Core

1. Core

   - A set of imputations C is the **core** $\Leftrightarrow$
     $C = \{x \in A | \sum_{i \in S} x_i \geq v(S) \ \forall S \subseteq N\}$
   - $\sum_{i \in S} x_i \geq v(S)$: coalitional rationality
   - $e(S, x) = v(S) - \sum_{i \in S} x_i$: **excess** of coalition S at imputation x
   - core $\Leftrightarrow$ a set of imputations in which no coalition S has a positive excess value

2. Dominance Core

   - Dominance:
     For two imputations $x, y \in A$, if there is a coalition $S \subseteq N$ such that the two conditions below are satisfied, then x is said to dominate y via coalition S, (noted as $x \ dom_S \ y$)
     - $x_i > y_i \ \forall i \in S$
     - $\sum_{i \in S} x_i \leq v(S)$

     If there exists some S such that $x \ dom_S \ y$, then x is said to dominate y, written as $x \ dom \ y$.
   - The set of imputations that are not dominated DC is called the **dominance core**. That is,

$$DC = \{x \in A | \text{there does not exist } y \in A \text{ such that } y \ dom \ x\}$$

3. $C \subseteq DC$ always holds.

4. If (N, v) is superadditive, $DC \subseteq C$ also holds, and $C = DC$.