Structural Analysis II
構造力学第二
(2)

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9.3 REQUIREMENTS AND LIMITATIONS OF COMPATIBILITY (適合性､ 適合条件)

1) What is compatibility?

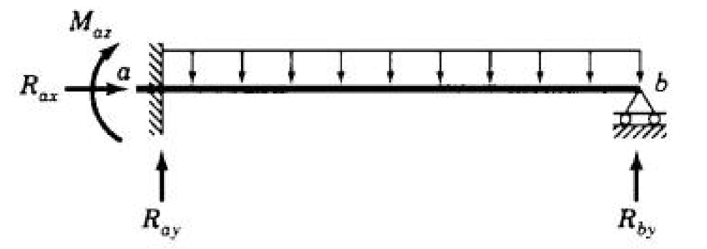
- Compatibility is constraints (拘束) on the displacements of a structure to ensure that its individual elements fit together properly and that the structure conforms to the displacement boundary conditions prescribed at the supports (支点における変位の境界条件).

- Compatibility is a requirement (要件) that must be satisfied.

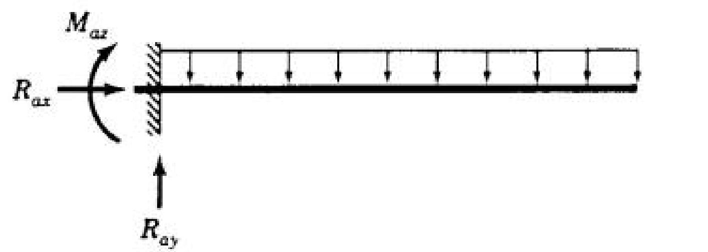
2) Example of primary structure （主構造）

●Analysis of a structure as shown in Fig. 9.3(a) could not be completed by statics alone because of statically indeterminate nature of the structure.

●If the reaction component R_{by} is removed as shown in Fig. 9.3(b), the resulting primary structure is statically determinate.



(a) statically indeterminate structure



(c) Stable and statically determinate primary structure

Fig. 9.3 Selection of redundant reaction

- The reactions R_{ay} and M_{az} can now be determined from statics

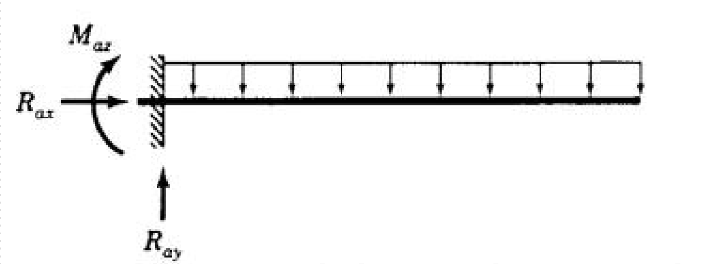


Fig. 9.3(c) Stable and statically determinate primary structure

- The deflection, which include a deflection of point b, Δ_{b1}, is

$$\Delta_{b1} = -\frac{wl^4}{8EI}$$

$$= -\frac{62500kN \cdot m^3}{EI}$$

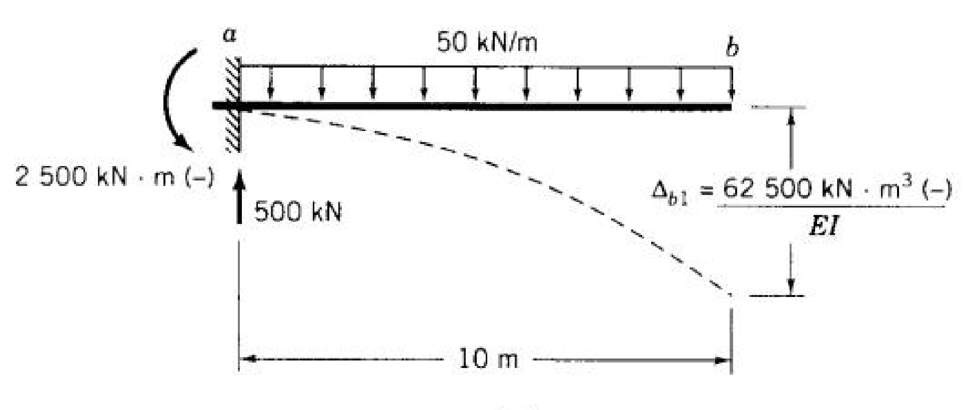
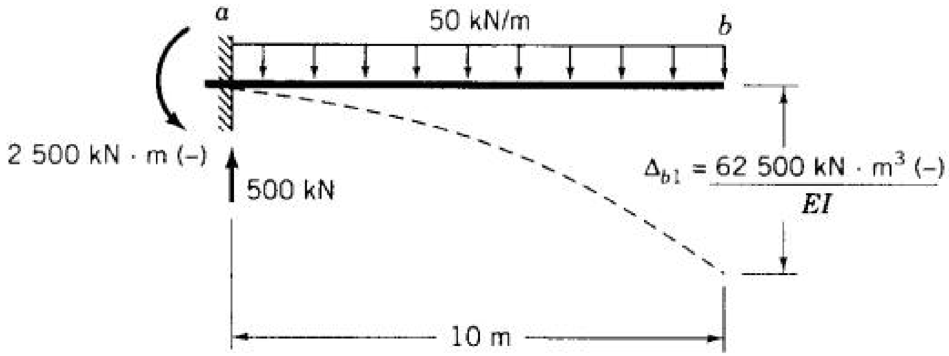


Fig. 9.4 (a) Primary structure subjected to given loading

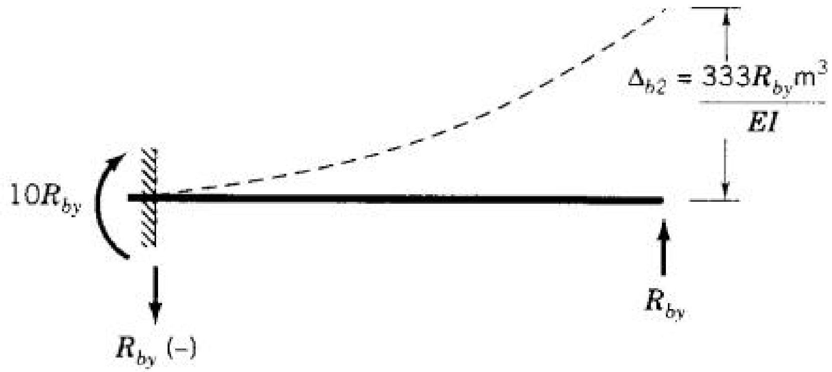
●However the deformation of the structure does not satisfy the boundary condition (境界条件) at point b.

● That is, the vertical restraint required by the support point b is not maintained.

●To remedy this problem, we allow the primary structure to be acted upon by the redundant reaction R_{by} as shown in Fig. 9.4 (b).



(a) Primary structure subjected to given loading



(b) Primary structure subjected to redundant reaction

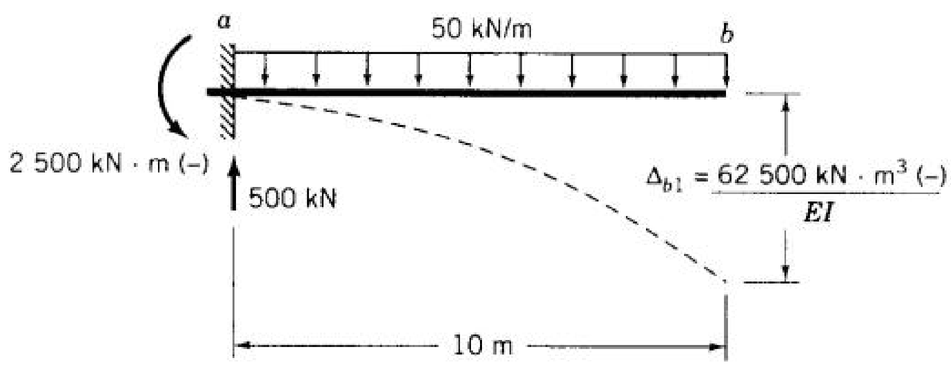
Fig. 9.4 Primary structure of Fig. 9.3

●The deflection of point b due to a point load of R_{by} is

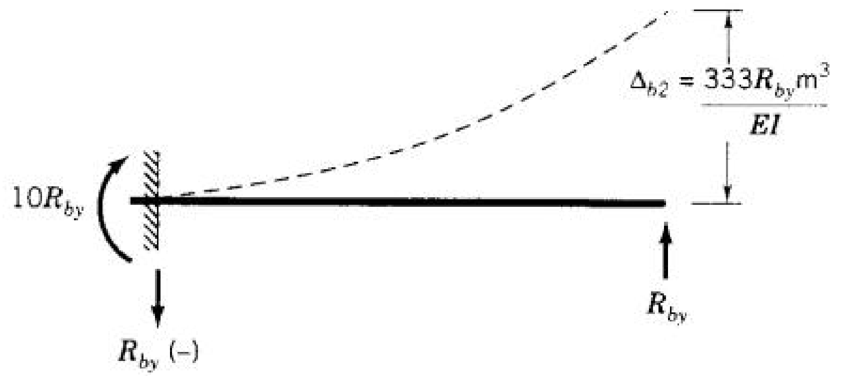
$$\Delta_{b2} = \frac{R_{by} l^3}{3EI}$$

$$= \frac{333R_{by}}{EI}$$

●For a solution that includes the proper loading and also satisfies the designated boundary condition, the solutions shown in Fig. 9.4(a) and 9.4(b) must be superposed.



(a) Primary structure subjected to given loading

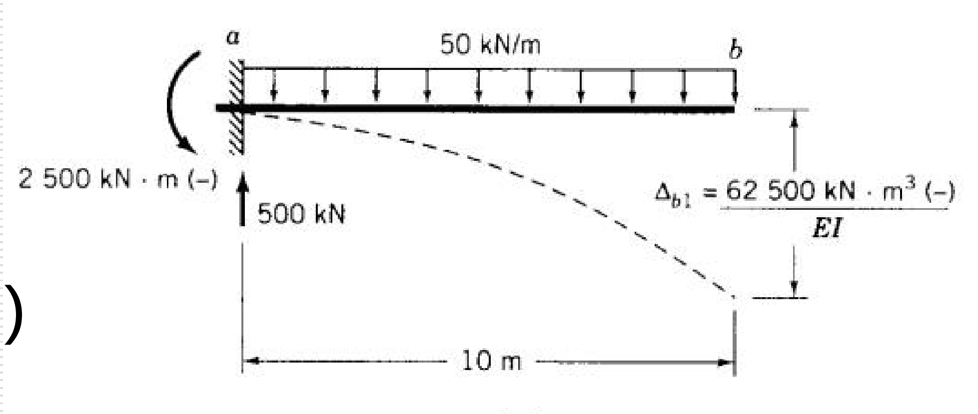


(b) Primary structure subjected to redundant reaction

Fig. 9.4 Primary structure of Fig. 9.3

●Thus, we have the compatibility condition (変位適合条件)

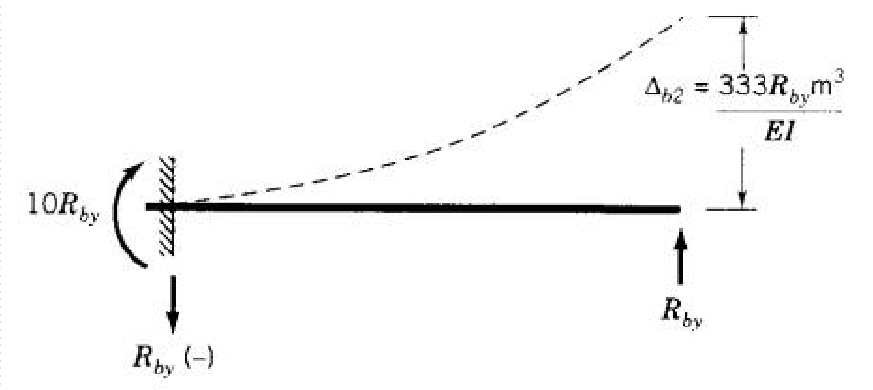
$$\Delta_b = \Delta_{b1} + \Delta_{b2} \qquad (9.3)$$



(a) Primary structure subjected to given loading

●Upon substitution and rearrangement,

$$\frac{333R_{by}}{EI} = \frac{62500}{EI} \qquad (9.4)$$



(b) Primary structure subjected to redundant reaction

●From which

$$R_{by} = 187.5kN \qquad (9.5)$$

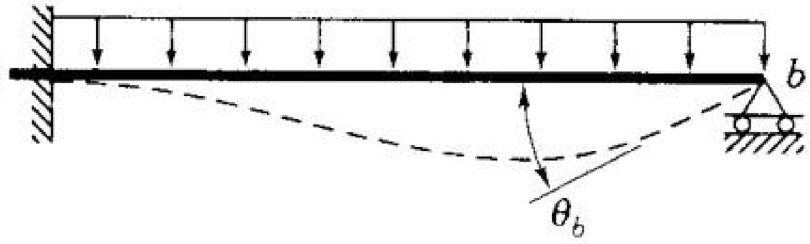
Fig. 9.4 Primary structure of Fig. 9.3

9.4 KINEMATIC INDETERMINACIES (幾何学的不静定)；REDUNDANCIES

1) What is kinematic indeterminacies?

- static indeterminacy refer to number of force quantities that must be determined in order to render the equilibrium solution complex.
- On the other hand, kinematic indeterminacy refers to the number of displacement quantities (kinematic degree of freedom) that are necessary to define the deformation response of the structure.

2) Example of kinematic indeterminacy



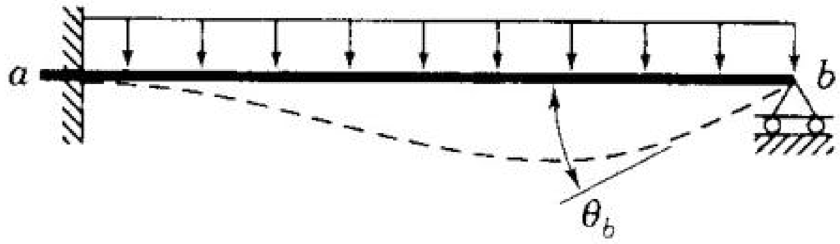
(a) Kinematically indeterminate structure with redundant displacement

Fig. 9.5 Selection of redundant displacement

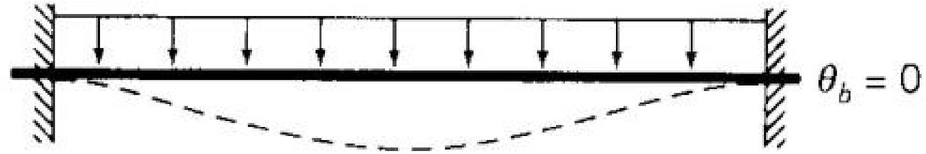
- Since the structure is fixed at point a and vertically restrained at point b and the axial deformation is zero, there is only one kinematic degree of freedom, θ_b
- We identify the rotation θ_b as a kinematic redundant.

● If the rotation θ_b is removed from the structure ($\theta_b = 0$), the resulting primary structure is called kinematically determinate (**幾何学的静定**).

● Fig. 9.5(b) shows the kinematically determinate primary structure.



(a) Kinematically indeterminate structure with redundant displacement



(b) Kinematically determinate primary structure

Fig. 9.5 Selection of redundant displacement

9.5 ALTERNATIVE FORM OF ANALYSIS

- Consider again the structure of Fig. 9.1 as shown in Fig. 9.6(a).
- An end moment M_{bz} has been added which must be eventually be set equal to zero.

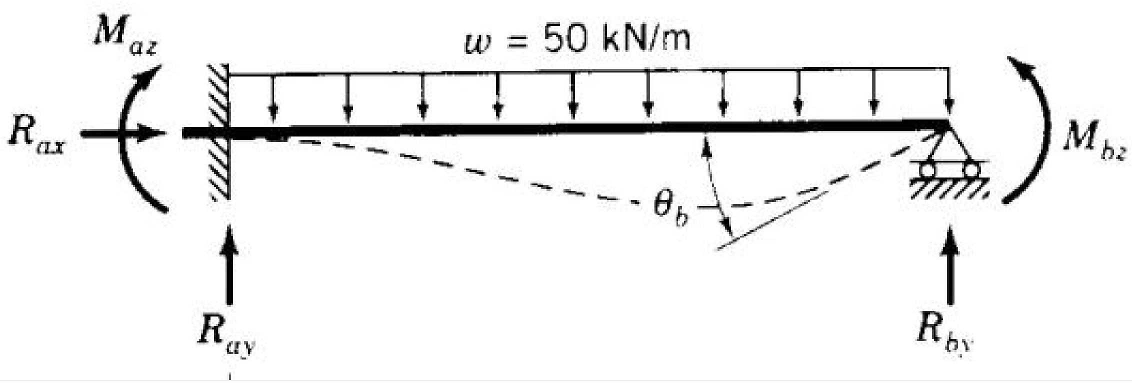


Fig. 9.6(a) Loading & deformation of a statically indeterminate beam

●Next, the statically determinate primary structure of Fig. 6.3(c) is separately loaded with w, R_{by} and M_{bz} as shown in Fig. 9.6(b)

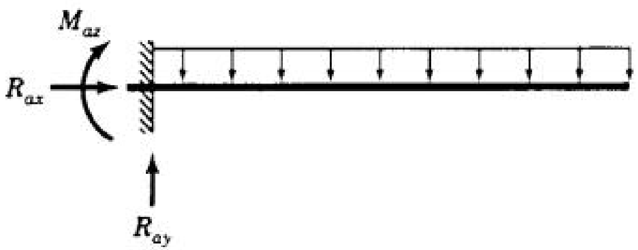


Fig. 9.3(c) Stable and statically determinate primary structure

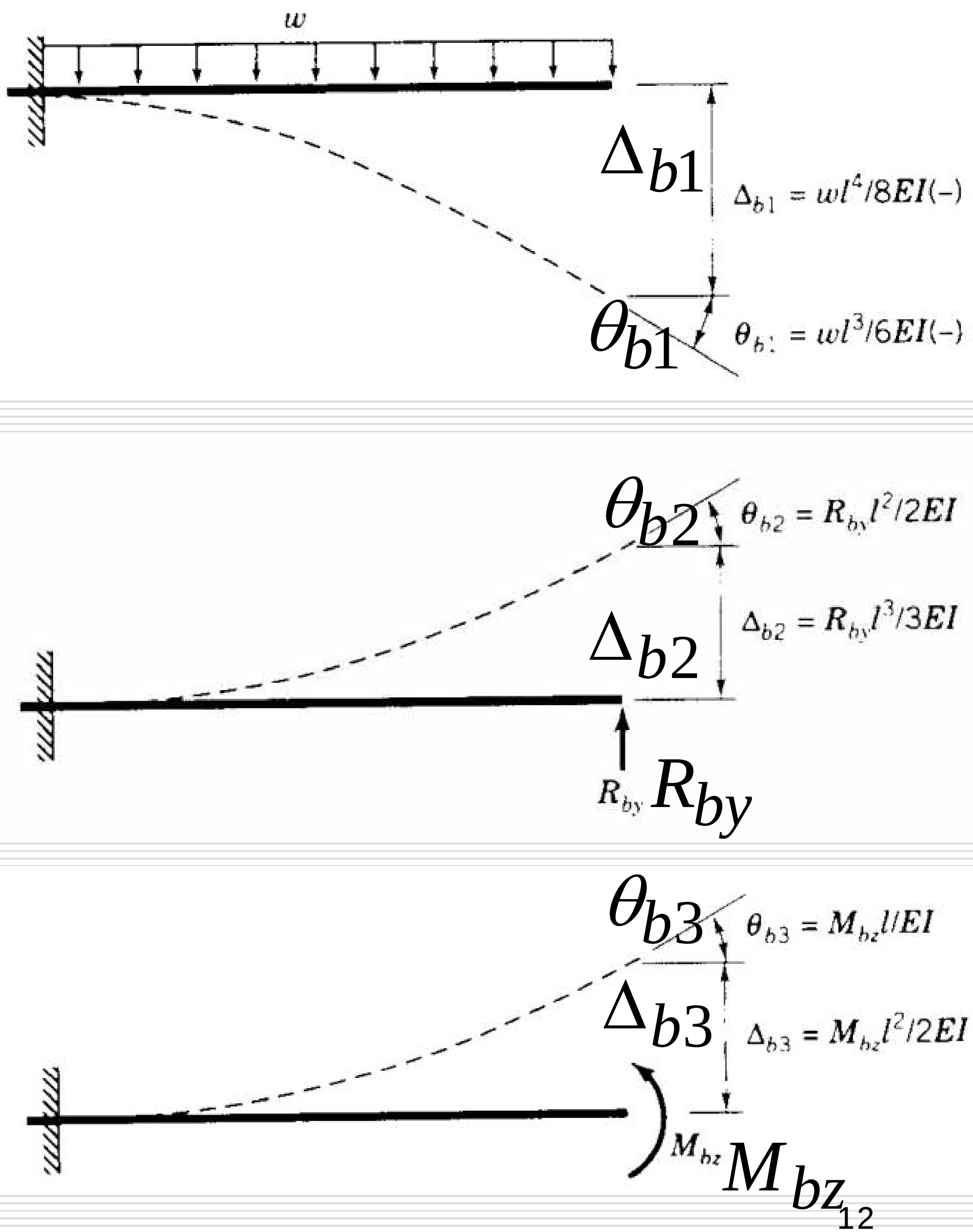


Fig. 9.6 (b)

● The separate solutions of Fig. 9.6(b) must be superimposed to obtain the correct boundary conditions for the given structure of Fig. 9.6(a) as

$$\Delta_{b1} + \Delta_{b2} + \Delta_{b3} = \Delta_b = 0$$

$$\theta_{b1} + \theta_{b2} + \theta_{b3} = \theta_b \qquad (9.6)$$

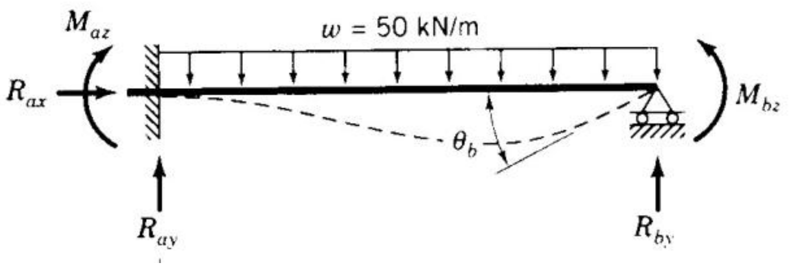


Fig. 9.6(a)

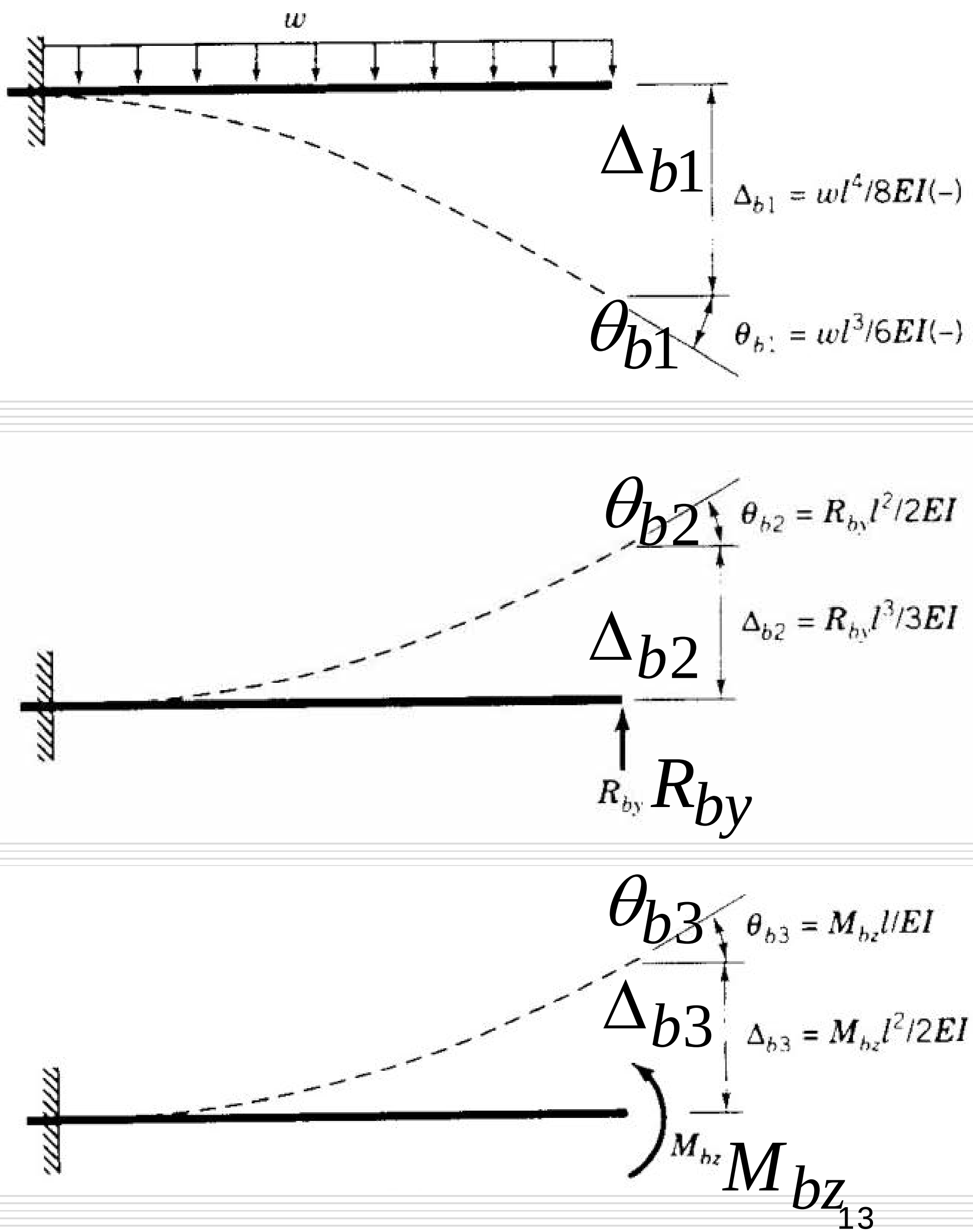


Fig. 9.6 (b)

●Substitution of the displacement quantities into Eq. (9.6) yields

$$\frac{R_{by}l^3}{3EI}+\frac{M_{bz}l^2}{2EI}=\frac{wl^4}{8EI}$$

$$\frac{R_{by}l^2}{2EI}+\frac{M_{bz}l}{EI}=\frac{wl^3}{6EI}+\theta_b$$

(9.7)

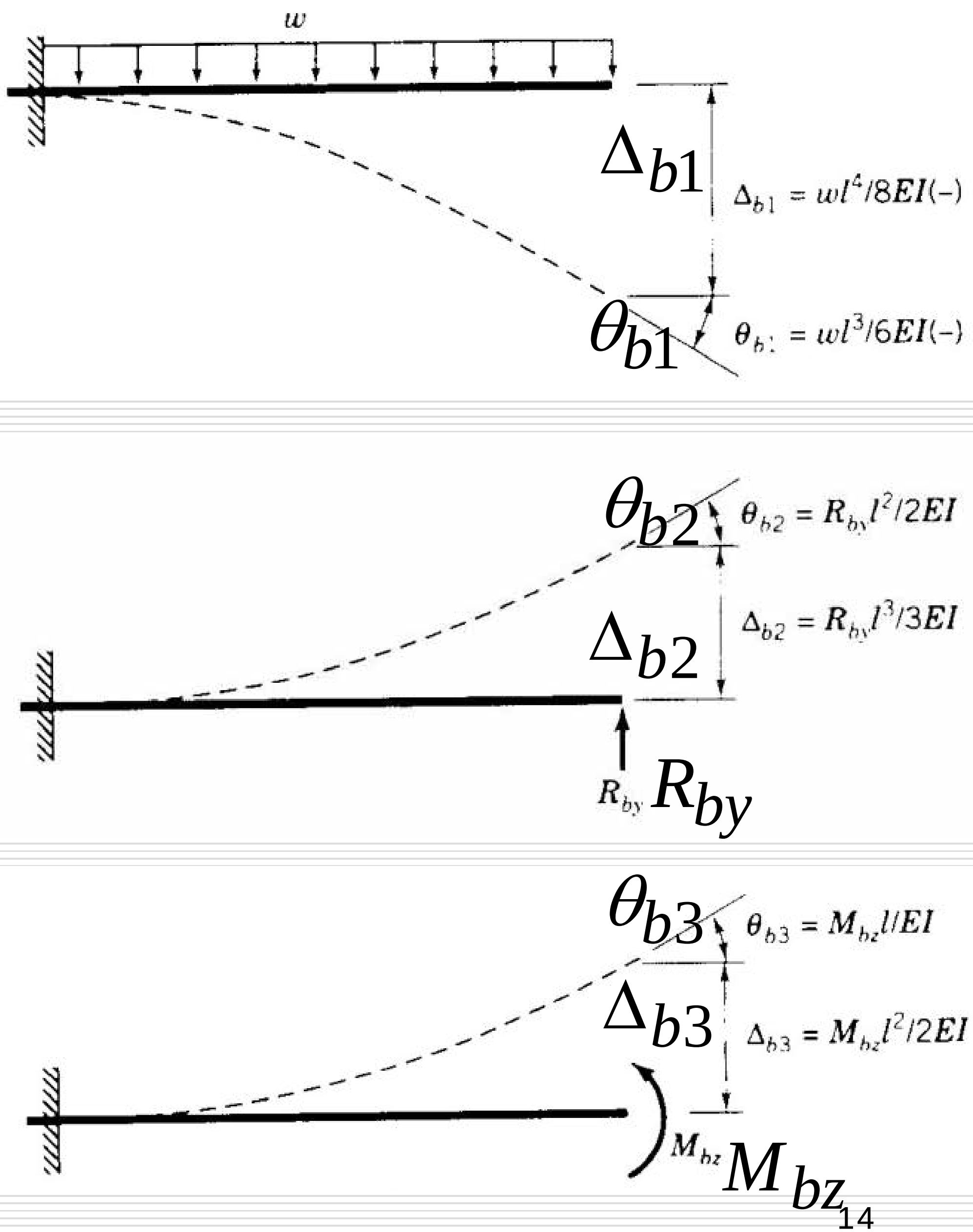


Fig. 9.6 (b)

●Static equilibrium can be written

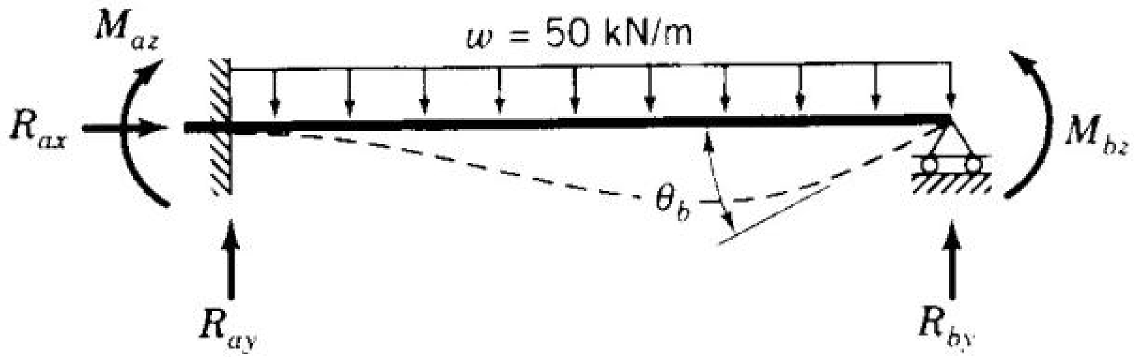


Fig. 9.6(a) Loading & deformation of a statically indeterminate beam

$$R_{ay} + R_{by} = wl$$

$$M_{az} + R_{ay}l - wl\frac{l}{2} - M_{bz} = 0$$

$$\boxed{\begin{aligned} \frac{R_{by}l^3}{3EI} + \frac{M_{bz}l^2}{2EI} &= \frac{wl^4}{8EI} \\ \frac{R_{by}l^2}{2EI} + \frac{M_{bz}l}{EI} &= \frac{wl^3}{6EI} + \theta_b \end{aligned} \qquad (9.7)}$$

●The static equilibrium and Eq. (9.7) can be solved as

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ l & 1 & 0 & -1 \\ 0 & 0 & l^3/3EI & l^2/2EI \\ 0 & 0 & l^2/2EI & l/EI \end{bmatrix} \begin{Bmatrix} R_{ay} \\ M_{az} \\ R_{by} \\ M_{bz} \end{Bmatrix} = \begin{Bmatrix} wl \\ wl^2/2 \\ wl^4/8EI \\ wl^3/6EI + \theta_b \end{Bmatrix}$$

$$\boxed{\begin{aligned} & R_{ay} + R_{by} = wl \\ & M_{az} + R_{ay} l - wl \frac{l}{2} - M_{bz} = 0 \\ & \frac{R_{by} l^3}{3EI} + \frac{M_{bz} l^2}{2EI} = \frac{wl^4}{8EI} \\ & \frac{R_{by} l^2}{2EI} + \frac{M_{bz} l}{EI} = \frac{wl^3}{6EI} + \theta_b \qquad (9.7) \end{aligned}}$$

●Solving the equations shown in the previous page, we have

$$R_{ay} = \frac{wl}{2} + \frac{3k_b}{2l}\theta_b \qquad R_{by} = \frac{wl}{2} - \frac{3k_b}{2l}\theta_b$$

$$M_{az} = -\frac{wl^2}{12} - \frac{k_b}{2}\theta_b \qquad M_{bz} = -\frac{wl^2}{12} + k_b\theta_b \tag{9.8}$$

where,

$$k_b = \frac{4EI}{l}$$

●Eq. (9.8) expresses all of the response quantities in terms of the single kinematic degree of freedom θ_b

●Solutions of the kinemetically determinate primary structure can be obtained from Eq. (9.8) by substituting $\theta_b = 0$

$$R_{ay1} = \frac{wl}{2} \quad R_{by1} = \frac{wl}{2} \quad M_{az1} = -\frac{wl^2}{12} \quad M_{bz1} = -\frac{wl^2}{12}$$

(9.8)

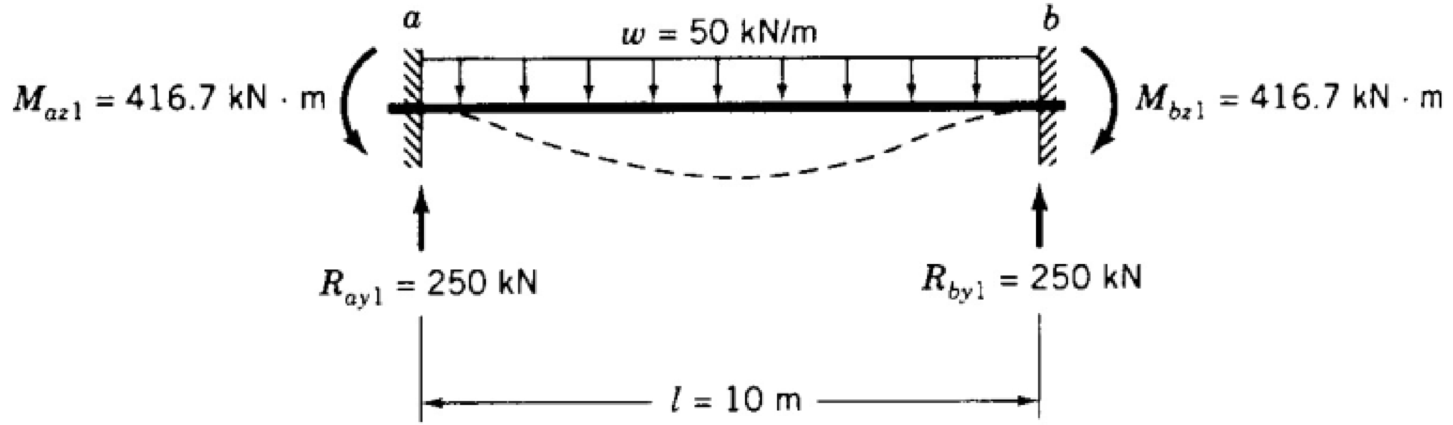


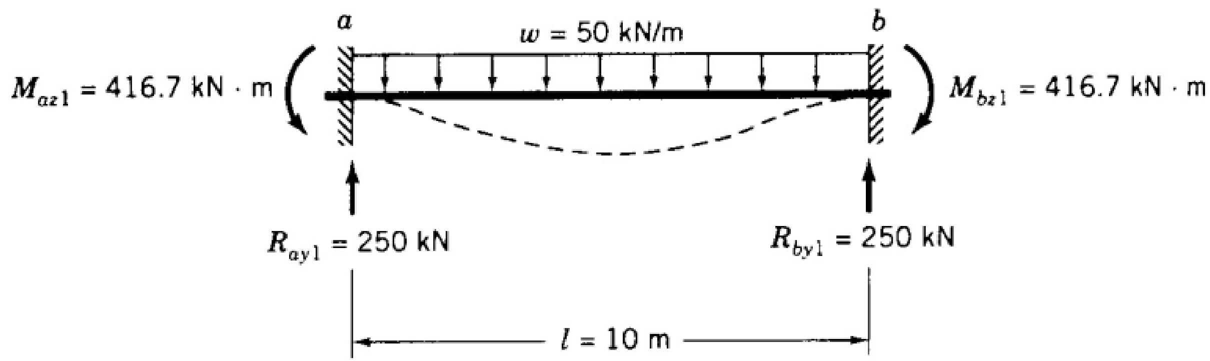
Fig. 9.8 (a) Primary structure subjected to given loading

$$R_{ay} = \frac{wl}{2} + \frac{3k_b}{2l}\theta_b \qquad R_{by} = \frac{wl}{2} - \frac{3k_b}{2l}\theta_b$$

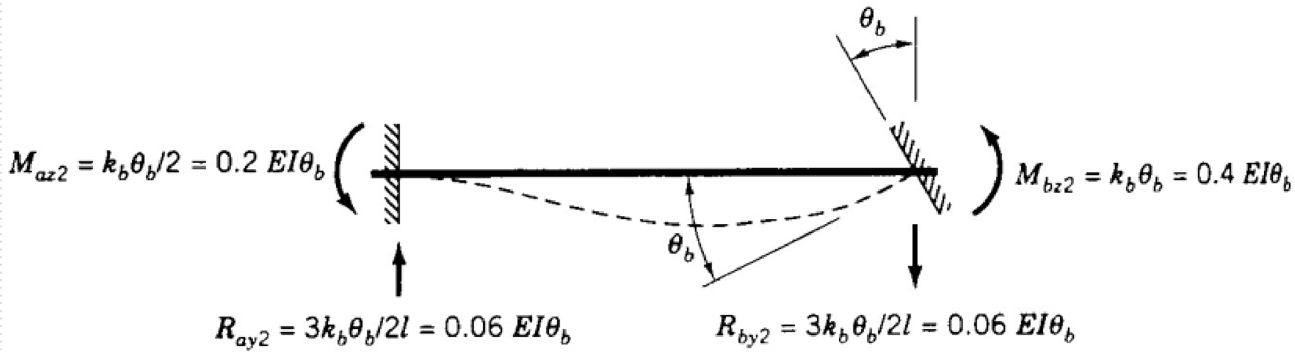
$$M_{az} = -\frac{wl^2}{12} - \frac{k_b}{2}\theta_b \qquad M_{bz} = -\frac{wl^2}{12} + k_b\theta_b \qquad (9.8)$$

- Although this solution is a valid equilibrium solution, it violates the required force boundary condition on the moment at point b.

- To remedy this problem, the primary structure is subjected to the redundant displacement θ_b



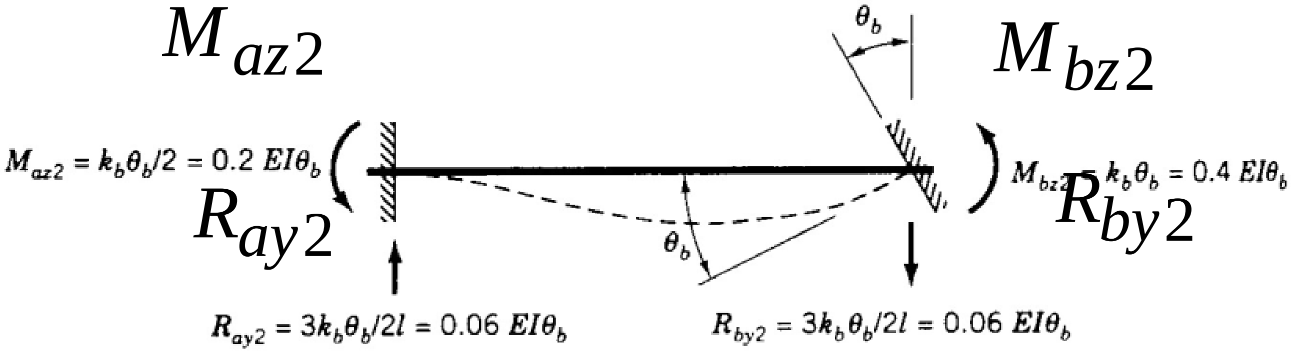
(a) Primary structure subjected to given load



(b) Primary structure subjected to redundant rotation

Fig. 9.8 Kinematic primary structure

●Assuming that $w=0$ in Eq. (9.8), and based on the associated end moments and reactions take on the values as



(b) Primary structure subjected to redundant rotation

$$R_{ay2} = \frac{3k_b}{2l}\theta_b = 0.06EI\theta_b \qquad R_{by2} = \frac{3k_b}{2l}\theta_b = 0.06EI\theta_b$$

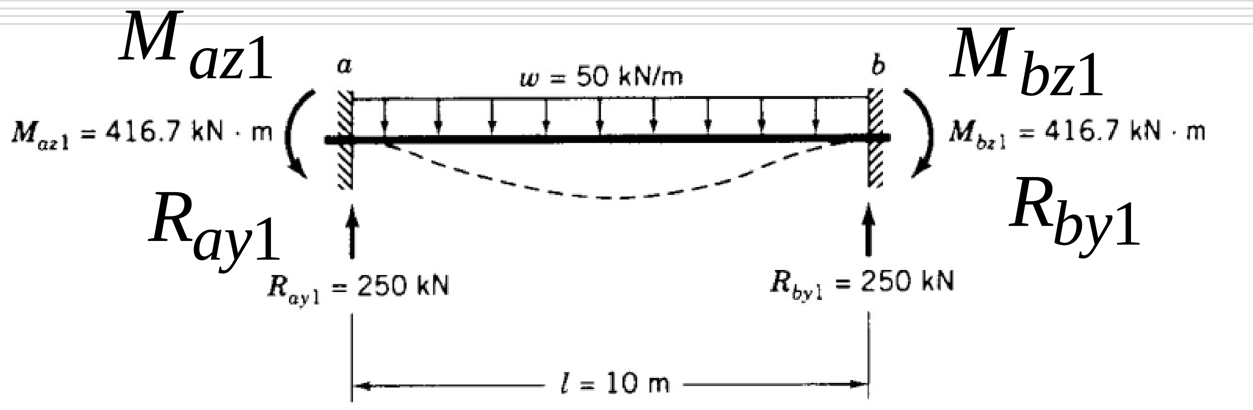
$$M_{az2} = -\frac{k_b}{2}\theta_b = -0.2EI\theta_b \qquad M_{bz2} = k_b\theta_b = 0.4EI\theta_b$$

$$\boxed{\begin{aligned} R_{ay} &= \frac{wl}{2} + \frac{3k_b}{2l}\theta_b & R_{by} &= \frac{wl}{2} - \frac{3k_b}{2l}\theta_b \\ M_{az} &= -\frac{wl^2}{12} - \frac{k_b}{2}\theta_b & M_{bz} &= -\frac{wl^2}{12} + k_b\theta_b \end{aligned} \quad (9.8)}$$

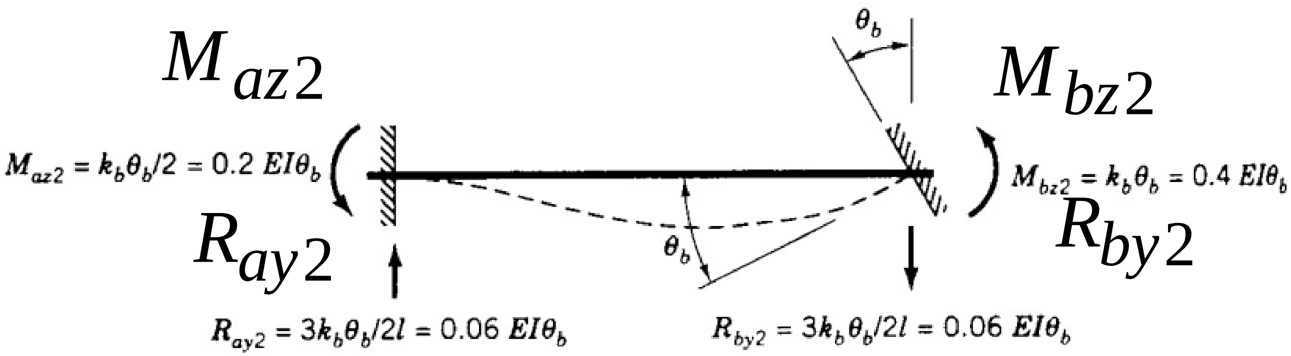
- For a solution that reflects the proper loading and satisfies the designated force boundary conditions at point b, the solution shown in Figs. 9.8(a) and 9.8(b) must be superimposed so that the final moment at point b is zero.

- Thus,

$$M_{bz1} + M_{bz2} = 0$$



(a) Primary structure subjected to given load



(b) Primary structure subjected to redundant rotation

Fig. 9.8 Kinematic primary structure

- Upon substitution,

$$M_{bz1} + M_{bz2} = -\frac{wl^2}{12} + \frac{4EI}{l}\theta_b = 0$$

- from which

$$\theta_b = \frac{wl^3}{48EI}$$

- Retuning to Eq. (9.8), we have

$$R_{ay} = \frac{5wl}{8} = +312.5kN \qquad R_{by} = \frac{3wl}{8} = 187.5kN$$

$$M_{az} = -\frac{wl^2}{8} = -625kNm \qquad M_{bz} = 0$$

$$R_{ay} = \frac{wl}{2} + \frac{3k_b}{2l}\theta_b \qquad R_{by} = \frac{wl}{2} - \frac{3k_b}{2l}\theta_b$$

$$M_{az} = -\frac{wl^2}{12} - \frac{k_b}{2}\theta_b \qquad M_{bz} = -\frac{wl^2}{12} + k_b\theta_b \qquad (9.8)$$