## WPBE in Ex.9.C. 3


$\mathrm{I}_{2}$ dominates $\mathrm{O} \rightarrow \sigma_{0}=0$
$\chi \geq 0$
(-1< $\ll 0 \rightarrow$ Ex.9.C.2)
E's strategy: $\left(\sigma_{0}, \sigma_{1}, \sigma_{2}\right)$
I's strategy: $\left(\sigma_{\mathrm{F}}, 1-\sigma_{\mathrm{F}}\right)$
I's belief: $\left(\mu_{1}, 1-\mu_{1}\right)$

|  | F | A |
| :--- | :--- | :--- |
| O | 0,2 | 0, |
| $\mathrm{I}_{1}$ | $-1,-1$ | $3,-2$ |
| $\mathrm{I}_{2}$ | $\gamma, 1$ | 2, |

## WPBE in Ex.9.C. 3


$\mu_{1}>2 / 3$
I plays $\mathrm{F}\left(\sigma_{\mathrm{F}}=1\right)$
$\rightarrow$ E plays $\mathrm{I}_{2}$ since $\gamma>0>-1$
$\rightarrow \mu=(0,1) \quad C!$ to $\mu_{1}>2 / 3$

## WPBE in Ex.9.C. 3


$\mu_{1}<2 / 3$
I plays $\mathrm{A}\left(\sigma_{\mathrm{F}}=0\right)$
$\rightarrow$ E plays $\mathrm{I}_{1}$ since $3>2>0$
$\rightarrow \quad \mu=(1,0) \quad \mathrm{C}!$ to $\mu_{1}<2 / 3$

## WPBE in Ex.9.C. 3



E: $\sigma_{1}=2 / 3, \sigma_{2}=1 / 3$ since $\sigma_{0}=0, \quad \mu_{1}=2 / 3$ and $1-\mu_{1}=1 / 3$
$\rightarrow \mathrm{E}: \mathrm{I}_{1}$ and $\mathrm{I}_{2}$ are indifferent under $\left(\sigma_{\mathrm{F}}, 1-\sigma_{\mathrm{F}}\right)$ since $\sigma_{1}, \sigma_{2}>0$

## WPBE in Ex.9.C. 3


$\underline{\mu}_{1}=2 / 3$
E: $I_{1}$ and $\mathrm{I}_{2}$ are indifferent under $\left(\sigma_{\mathrm{F}}, 1-\sigma_{\mathrm{F}}\right)$ since $\sigma_{1}, \sigma_{2}>0$.
E's payoff: $\mathrm{I}_{1} \rightarrow-\sigma_{\mathrm{F}}+3\left(1-\sigma_{\mathrm{F}}\right), \mathrm{I}_{2} \rightarrow \gamma \sigma_{\mathrm{F}}+2\left(1-\sigma_{\mathrm{F}}\right)$

$$
-\sigma_{\mathrm{F}}+3\left(1-\sigma_{\mathrm{F}}\right)=\gamma \sigma_{\mathrm{F}}+2\left(1-\sigma_{\mathrm{F}}\right) \rightarrow \sigma_{\mathrm{F}}=1 /(\gamma+2)
$$

I's strategy: $(1 /(\gamma+2),(\gamma+1) /(\gamma+2))$

## WPBE in Ex.9.C. 3



WPBE
$((0,2 / 3,1 / 3),(1 /(\gamma+2),(\gamma+1) /(\gamma+2)), \mu=(2 / 3,1 / 3))$

## Sequential Equilibrium (motivation, Ex.9.C.4)



P2 has an arbitrary belief since his information set is not reached in equilibrium. ? ? ?

## Sequential Equilibrium (motivation, Ex.9.C.5)


$((\mathrm{O}, \mathrm{A}), \mathrm{F},(1,0))$
$\rightarrow \mathrm{WPBE}$

| I | $F$ | $A$ |
| :---: | :---: | :---: |
| F | $-3, \underline{-1}$ | $1,-2$ |
| $A$ | $\underline{-2},-1$ | $\underline{3}, \underline{1}$ |

Nash eq $\rightarrow$ (A, A)

## Sequential Equilibrium (definition)

Def. 9.C.4: $(\sigma, \mu)$ is a sequential equilibrium (SE) if
(i) $\sigma$ is sequentially rational given $\mu$;
(ii) $\exists$ a sequence of completely mixed strategies $\left\{\sigma^{k}\right\}_{k=1}{ }^{\infty}$
with $\lim _{\mathrm{k} \rightarrow \infty} \sigma^{\mathrm{k}}=\sigma$ such that $\mu=\lim _{\mathrm{k} \rightarrow \infty} \mu^{\mathrm{k}}$
where $\mu^{\mathrm{k}}$ is the set of beliefs derived from $\sigma^{\mathrm{k}}$ using Bayes' rule.

## Sequential Equilibrium (Ex. 9.C.4)



For any comp. mixed strategy $\left(\sigma_{x}, \sigma_{y}\right)$, P 2 's belief $=(.5, .5)$
P2's choice must be " $r$ " since $5<2 \times .5+10 \times .5=6$
P1's choice must be "y" since $2<5$

$$
\mathrm{SE} \rightarrow(\mathrm{y}, \mathrm{r},(.5, .5),(.5, .5))
$$

## Sequential Equilibrium (Ex. 9.C.5)



SE must contain (A, A). ( $\rightarrow$ next slide)

## Sequential Equilibrium (Ex. 9.C.5)



## Sequential Equilibrium (Ex. 9.C.5)


$\operatorname{Prob}\left(\mathrm{H} \mid \sigma^{\mathrm{k}}\right)=\sigma_{\mathrm{E}}^{\mathrm{k}}(\mathrm{I})=1-\varepsilon, \operatorname{Prob}\left(\mathrm{x} \mid \sigma^{\mathrm{k}}\right)=\sigma_{\mathrm{E}}^{\mathrm{k}}(\mathrm{I}) \times \sigma_{\mathrm{E}}^{\mathrm{k}}(\mathrm{F})=(1-\varepsilon) \varepsilon^{\prime}$

$$
\mu^{\mathrm{k}}(\mathrm{x})=\varepsilon^{\prime} \rightarrow \mu(\mathrm{x})=0 \quad \mu^{\mathrm{k}}(\mathrm{y})=1-\varepsilon^{\prime} \rightarrow \mu(\mathrm{y})=1
$$

## Sequential Equilibrium and SPNE

Prop. 9.C.2: In every SE $(\sigma, \mu), \sigma$ is an SPNE.

## Assignments

Problem Set 10 (due July 19)
Exercises (pp.301-305)

$$
\text { 9.C.2, 9.C.6 (only 9.C. } 3 \text { part) }
$$

Reading Assignment:
Text, Chapter 9, pp.292-300

