

Fading Theory

- In many circumstances, it is too complicated to describe all reflection, diffraction, and scattering processes that determine the different Multi-path Components. Rather, it is often preferable to describe the probability that a channel parameter attains a certain value.

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Deterministic vs. Stochastic

- Deterministic case : “ $x=y$ ” means $2=2$.
- Stochastic case : “ $x=y$ ” means “ $p(x)=p(y)$ ”.
- For example, $x = 1-x$ holds when x is a uniform distributed random variable in the interval $[0,1]$
 z : zero-mean Complex Gaussian Noise
 $\therefore “z=-z^*= -z^*”$
 Z : zero-mean Complex Gaussian Independent Vector
 $\therefore “Z=UZ”$ where U : Unitary matrix

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Stochastic Signal Processing

- Realized value of random number is known \rightarrow e.g. MRC
- Pdf of random number is known \rightarrow e.g. Wiener Filtering
- Moments of random number is known \rightarrow e.g. Decoupling Circuit Design

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Stochastic Process in Wireless Communication

- Noise (white spectrum)
- Signal (modulated bandwidth)
- Channel (Doppler frequency)
- Noise > Signal > Channel

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Contents

- Path Loss Formula
- Log-normal distribution
- Rayleigh/Rice distribution
- Envelope/Phase distribution
- Power Spectrum & Doppler effect
- Fading Coefficient
- MAP Estimation of Fading Channel in PHS

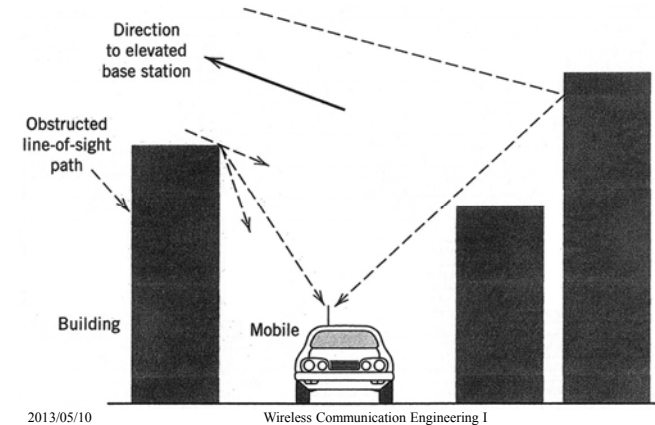
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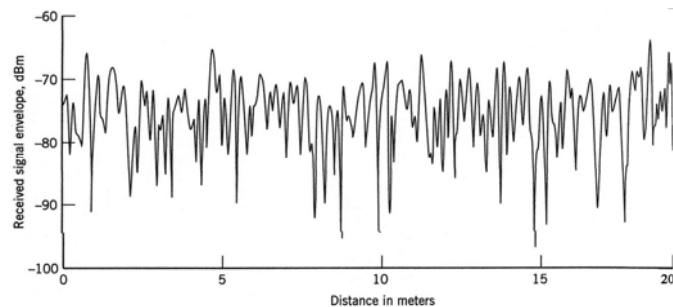
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Mobile Communication Channel

In addition to Direct wave, there are many Reflection, Refraction and Diffraction waves.



- Received signal fluctuates dramatically
- Fading (Long-range, Medium-range, Short-range)



Hierarchical stochastic structure

- **Path loss** : The large-scale mean itself depends on the “distance” between transmitter and receiver.
- **Log-normal** : Mean power, averaged over about 10 wavelengths, itself shows fluctuations due to “shadowing” by large objects.
- **Rayleigh and Nakagami-Rice** : On a very-short-distance scale, power fluctuates around a local mean value due to “interference” between different MPCs.

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Path loss and Power Control

- For 3G Wireless Communication System, i.e. W-CDMA (Wideband Code Division Multiple Access) **Power Control** is used in order to alleviate “**Near-Far Problem**”.
Dynamic Range for Power Control is required more than **74dB**.

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Path Loss Formula

- Land mobile electromagnetic wave propagation**
Propagation characteristics are important in designing a cell size, a transmitter and a receiver.
 - Long** distance variation (Okumura curve): The CCIR adopted the basic formula for the median path loss, based on Okumura's measurements.

$$L = 69.55 + 26.16 \log(f) - 13.82 \log(H_b) + [44.9 - 6.55 \log(H_b)] \log(d) + a_x(H_m)$$

f : frequency in MHz

H_b : Base station antenna height in meter

d : Range in Km

H_m : Mobile station antenna height in meter

$a_x(H_m)$: Correction factor

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- Middle** distance variation (**Log-normal** distribution: Shadowing) Median over several ten or hundred wavelengths obeys a log-normal distribution.

$$E_r = T_1 \times T_2 \times T_3 \times \cdots \times E_s$$

E_r : Signal Strength at the receiver

E_s : Signal Strength at the transmitter

T_i : Transmission coefficient at the i -th obstacle

$$\therefore \log E_r = \log T_1 + \log T_2 + \cdots + \log E_s$$

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Central Limit Theorem

- The sum of statistically independent and identically distributed random variables with finite mean and variance approaches a **Gaussian distribution** as the number of variables increases.
- Gaussian distribution** is characterized only by mean and variance (2 parameters).
- An instantaneous complex amplitude of OFDM signal can be also approximated by Gaussian variable.

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Shadowing effect

- Typical shadowing range is around 4-10dB
- 3GPP Channel model:
 - Suburban Macro 8dB
 - Urban Macro 8dB
 - Urban Micro 10dB(NLOS) 4dB(LOS)

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Rayleigh Fading

- **Short** distance variation (Rayleigh Fading) There are so many reflection and diffraction waves to generate a complicate standing wave pattern. The mobile station moves through there.

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BER Performance in Rayleigh Fading Channel

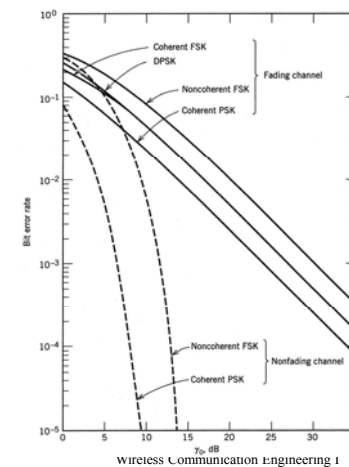
- BER (Bit Error Rate) is proportional to an **exponential function of SNR in non-fading channel** (AWGN channel).
- BER is proportional to an **inverse of SNR in fading channel**.
- Because SNR in fading channel is a random variable of which **PDF (probability density function) is an exponential function**.

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- Fading significantly deteriorates **QoS** (i.e. bit error rate).



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BER in Rayleigh channel

- Instantaneous BER: $Pe(\gamma) \cong \exp(-\gamma)/2$
- Averaged BER: $\overline{Pe(\gamma)} = \int Pe(\gamma) \times P(\gamma) d\gamma = 1/\{2(\Gamma + 1)\}$
- Pdf of SNR: $P(\gamma) = \exp(-\gamma/\Gamma)/\Gamma$
where Γ : average SNR

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Interference between Multi-path Components

• Rayleigh Fading Model

The n -th elementary arriving wave $e_n(t)$ at an angle of ϕ_n

$$e_n(t) = \text{Re}[z_n(t) \exp(j2\pi[f_c + f_D \cos(\phi_n)]t)]$$

$\text{Re}[\]$: Real part complex number

$z_n(t)$: Complex envelope = $R_n e^{j\theta_n}$

f_c : Carrier frequency

f_D : Maximum Doppler frequency shift ($= v/\lambda$)

v : Velocity of mobile station

λ : Wavelength ($= c/f_c$)

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– Envelope and phase distribution

Received signal $e(t)$ is composed of N elementary waves.

$$\begin{aligned} e(t) &= \sum_{n=1}^N e_n(t) \\ &= \text{Re} \left[\sum_{n=1}^N z_n(t) \exp(j2\pi f_c t) \right] \\ z(t) &= \sum_{n=1}^N z_n(t) \\ &= x(t) + jy(t) \end{aligned}$$

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$x(t)$: In - phase component = $R(t) \cos(\theta(t))$

$y(t)$: Quadrature component = $R(t) \sin(\theta(t))$

In the limit ($N \rightarrow \infty$), $x(t)$ and $y(t)$ become an independent Gaussian random variable with zero mean.

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Thus, a joint pdf (probability density function) of x and y

$$p(x, y) = \exp\left(-\frac{x^2 + y^2}{2b_0}\right) / 2\pi b_0$$

where $2b_0$: average received power = $E[x^2 + y^2] = E[R^2]$

A joint pdf of R and θ is

$$p(R, \theta) = \frac{R}{2\pi b_0} \exp\left(-\frac{R^2}{2b_0}\right) = p(R) p(\theta)$$

where R : envelope

θ : phase

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Rayleigh Distribution

A pdf of envelope R is a **Rayleigh** distribution

$$p(R) = \frac{R}{b_0} \exp\left(-\frac{R^2}{2b_0}\right)$$

A pdf of phase θ is a **uniform** distribution on

$$p(\theta) = 1/2\pi$$

CNR (Carrier - to - noise ratio), $\gamma = R^2/p_n$ is **exponential** distribution with noise power of p_n

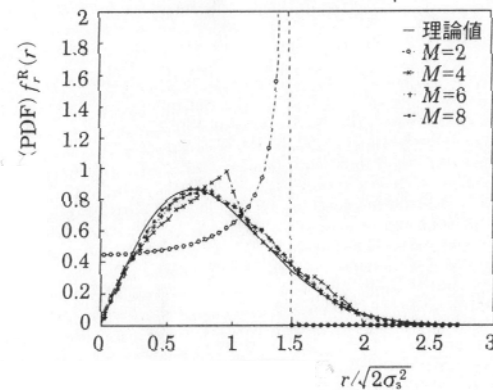
$$p(\gamma) = \frac{1}{\Gamma} \exp\left(-\frac{\gamma}{\Gamma}\right)$$

where Γ : Average CNR

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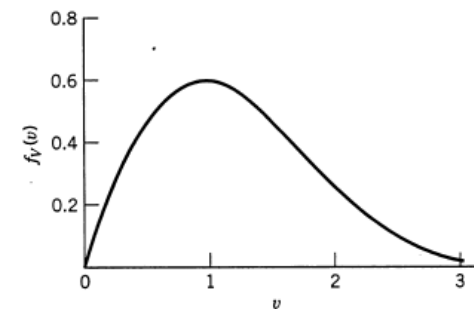


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Normalized Rayleigh Distribution

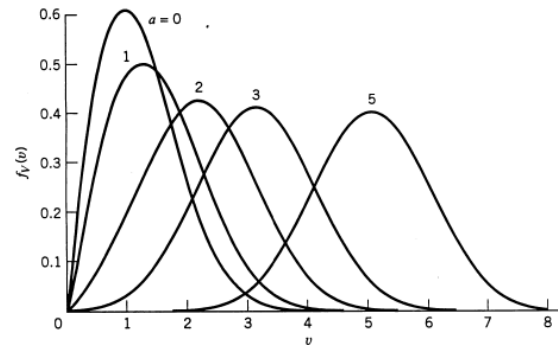


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Normalized Nakagami/Rice Distribution



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Rician Distribution

$$f_V(v) = v \exp\left(-\frac{v^2 + a^2}{2}\right) I_0(av)$$

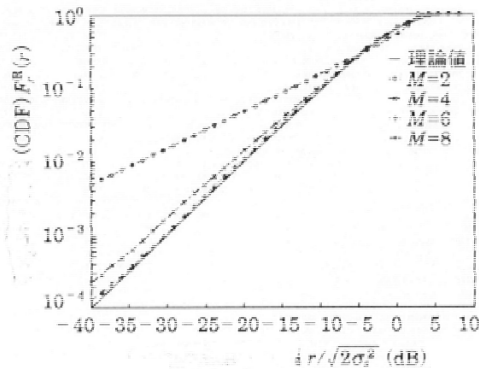
$I_0(\)$: 0-th Modified Bessel Function

a : Rice factor = LOS / NLOS

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• Power spectrum & Doppler effect

Elementary wave of arrival angle ϕ has a different frequency from f_c due to the Doppler effect.

$$f = f_c + f_D \cos \phi$$

Arriving angle is uniformly distributed so that received power $S(f)df$ in the range $[f, f + df]$ is

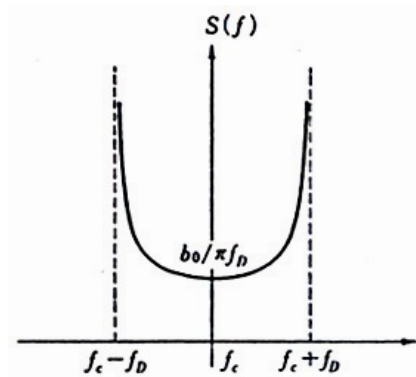
$$\begin{aligned} S(f)df &= 2 \times \frac{b_0}{2\pi} df \\ &= \frac{b_0}{\pi f_D \sqrt{1 - [(f - f_c)/f_D]^2}} df \end{aligned}$$

(cf. $f_c = 1.5\text{GHz}$, $v = 50\text{km/h}$, $f_D = 135\text{Hz}$)

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Power Spectrum

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Time derivative of random variables

$$dx(t)/dt = dR(t)/dt \times \cos(\theta(t)) - R(t) \times \sin(\theta(t)) \times d\theta(t)/dt$$

$$dy(t)/dt = dR(t)/dt \times \sin(\theta(t)) + R(t) \times \cos(\theta(t)) \times d\theta(t)/dt$$

$$pdf(x, y, dx/dt, dy/dt) \rightarrow pdf(R, \theta, dR/dt, d\theta/dt)$$

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Joint pdf

$$p(x, y, \dot{x}, \dot{y}) = \frac{1}{2\pi b_0} e^{-\frac{x^2 + y^2}{2b_0^2}} \frac{1}{2\pi b_2} e^{-\frac{\dot{x}^2 + \dot{y}^2}{2b_2^2}}$$

$$p(R, \dot{R}, \dot{\theta}) = \frac{R^2}{b_0} e^{-\frac{R^2}{2b_0^2}} \frac{1}{2\pi b_2} e^{-\frac{\dot{R}^2 + R^2 \dot{\theta}^2}{2b_2^2}}$$

$$p(R, \dot{R}) = \frac{R}{b_0} e^{-\frac{R^2}{2b_0^2}} \frac{1}{\sqrt{2\pi b_2}} e^{-\frac{\dot{R}^2}{2b_2^2}} = p(R) \cdot p(\dot{R})$$

$$p(R, \dot{\theta}) = \frac{R^2}{b_0} e^{-\frac{R^2}{2b_0^2}} \frac{1}{\sqrt{2\pi b_2}} e^{-\frac{R^2 \dot{\theta}^2}{2b_2^2}}$$

$$p(\theta) = \frac{1}{2} \sqrt{\frac{b_0}{b_2}} \frac{1}{\left(1 + \frac{b_0}{b_2} \dot{\theta}^2\right)^{3/2}}$$

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• Level crossing number & Fade duration

They are important parameters for mobile communication quality.

– Level crossing number

\dot{R} : time derivative of envelope R

A joint pdf of R and \dot{R} , $p(R, \dot{R})$ is

$$p(R, \dot{R}) = \frac{R}{b_0} \exp\left[-\frac{R^2}{2b_0^2}\right] \frac{1}{\sqrt{2\pi b_2}} \exp\left[-\frac{\dot{R}^2}{2b_2^2}\right]$$

Level crossing number of envelope per unit time $N(R_s)$ at the level R_s

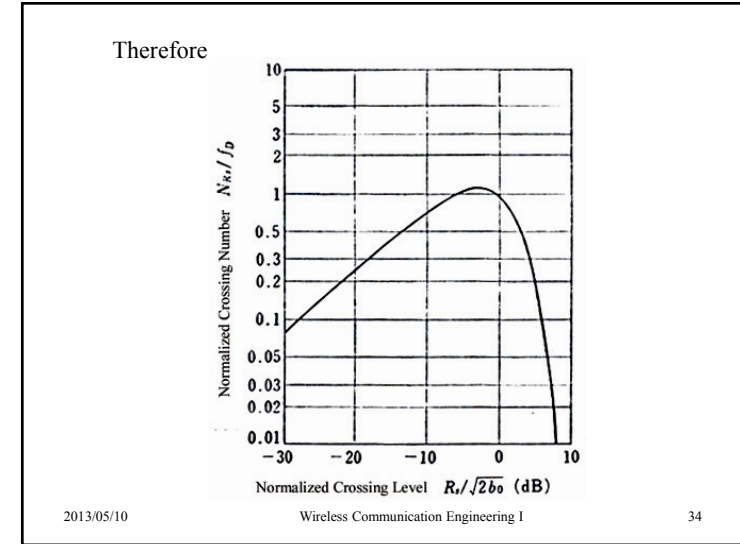
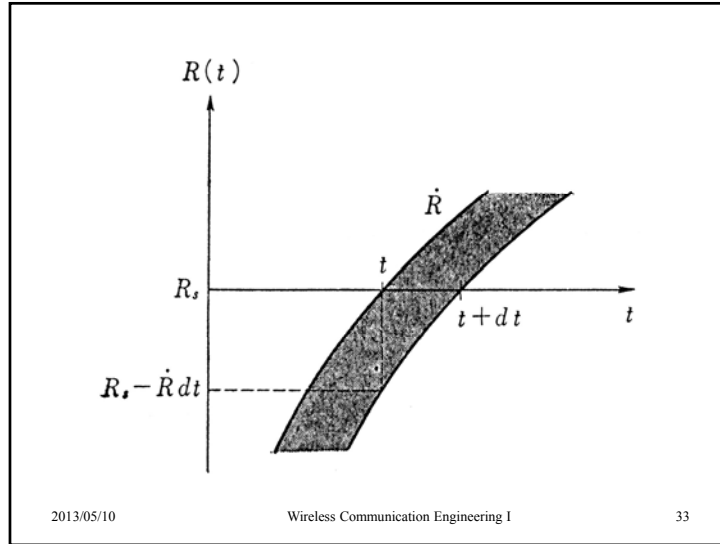
$$N(R_s) = \int_0^\infty \dot{R} \cdot P(R_s, \dot{R}) d\dot{R}$$

where $b_2 = E[\dot{R}^2]$

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$$\begin{aligned}
 N(R_s) &= \sqrt{\frac{b_2}{\pi b_0}} \frac{R_s}{\sqrt{2b_0}} \exp\left[-\frac{R_s^2}{2b_0}\right] \\
 &= \sqrt{2\pi} f_D \frac{R_s}{\sqrt{2b_0}} \exp\left[-\frac{R_s^2}{2b_0}\right] \\
 N(\sqrt{b_0})_{\max} &= f_D \sqrt{\pi/e} \\
 2\pi f_0 &= \sqrt{2b_2/b_0}
 \end{aligned}$$

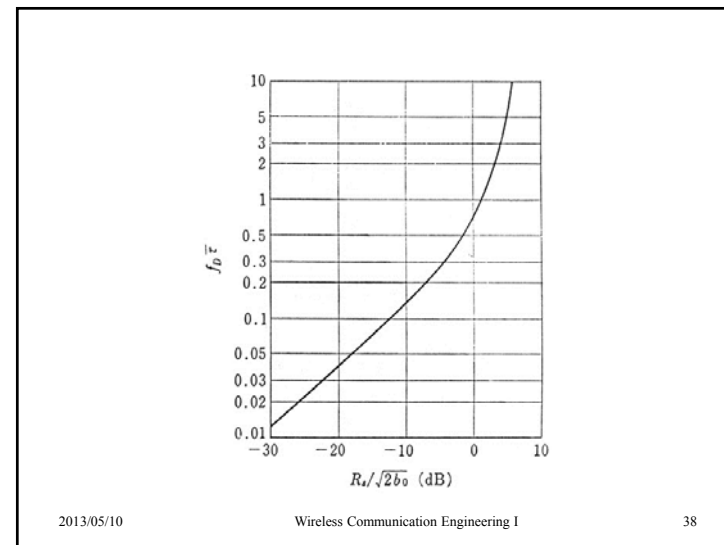
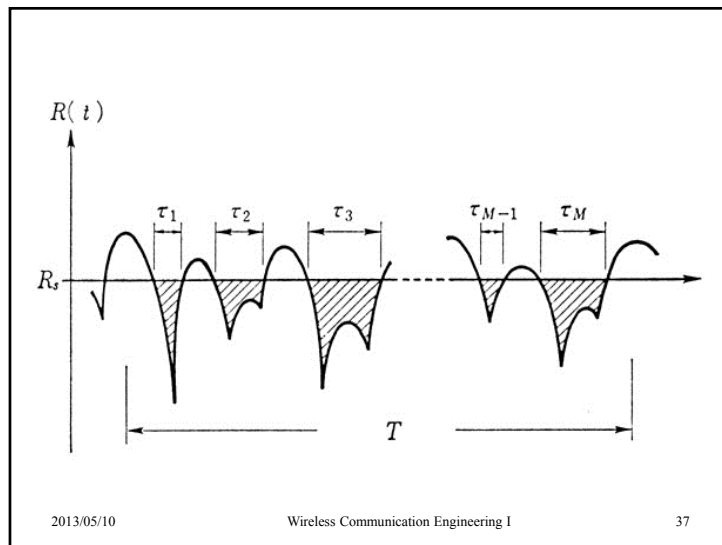
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– Average fade duration time at the level R_s , $\bar{\tau}$

$$\begin{aligned}
 \bar{\tau} &= \frac{\Pr[R(t) \leq R_s]}{N(R_s)} \\
 &= \frac{\sqrt{2b_0}}{\sqrt{2\pi} f_D R_s} \left[\exp\left(\frac{R_s^2}{2b_0}\right) - 1 \right]
 \end{aligned}$$

(cf. When $R_s/\sqrt{2b_0} = 0.1$ (20dB down), $f_c = 1.5\text{GHz}$, $v = 50\text{km/h}$, $\bar{\tau} = 2\text{ms}$)

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- Random FM noise

$\theta(t)$ fluctuates randomly \rightarrow FM noise

A pdf of $\dot{\theta}$, $p(\dot{\theta})$ is

$$p(\dot{\theta}) = \frac{1}{2} \sqrt{\frac{b_0}{b_2}} \left[1 + \frac{b_0}{b_2} \dot{\theta}^2 \right]^{-3/2}$$

Random FM noise is independent on average received power.

This determines a lower bound of bit error rate.

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- **Fading correlation**

The correlation characteristics are necessary for the design of diversity system.

– Time correlation

$$\begin{aligned} \rho(\tau) &= \frac{E[z^*(t)z(t+\tau)]}{E[z(t)^*z(t)]} \\ &= J_0(2\pi f_D \tau) \end{aligned}$$

$J_0(\quad)$: 0 - th order Bessel function of the first kind

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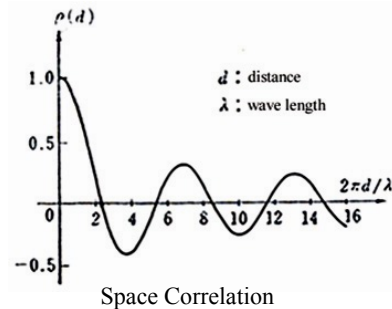
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– Space correlation

Space distance $d = v\tau$

$$\rho(d) = J_0(2\pi d/\lambda)$$

Around half wavelength spacing ($d \sim \lambda/2$) → no correlation



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MIMO Transmission and Antenna correlation

- Antenna correlation decreases MIMO channel capacity if average SNR at RX antenna is equal to each other.

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– Frequency correlation

This is important parameter for Wide-band transmission.

$$\rho(\Omega) = \frac{1}{1 + j2\pi\Omega(\delta\ell/c)} \exp(j2\pi\ell_0/c)$$

ℓ_0 : minimum path length

$\delta\ell$: deviation in path length

(cf. For $\delta\ell = 200\text{m}$, coherent bandwidth is 400kHz)

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A MAP Estimation of Rayleigh Fading Channel

- A Filter Theory of Complex Gaussian Process – and Its Application to PHS SDMA

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Contents

- Background & Motivation
- Complex Gaussian Stochastic Process
- Noisy Rayleigh Fading Channel
- MAP Estimation of Channel Transfer Coefficient
- Numerical Results
- Conclusion
- Future Work

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Background & Motivation

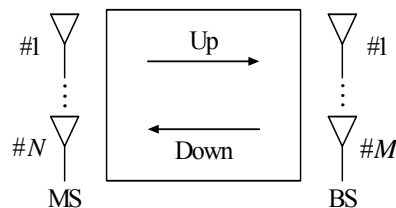
- Recursive Simulation Method for Rayleigh Fading Channel.
 - How to write a computer program ?
- Fading Channel Coefficients should be estimated in SDMA PHS Systems

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- Mobile Communication Channel with MIMO Systems
 - Time Variant Linear Reciprocal System



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For $(N + M)$ -port Circuit, a $(N + M) \times (N + M)$ scattering matrix S is defined;

$$S(f, t) = \begin{bmatrix} \overset{\leftarrow N \rightarrow}{S_{MM}} & \overset{\leftarrow M \rightarrow}{S_{BM}} \\ S_{MB} & S_{BB} \end{bmatrix} \begin{matrix} \uparrow N \\ \downarrow M \end{matrix}$$

where

$S_{BM} : M \times N$ Transfer Matrix of **Up-Link** from MS to BS

$S_{MB} : N \times M$ Transfer Matrix of **Down-Link** from BS to MS

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By the reciprocity,

$$S = S^t$$

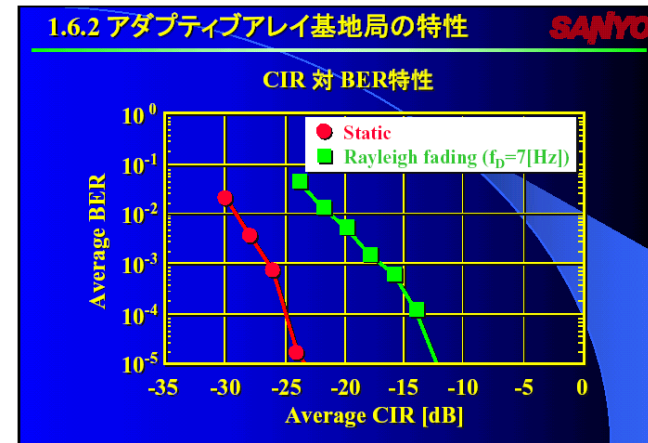
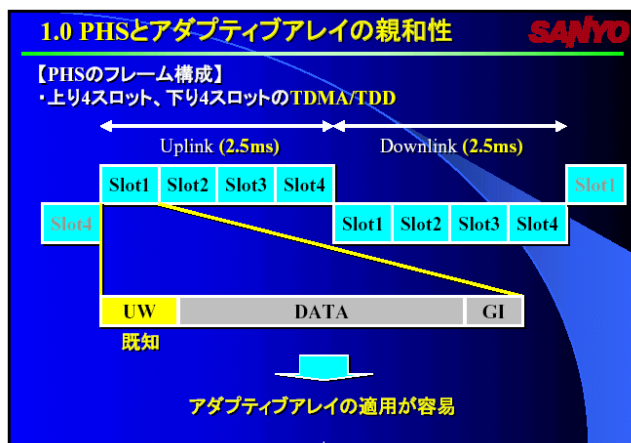
$$\therefore S_{MB}(f, t) = S_{BM}(f, t)^t$$

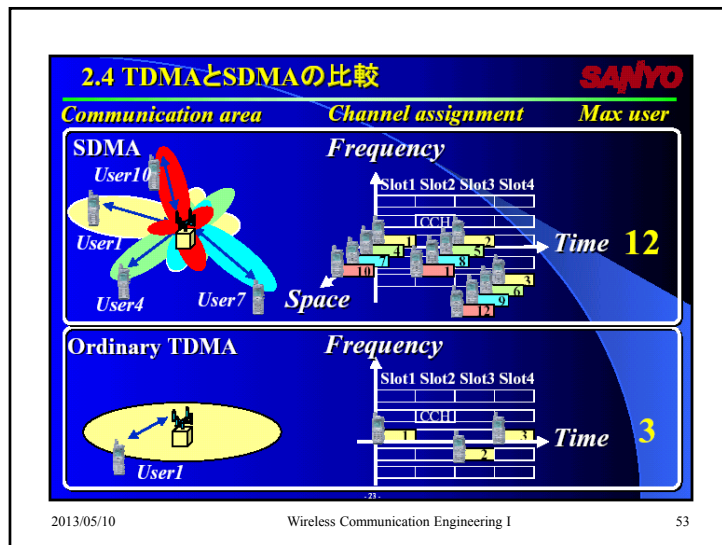
Thus, the **Down-Link** Transfer Characteristics can be determined by the **Up-Link** one.

The above equality, however, holds only for the same **frequency and time**.

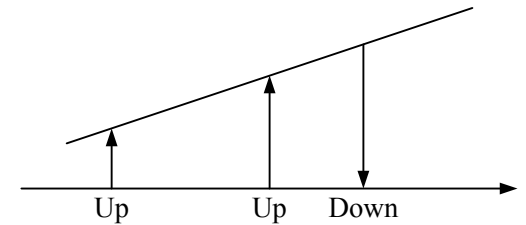
PHS system

- TDD (Time Domain Duplex)
- TDMA (Time Domain Multiple Access)
- 4 Time Slot Segmentation
- Introduction of SDMA increases a channel capacity by 3 times or more.
- At the PHS base station, 4 antennas are installed.
- At most 4 data streams can be transmitted simultaneously by pre-coding at BS for down link.
- The idea is used in “i-Burst” system (IEEE802.20)





- Conventionally
 - Linear Extrapolation for Channel coefficient is used.



- Noise Filtering is not taken into account.

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Complex Gaussian Stochastic Process

- 1) Rayleigh (or Rice) Fading Coefficient : $X(t)$
 - 2) Random White Gaussian Noise : $Y(t)$
- ↓
- 3) Rayleigh Fading Coefficient contaminated with Noise:

$$Z(t) = X(t) + Y(t)$$

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Stationary Gaussian Process can be characterized only by Autocorrelation Function

$$R_{ZZ}(\tau) = \overline{Z(t)Z(t+\tau)} \\ = R_{XX}(\tau) + R_{YY}(\tau)$$

where

$$R_{XX}(\tau) = A J_0(2\pi f_D \tau)$$

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$A = \overline{|X(t)|^2}$: Average Fading Level

J_0 : 0th Order Bessel Function of First Kind

f_D : Maximum Doppler Frequency ($= f_c \frac{v}{c}$)

f_c : Carrier Frequency

v : velocity of MS

c : velocity of Light

$$R_{YY}(\tau) = \begin{cases} N & (\tau = 0) \\ 0 & (\tau \neq 0) \end{cases}$$

$N = \overline{|Y(t)|^2}$: Average Noise Level

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For MAP Estimation, **Cross-correlation** Function is also needed

$$\begin{aligned} R_{ZX}(\tau) &= \overline{Z(t)X(t+\tau)} = \overline{(X(t)+Y(t))X(t+\tau)} \\ &= \overline{X(t)X(t+\tau)} = R_{XX}(\tau) \end{aligned}$$

$\therefore X(t)$ and $Y(t)$ are independent.

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- MAP (LS) Estimation and Optimal Noise Reduction
– Wiener-Hopf Equation

Optimal Linear Combination Estimator Vector : \mathbf{b}

$$\begin{bmatrix} 1 + \frac{N}{A} & J_0(2\pi f_D(t_1 - t_0)) & \cdots & J_0(2\pi f_D(t_{n-1} - t_0)) \\ & \ddots & \ddots & \vdots \\ & & 1 + \frac{N}{A} & J_0(2\pi f_D(t_n - t_{n-1})) \end{bmatrix} \mathbf{b} = \begin{bmatrix} J_0(2\pi f_D(t_n - t_0)) \\ \vdots \\ J_0(2\pi f_D(t_n - t_{n-1})) \end{bmatrix}$$

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MAP Estimator for $x(t_n)$

$$x(t_n)_{\text{MAP}} = \mathbf{b}^T \mathbf{Z}$$

where

$\mathbf{Z} = (Z(t_0), \dots, Z(t_{n-1}))$: Observed Noisy Data

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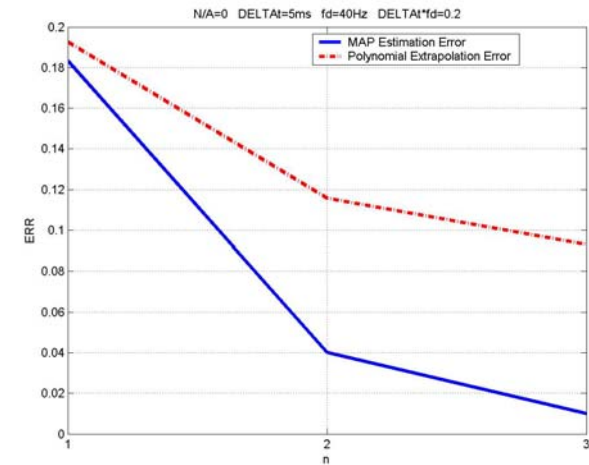
Numerical Results

- (1) NoiseLevel : $N/A = 0, 0.1, 1$
 (2) DopplerFrequency: $f_D = 10, 40$ [Hz]
 (3) No. of Data : $n = 1, 2, 3$

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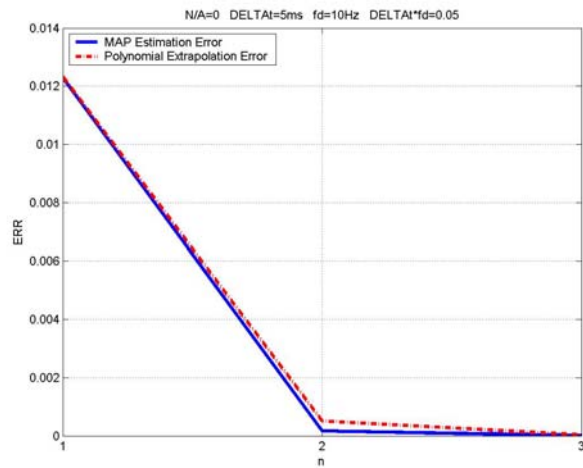
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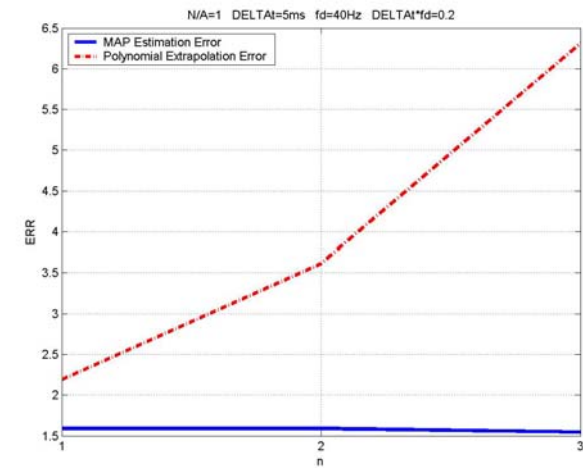
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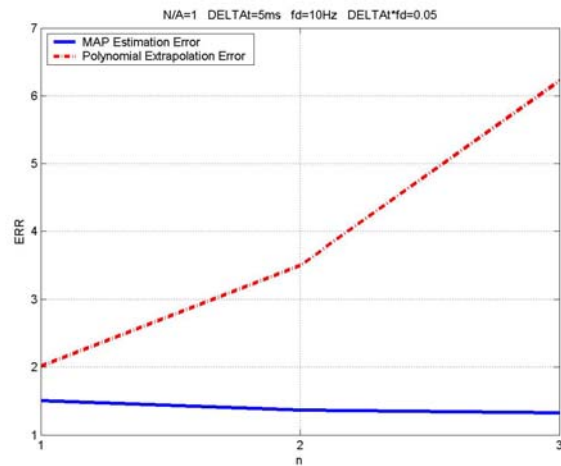
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Conclusion

- Estimation of Fading Coefficient is useful for TDMA/ TDD.
- Conventional Estimation is not satisfactory.
- Estimation Error can be greatly reduced by MAP Estimation.

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