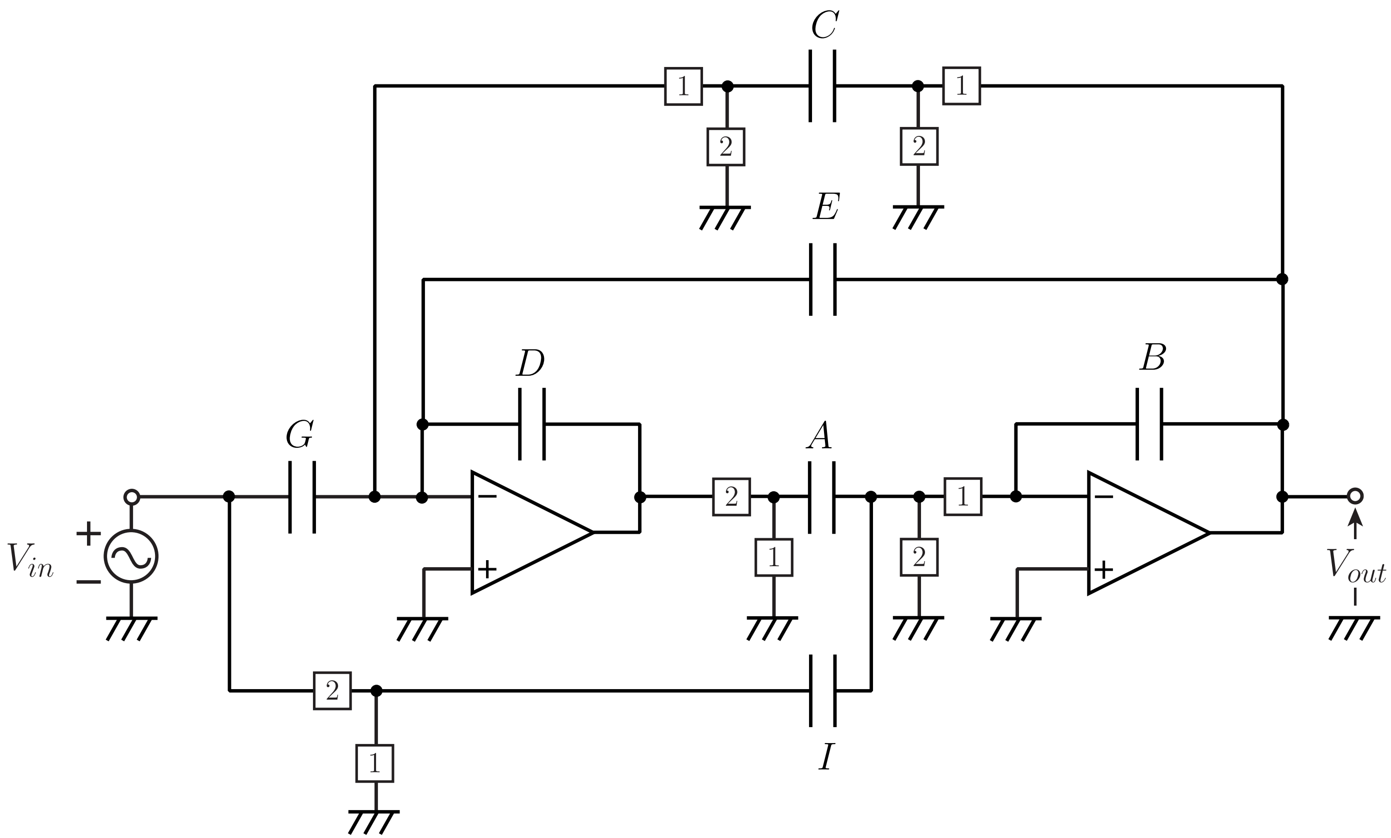
$$T_{SC2}(z)=\frac{\dfrac{\dfrac{2K\Omega_0}{TQ}}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}(1-z^{-2})}{1-2\dfrac{\dfrac{4}{T^2}-\Omega_0{}^2}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}z^{-1}+\dfrac{\dfrac{4}{T^2}-\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}z^{-2}}$$

$A=B=D=1$とすると

$$\boxed{T_{FL}(z)=\frac{I+(G-I-J)z^{-1}+(J-H)z^{-2}}{1+(C+E-2)z^{-1}+(1-E)z^{-2}}}$$

$$C=\frac{4\Omega_0{}^2}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}\quad E=\frac{\dfrac{4\Omega_0}{TQ}}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}\quad I=G=H=\frac{\dfrac{2K\Omega_0}{TQ}}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}\quad J=0$$

2次帯域通過フィルタ



例題　中心周波数1kHz，Q=5，T=1/20000s，K=1

$$\boxed{\Omega \to \frac{2}{T}\tan\left(\frac{\omega T}{2}\right)}$$

$$\Omega_0 = 40000\tan\left(\frac{2\pi\times 1000}{2}\times\frac{1}{20000}\right) = 2\pi\times 1008$$

$$T(z^{-1}) = \frac{\dfrac{2K\Omega_0}{TQ}(1-z^{-2})}{\left(\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\omega_0{}^2\right) - 2\left(\dfrac{4}{T^2}-\Omega_0{}^2\right)z^{-1} + \left(\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2\right)z^{-2}}$$

$$= \frac{0.0300(1-z^{-2})}{1-1.85z^{-1}+0.940z^{-2}}$$

$$\boxed{\Omega_0 = 2\pi\times1008,\quad Q=5,\quad T=1/20000\text{s},\quad K=1}$$

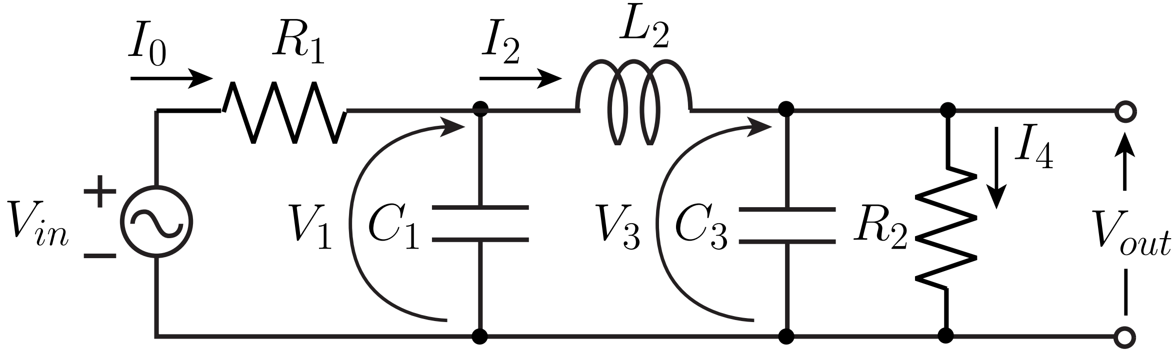
$$A=B=D=1 \qquad \to 33.3\text{pF}$$

$$C=\frac{4\Omega_0^{\ 2}}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^{\ 2}}=0.0950 \qquad \to 3.17\text{pF}$$

$$E=\frac{\frac{4\Omega_0}{TQ}}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^{\ 2}}=0.0600 \qquad \to 2.00\text{pF}$$

$$I=G=H=\frac{K\frac{2\Omega_0}{TQ}}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^{\ 2}}=0.0300 \qquad \to 1.00\text{pF}$$

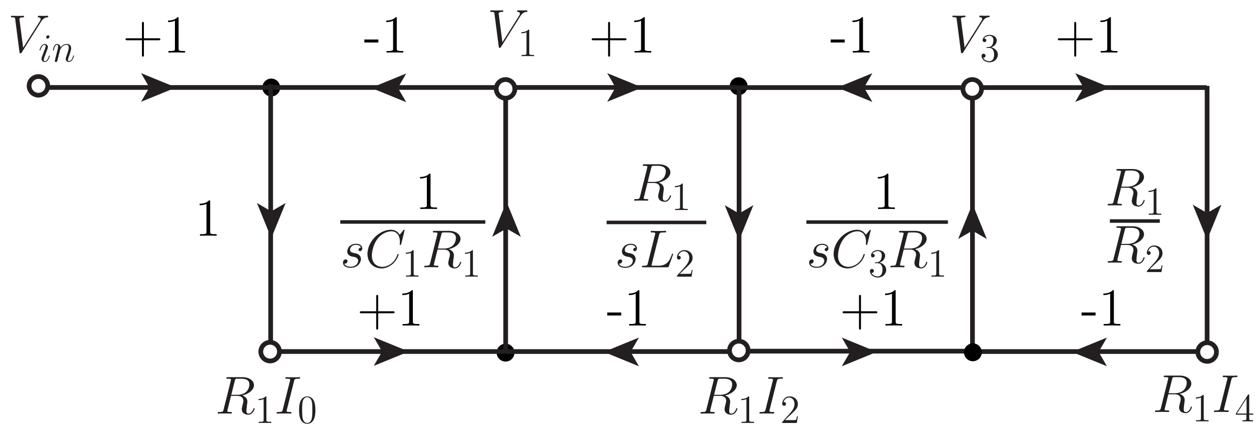
スイッチトキャパシタ回路による リープフロッグ・シミュレーション



$$R_1 I_0 = V_{in} - V_1$$

$$V_1 = \frac{1}{sC_1R_1}(R_1I_0 - R_1I_2)$$

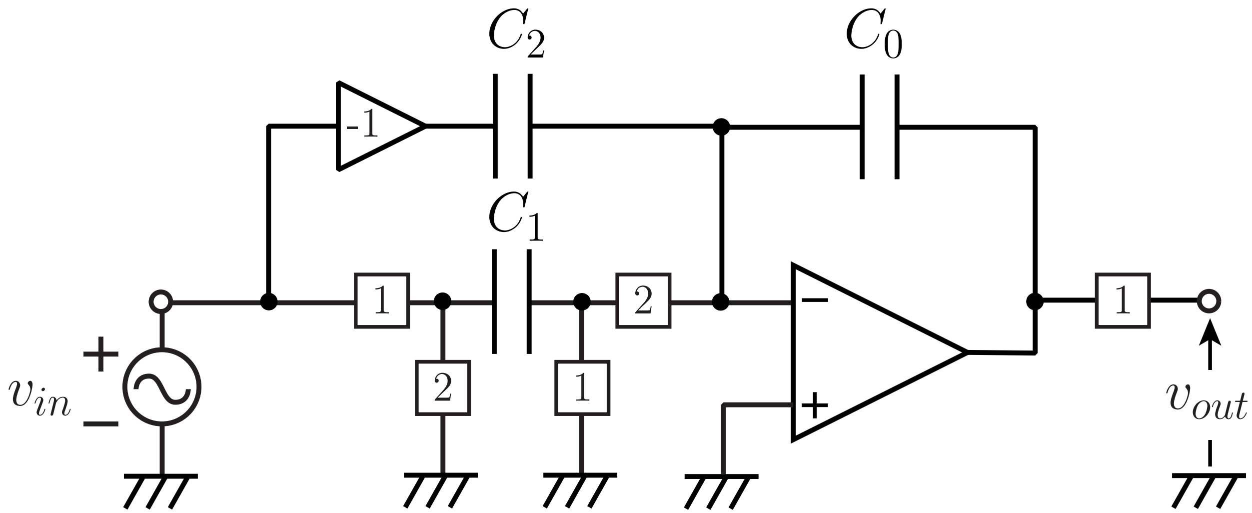
$$R_1I_2 = \frac{R_1}{sL_2}(V_1 - V_3) \quad V_3 = \frac{1}{sC_3R_1}(R_1I_2 - R_1I_4) \quad R_1I_4 = \frac{R_1}{R_2}V_3$$



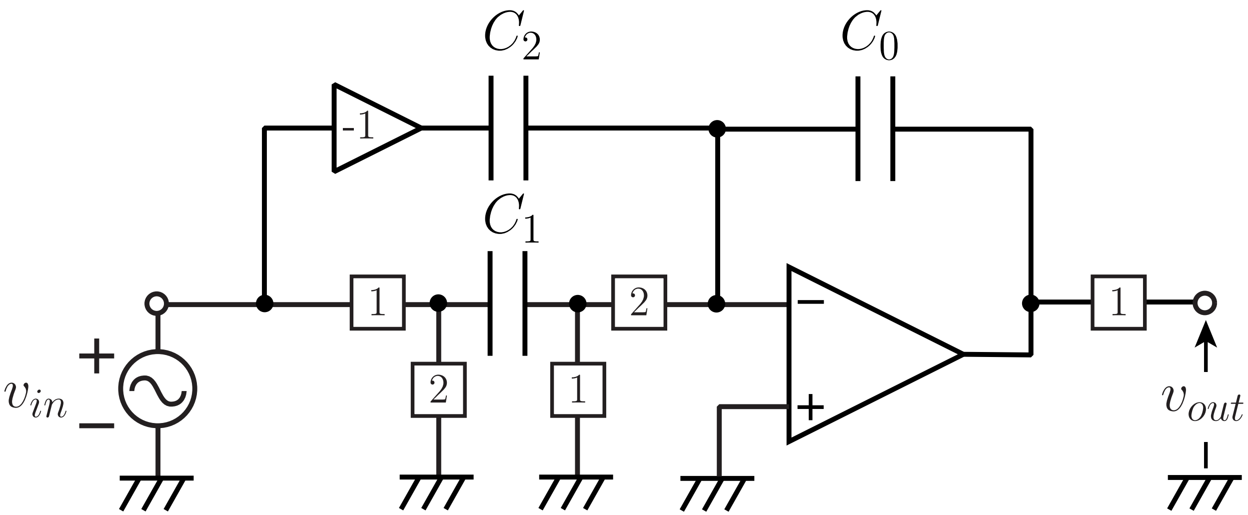
(a)

積分回路の選択

双一次積分回路

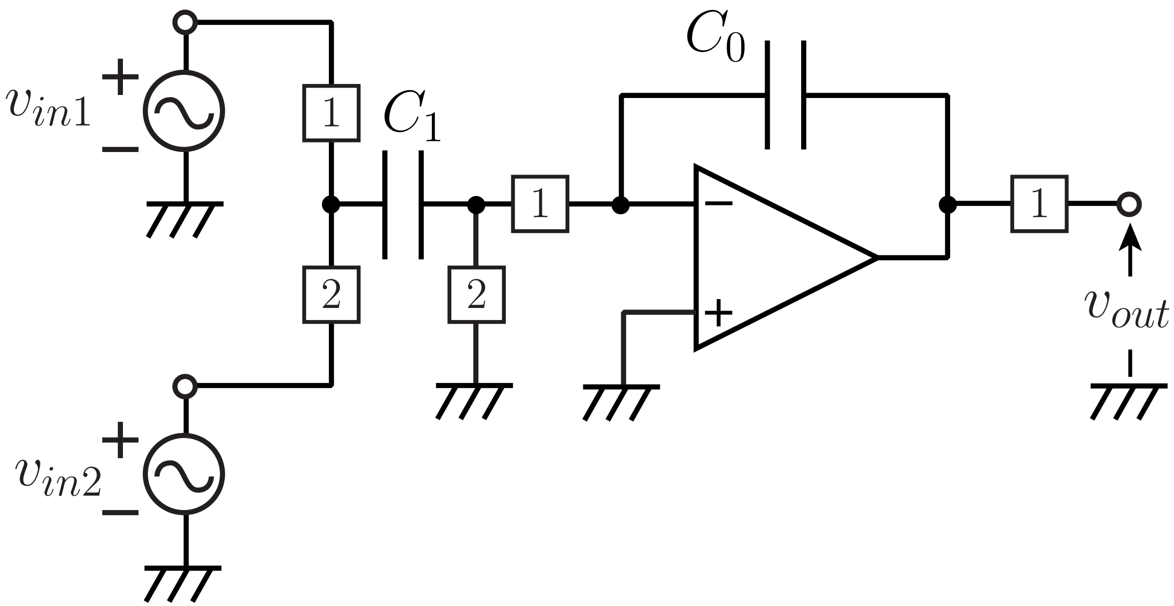


問　$C_1 = 2C_2$のとき，上の回路の伝達特性$\dfrac{v_{out}}{v_{in}}$をC_0とC_1を用いて表せ.

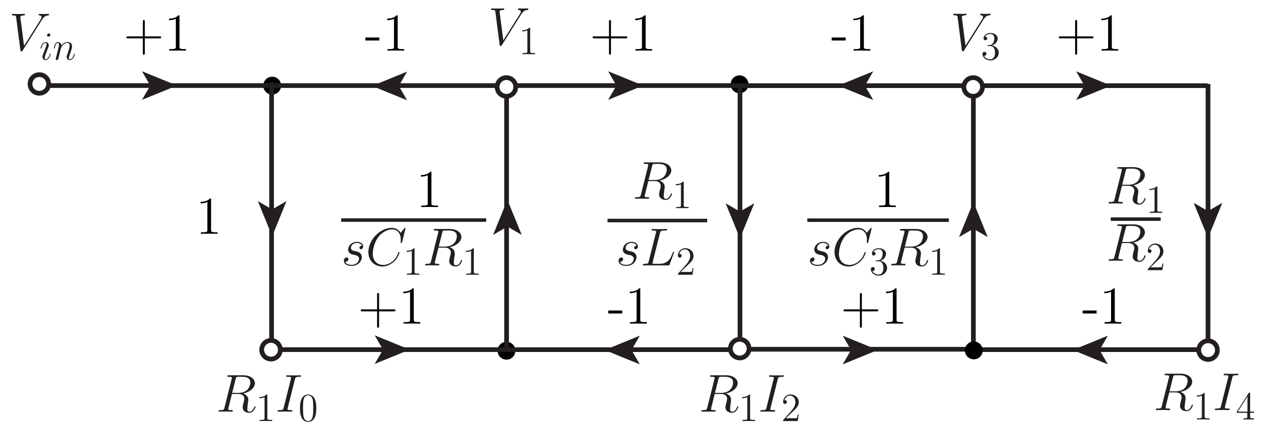


C_1=2C_2のとき

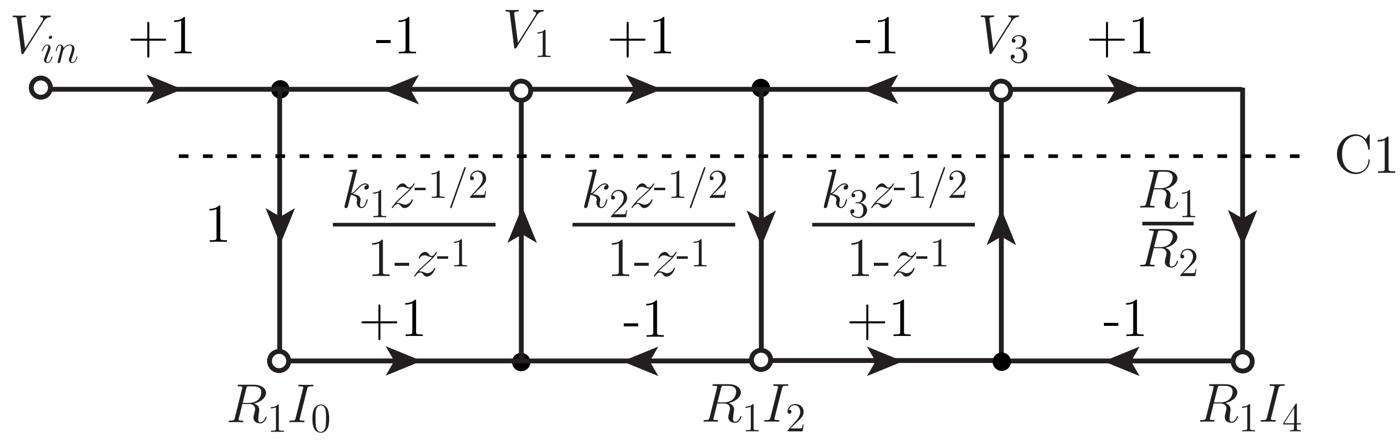
双一次積分回路
複　雑



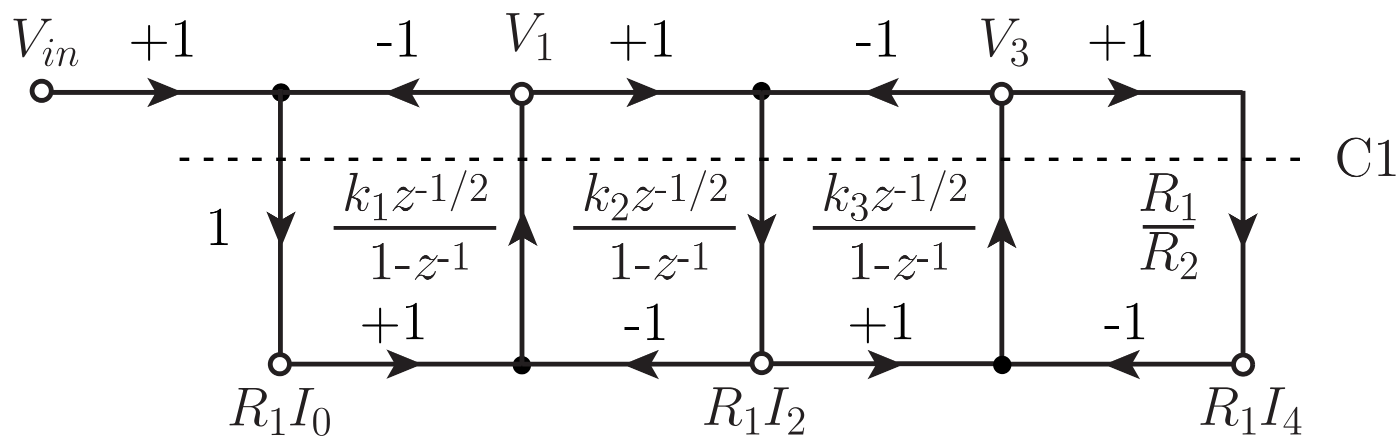
LDI積分回路
簡　単


V_{in}
+1
-1
V_1
+1
-1
V_3
+1
1
$\frac{1}{sC_1R_1}$
$\frac{R_1}{sL_2}$
$\frac{1}{sC_3R_1}$
$\frac{R_1}{R_2}$
+1
-1
+1
-1
R_1I_0
R_1I_2
R_1I_4

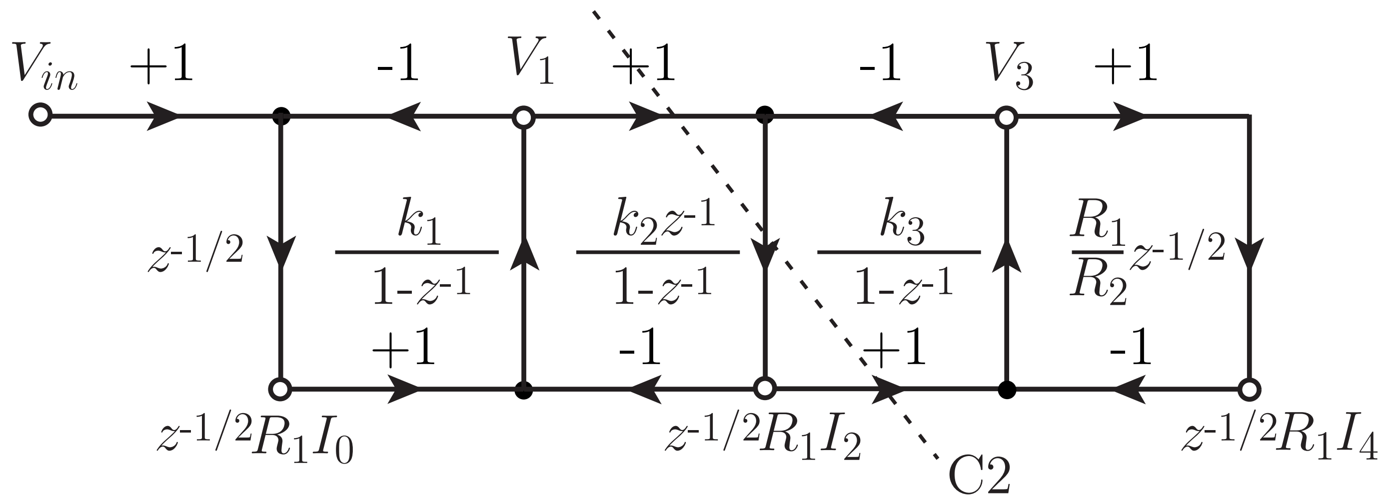
(a)


V_{in}
+1
-1
V_1
+1
-1
V_3
+1
C1
1
$\frac{k_1z^{-1/2}}{1-z^{-1}}$
$\frac{k_2z^{-1/2}}{1-z^{-1}}$
$\frac{k_3z^{-1/2}}{1-z^{-1}}$
$\frac{R_1}{R_2}$
+1
-1
+1
-1
R_1I_0
R_1I_2
R_1I_4

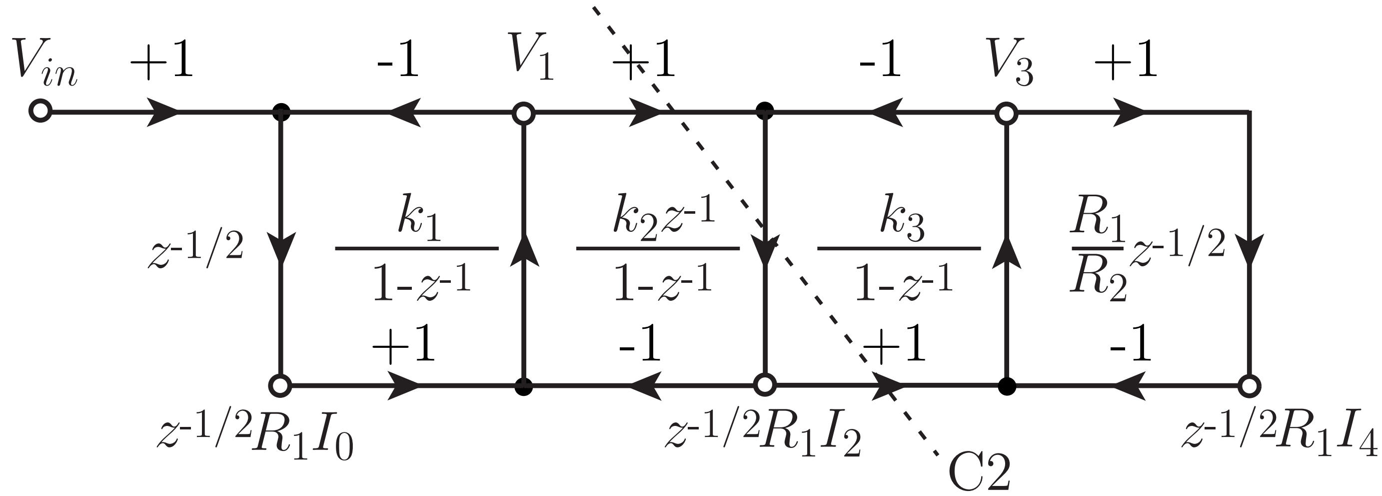
(b)


V_{in}
+1
-1
V_1
+1
-1
V_3
+1
C1
1
$\frac{k_1 z^{-1/2}}{1-z^{-1}}$
$\frac{k_2 z^{-1/2}}{1-z^{-1}}$
$\frac{k_3 z^{-1/2}}{1-z^{-1}}$
$\frac{R_1}{R_2}$
+1
-1
+1
-1
$R_1 I_0$
$R_1 I_2$
$R_1 I_4$

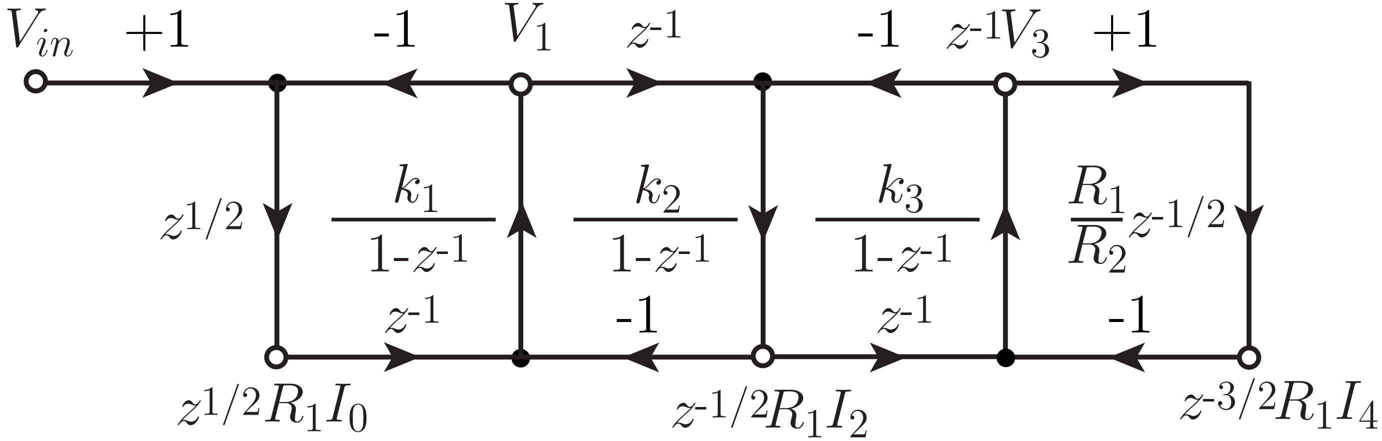
(b)


V_{in}
+1
-1
V_1
+1
-1
V_3
+1
$z^{-1/2}$
$\frac{k_1}{1-z^{-1}}$
$\frac{k_2 z^{-1}}{1-z^{-1}}$
$\frac{k_3}{1-z^{-1}}$
$\frac{R_1}{R_2} z^{-1/2}$
+1
-1
+1
-1
$z^{-1/2} R_1 I_0$
$z^{-1/2} R_1 I_2$
$z^{-1/2} R_1 I_4$
C2

(c)

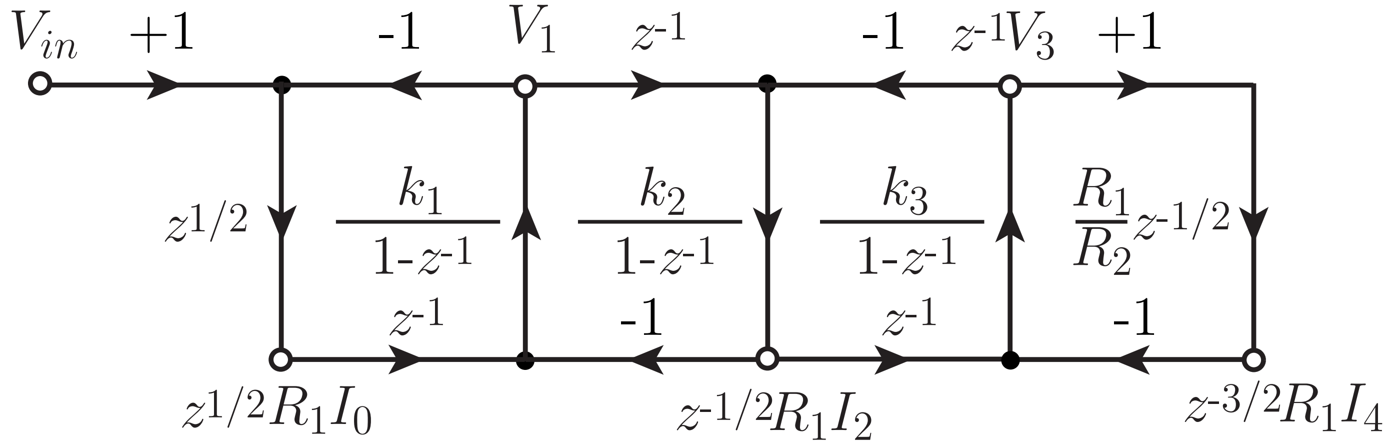


(c)



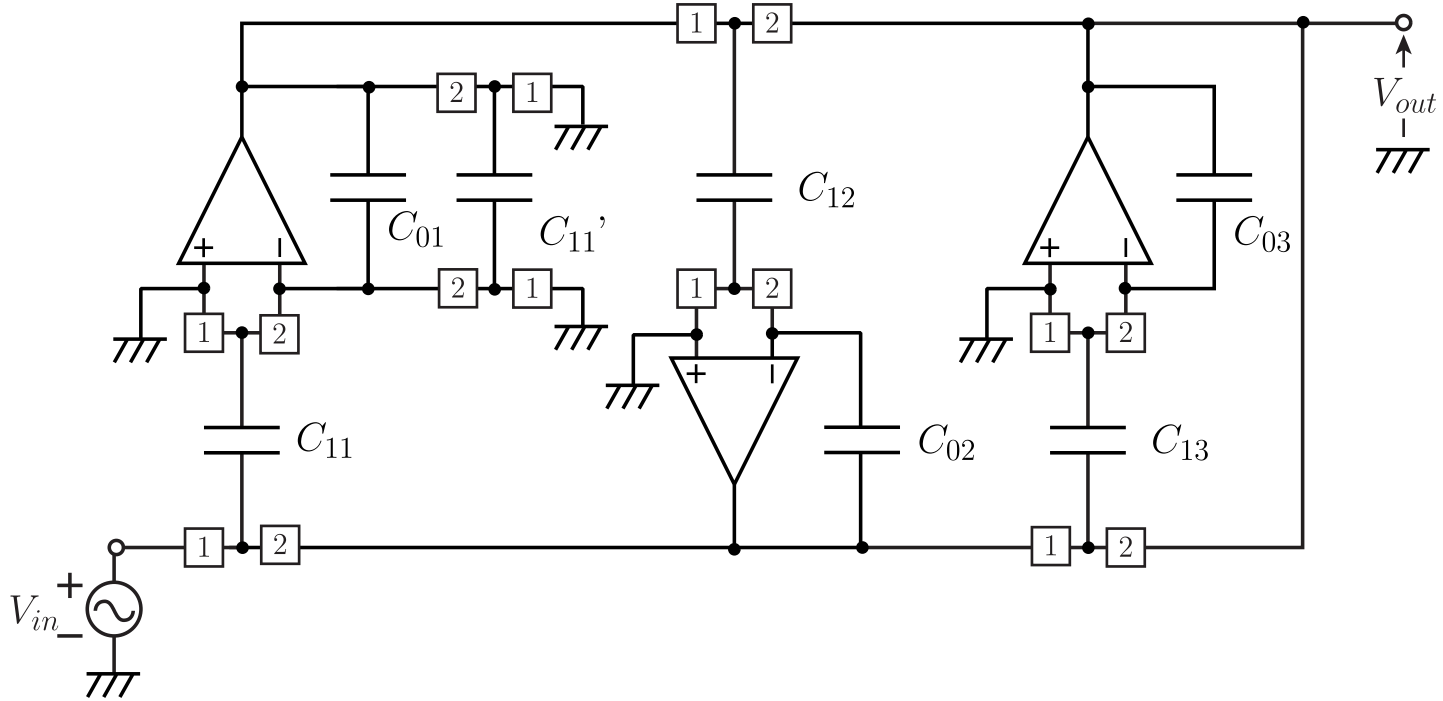
(d)

$$k_1 = \frac{T}{C_1 R_1} \qquad k_2 = \frac{R_1 T}{L_2} \qquad k_3 = \frac{T}{C_3 R_1}$$



(d)

$$k_1=\frac{T}{C_1R_1} \qquad k_2=\frac{R_1T}{L_2} \qquad k_3=\frac{T}{C_3R_1}$$



LDI変換の問題点

伝達関数$T(s)$の極： $s_i=\sigma_i+j\omega_i$

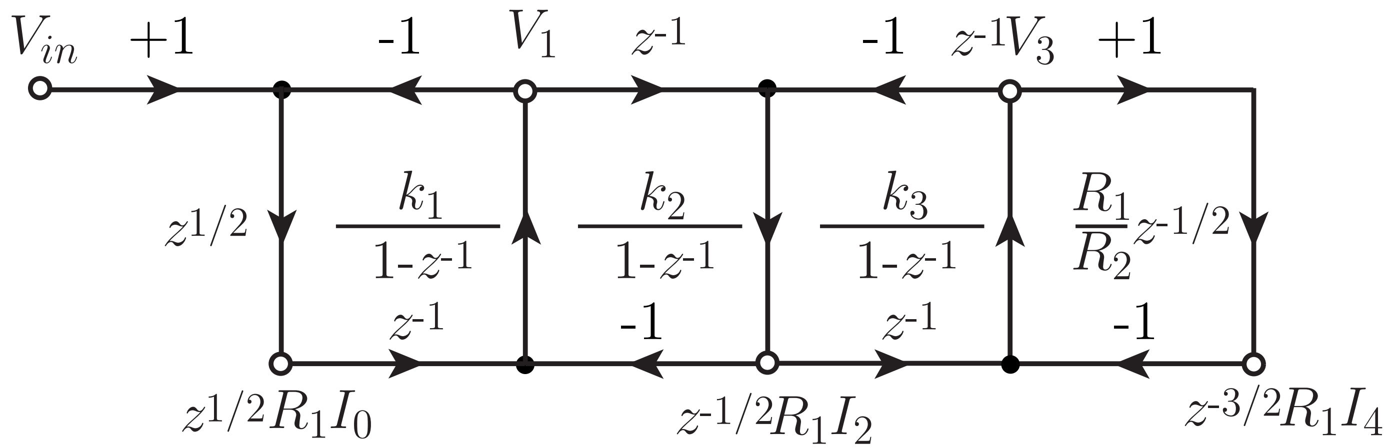
$$s_i=\frac{z^{1/2}-z^{-1/2}}{T} \longrightarrow s_i{}^2=\frac{z-2+z^{-1}}{T^2}$$

解: $z_i \longrightarrow z_i{}^{-1}$

$|z_i|<1 \longrightarrow$ 安定

$\downarrow$

$|z_i{}^{-1}|>1 \longrightarrow$ 不安定



(d)

$$k_1=\frac{T}{C_1R_1} \qquad k_2=\frac{R_1T}{L_2} \qquad k_3=\frac{T}{C_3R_1}$$

誤差要因

不安定性を除去

例題

遮断周波数1kHzの3次低域通過フィルタ

$$\boxed{\Omega \rightarrow \frac{2}{T}\sin(\frac{\omega T}{2})}$$

$$T=1/20000\mathrm{s}$$

$$\Omega_0=40000\sin(\frac{2\pi\times 1000}{2}\times\frac{1}{20000})=2\pi\times 996$$

$$R_1=R_2=1\mathrm{k}\Omega \qquad C_1=C_3=0.159\mu\mathrm{F} \qquad L_2=0.318\mathrm{H}$$

$$\downarrow$$

$$R_1=R_2=1\mathrm{k}\Omega \qquad C_1=C_3=0.160\mu\mathrm{F} \qquad L_2=0.319\mathrm{H}$$

$$k_1=\frac{T}{C_1R_1}=\frac{C_{11}}{C_{01}}=\frac{C'_{11}}{C_{01}}=0.319$$

$$k_2=\frac{TR_1}{L_2}=\frac{C_{12}}{C_{02}}=0.160$$

$$k_3=\frac{T}{C_3R_1}=\frac{C_{13}}{C_{03}}=0.319$$

$C_{01}=C_{02}=C_{03}=1$とすれば

$$C_{11}=C'_{11}=C_{13}=0.319,\quad C_{12}=0.160$$

$$C_{01}=C_{02}=C_{03}=6.25\text{pF},\quad C_{11}=C'_{11}=C_{13}=1.99\text{pF},\quad C_{12}=1.00\text{pF}$$

容量の総和 :25.7pF

$$k_1 = \frac{T}{C_1 R_1} = \frac{C_{11}}{C_{01}} = \frac{C'_{11}}{C_{01}} = 0.319$$

$$k_2 = \frac{T R_1}{L_2} = \frac{C_{12}}{C_{02}} = 0.160$$

$$k_3 = \frac{T}{C_3 R_1} = \frac{C_{13}}{C_{03}} = 0.319$$

$\boxed{C_{01}=C_{03}=1,\ C_{02}=2}$とすれば

$$C_{11}=C'_{11}=C_{12}=C_{13}=0.319$$

$$C_{01}=C_{03}=3.13\mathrm{pF},\ C_{02}=6.25\mathrm{pF},\ C_{11}=C'_{11}=C_{12}=C_{13}=1.00\mathrm{pF}$$

容量の総和 :13.4pF