### Structural Analysis II 構造力学第二 (6)

## Kazuhiko Kawashima Department of Civil Engineering Tokyo institute of Technology 東京工業大学大学院理工学研究科土木工学専攻 川島一彦

# 10.5 Support Settlements (支点沈下) and Elastic Supports (弹性支点)

• Support settlement in statically indeterminate structures induces forces throughout the structure.

• Consider the structure shown in Fig. 10.1 The compatibility equation given as Eq. (10.2) includes the displacement  $\Delta_1$ , which is the upward vertical displacement at point b for the actual structure.



 $\bullet$  If thee is a downward settlement of  $\Delta_{1s}$  , then

 $\Delta_1 = -\Delta_{1s}$ 

In such a case,  $R_1$  becomes from Eq. (10.2) as

$$R_1 = \frac{-\Delta_{1s} - \Delta_{10}}{D_{11}} \qquad (1)$$

• Thus, it is clear that the magnitude of redundant reaction is affected by the support settlement

 Once R<sub>1</sub> has been obtained, the final response quantities are evaluated by superposition



• In a similar manner, if downward settlements of  $\Delta_{1s}$ and  $\Delta_{2s}$  are prescribed for the structure of Fig. 10.2, Eq. (10.8) becomes

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} -\Delta_{s1} - \Delta_{10} \\ -\Delta_{2s} - \Delta_{20} \end{bmatrix}$$
(10.11)



• If there are prescribed settlements for the end supports (はりの両端で支点沈下が与えられる場合), then and  $\Delta_1$  and  $\Delta_2$  must be interpreted as the displacement relative to the chord connecting the ends of the beam (はりの両端を結ぶ直線からの距離) as shown in Fig. 10.3.



Figure 10.3 Settlement of continuous beam. Note that upward displacements are positive.

#### Elastic Supports (弹性支持)

• Frequently, structures are mounted on supports in which the support movement depends on the magnitude of the reaction force.

The simplest
representation of this kind of
support is shown in Fig. 10.4
and it is called elastic
support in which settlement
is a linear function of the
reaction force.







# • If the supports at points b and c of Fig. 10.2 (a) are provided by elastic supports as shown in Fig. 10.5,





• This arrangement would correspond to the case in which  $R_1$ and  $R_2$  are provided by columns. Here, the elastic constants  $k_{s1}$ and  $k_{s2}$  would be a function of the properties of the columns



 Redundant reactions can be determined by solving Eq. (10.15), and any other response quantities are determined from Eq. (10.9).

$$\begin{bmatrix} D_{11} + 1/k_s & D_{12} \\ D_{21} & D_{22} + 1/k_s \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} -\Delta_{10} \\ -\Delta_{20} \end{bmatrix}$$
(10.15)  
$$S = S_0 + S_1 R_1 + S_2 R_2$$
(10.9)

#### EXAMPLE 10.4

Consider the continuous beam of Example 10.2, and determine the moment diagram for the following set of support settlements:

$$\Delta_a = -27.5mm \qquad \Delta_b = -47.5mm$$
  
$$\Delta_c = -22mm \qquad \Delta_d = -10mm$$



#### Settlement pattern



**Structural Classification** 

• This structure is statically indeterminate to the second order. (same as Example 10.2)

Compatibility equations

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} \Delta_1 - \Delta_{10} \\ \Delta_2 - \Delta_{20} \end{bmatrix}$$
(10.8)

Redundant reactions

In this case,  $\Delta_{10} = \Delta_{20} = 0$ , and thus  $\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$ (a)

14



•Solving Eq. (b) for the redundant reactions, we obtain

$$\begin{cases} R_1 \\ R_2 \end{cases} = EI \begin{cases} -99.1 \times 10^{-6} \\ 69.5 \times 10^{-6} \end{cases} (\frac{EI}{kN \cdot m^2})$$

$$\frac{1}{EI}\begin{bmatrix} 595.1 & 488.4 \\ 488.1 & 595.1 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} -0.025 \\ -0.007 \end{bmatrix}$$
(b)

Final moments

 $S = S_0 + S_1 R_1 + S_2 R_2 \qquad (10.9)$ 

• In this case,  $M_0=0$  (because no load is applied to the structure), and  $M_1$  and  $M_2$  are the moment diagram ordinates given in Example 10.2.





Note that reaction forces and moments are dependent on the flexural stiffness (曲げ剛性) EI.
For a specific case of EI=3000x10<sup>-6</sup>m<sup>4</sup> and E=200x10<sup>9</sup>Pa, we obtain

