Properties of SPNE (Prop. 9.B.3)

Prop. 9.B.3 : Γ_E : an extensive form game, S : a subgame

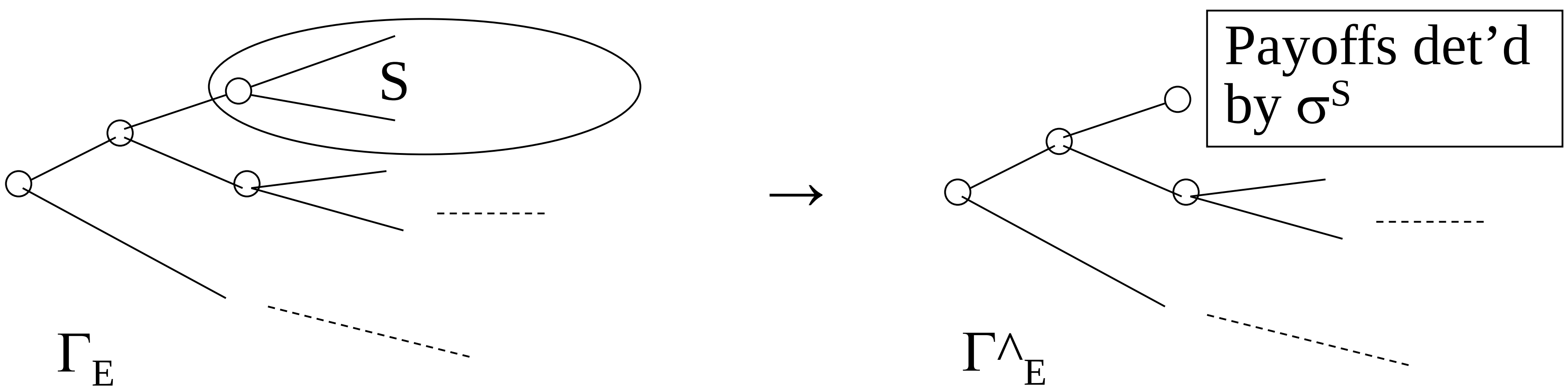
σ^S : an SPNE of subgame S

$\Gamma\wedge_E$: the reduced game replacing the subgame S by a terminal node with payoff determined by σ^S

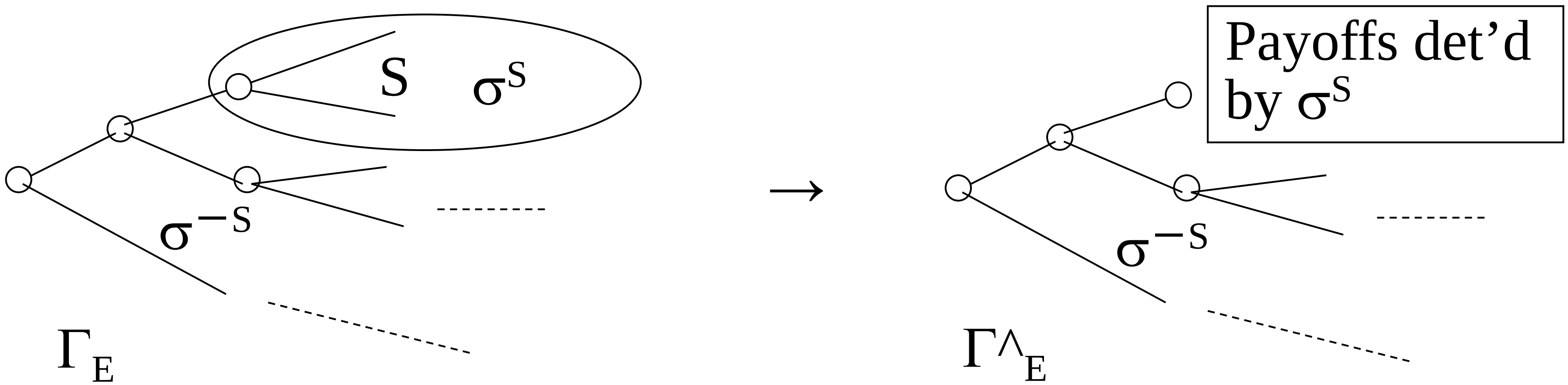
(1) σ : an SPNE of Γ_E s.t. restriction of σ to S is σ^S.

σ^{-S}, the restriction of σ to outside S $\rightarrow$ σ^{-S} is an SPNE of $\Gamma\wedge_E$

(2) $\sigma\wedge$: an SPNE of $\Gamma\wedge_E$ $\rightarrow$ $(\sigma\wedge, \sigma^S)$ is an SPNE of Γ_E



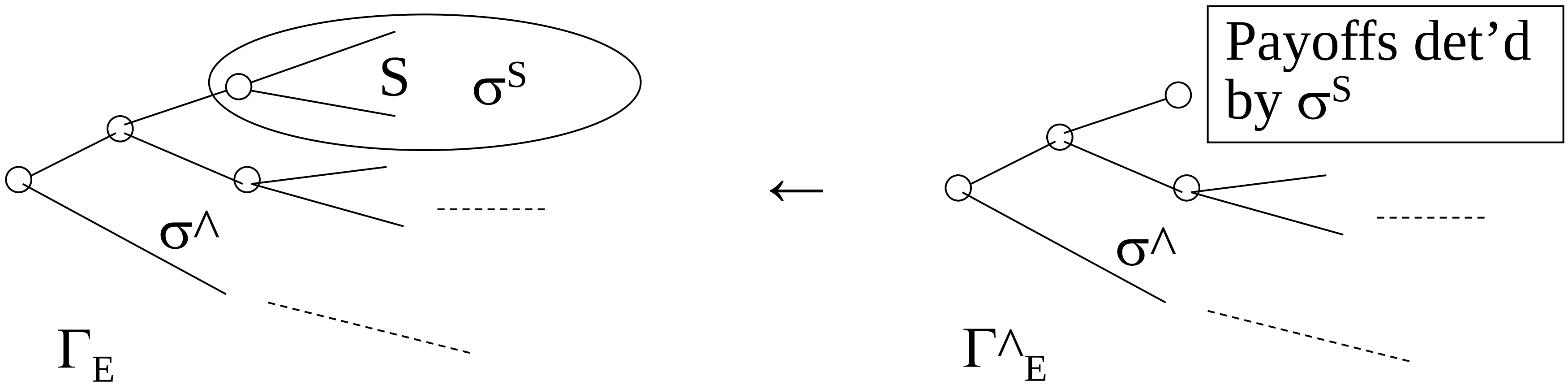
Proof of Prop. 9.B.3



(1) σ : an SPNE of Γ_E σ^S : restriction of σ to S
σ^{-S} : restriction of σ to outside S
$\rightarrow$ σ^{-S} is an SPNE of $\Gamma^\wedge_E$

Pf: Suppose σ^{-S} is not an SPNE of $\Gamma^\wedge_E$.
Then $\exists$ a subgame T of $\Gamma^\wedge_E$ s.t. σ^T is not a Nash eq. in $\Gamma^\wedge_E$.
$\exists$i who can increase his payoff by deviating from σ^T in $\Gamma^\wedge_E$.
i can increase his payoff in Γ_E by the same deviation.

Proof of Prop. 9.B.3



(2) $\sigma^\wedge$: an SPNE of $\Gamma^\wedge_E$ $\rightarrow$ $(\sigma^\wedge,\ \sigma^S)$ is and SPNE of Γ_E

<u>Pf</u>: Let $\sigma' = (\sigma^\wedge,\ \sigma^S)$. Take any subgame T.

If $T \subseteq S$ or $T \subseteq {}^{-}S$, then σ'^{T} is a Nash eq. of T.

If not, T contains S.

Suppose $\exists$ i who can gain more by deviating from σ'_i .

Since σ^S is an SPNE of S, i changes his choice outside S.

Then i can gain more also in $\Gamma^\wedge_E$. C! Q.E.D.

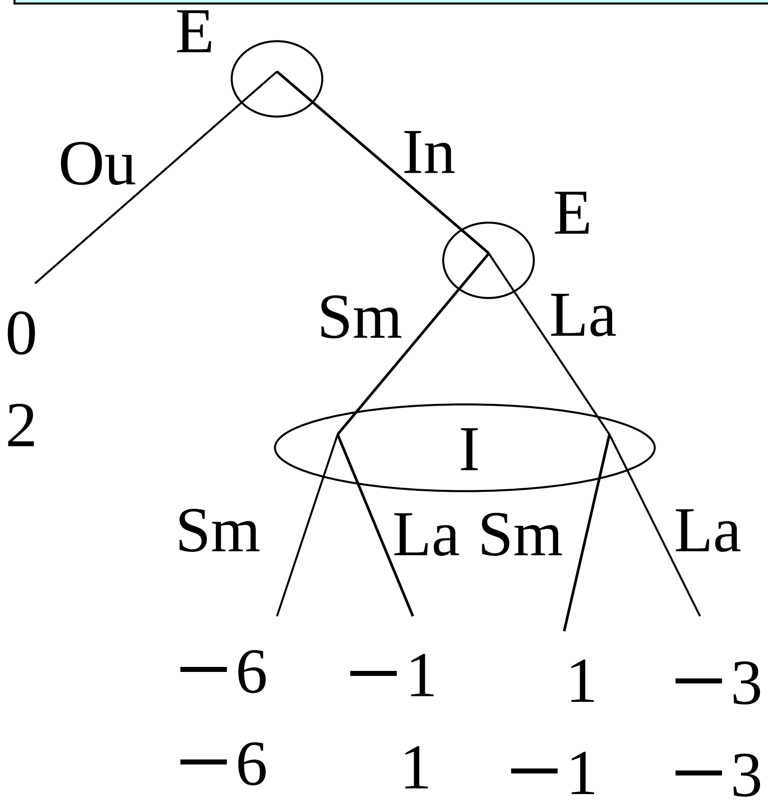
Generalized Backward Induction

1 Start at the end of the game tree. Identify Nash eq. in each of the final subgames.

2 Select one Nash eq. in each of the final subgames, and derive the reduced extensive form game by replacing each subgame by a terminal node with payoffs of the selected Nash eq.

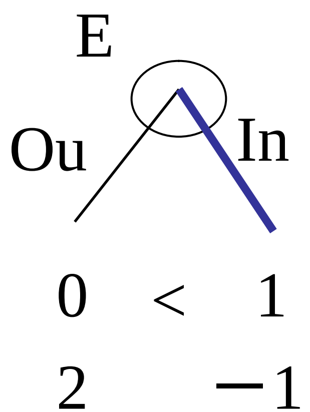
3 Repeat this procedure until every move in the original extensive form game is determined.

Example 9.B.4



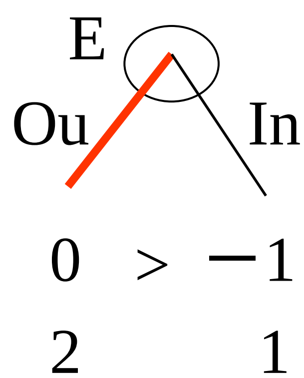
		I	
		Sm	La
E	Sm	$-6, -6$	$\underline{-1}, \underline{1}$
	La	$\underline{1}, \underline{-1}$	$-3, -3$

Nash eq. (La, Sm), (Sm, La)



SPNE

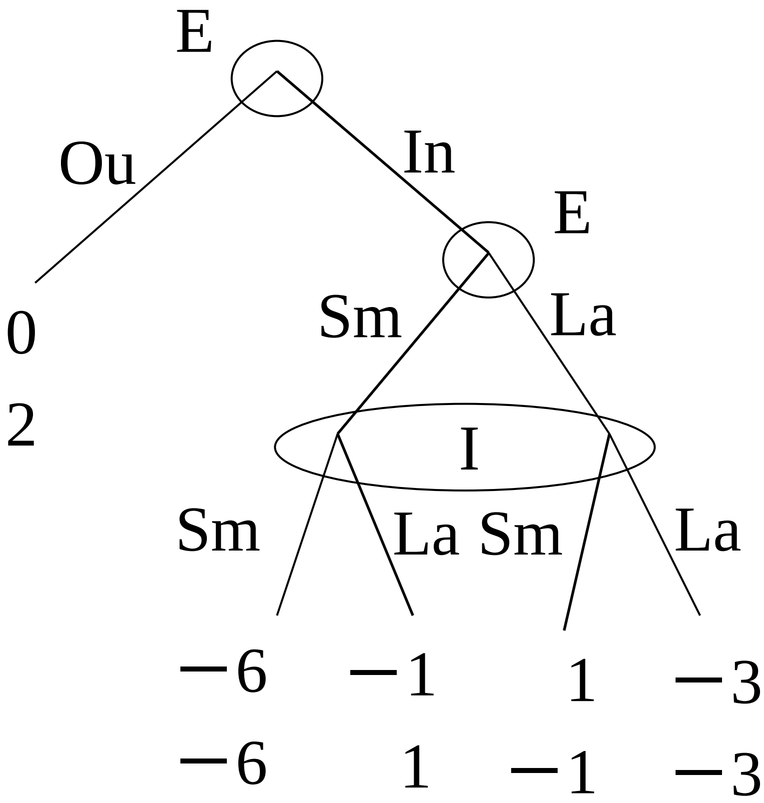
((In, La), Sm)



SPNE

((Ou, Sm), La)

Example 9.B.4 (Ex. 9.B.6)
Mixed strategy Nash eq. in the subgame



		I	
		Sm	La
E	Sm	$-6, -6$	$\underline{-1}, \underline{1}$
	La	$\underline{1}, \underline{-1}$	$-3, -3$

Nash eq. (La, Sm), (Sm, La)

Mixed strategy Nash eq. ?

Prop. 9.B.4

Prop. 9.B.4 : Γ^t_E : simultaneous move game, $t = 1, 2, \dots, T$.

Γ_E : successive play of Γ^t_E

Each player's payoff = sum of his payoffs in T periods

Each player knows others' choices just after each game is played.

If $\exists$ a unique Nash equilibrium σ^t in Γ^t_E,

then there is a unique SPNE in Γ_E

in which each player i plays σ^t_i in $t = 1, 2, \dots, T$.

Pf: Induction on T. If $T = 1$, clear.

Suppose the claim is true for all $T \leq n - 1$.

Show the claim holds when $T = n$.

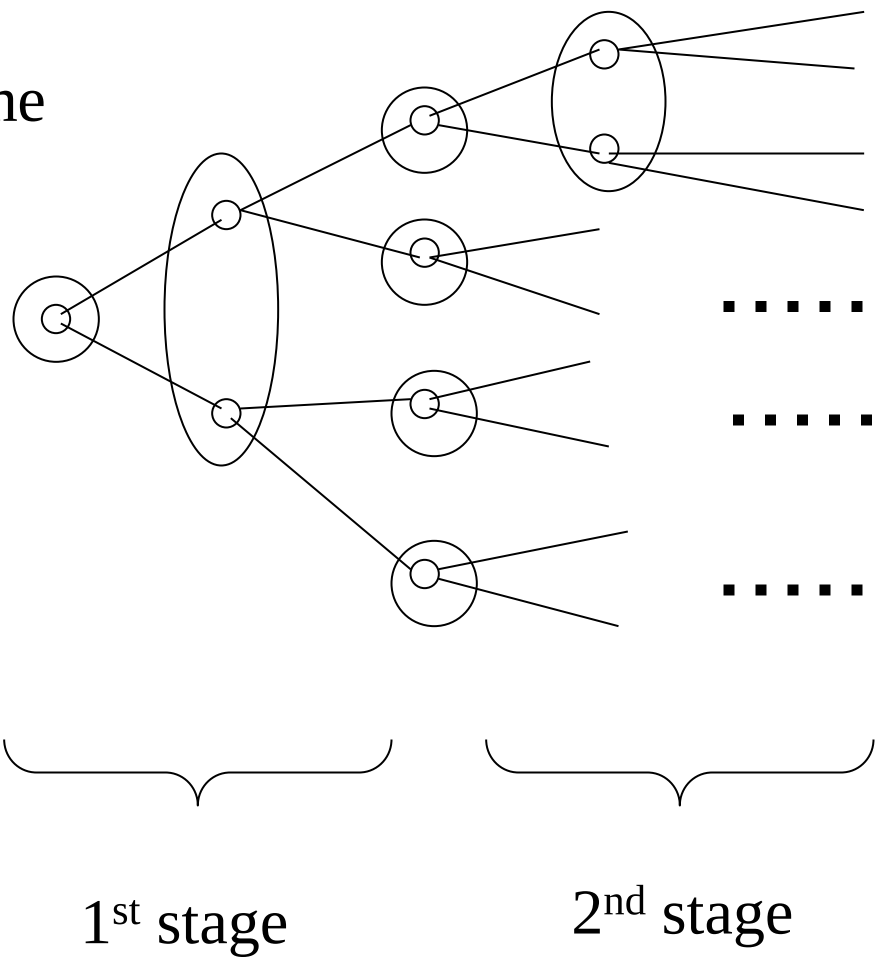
After the first period is over, we have $n - 1$ period game.

Thus from the induction hypothesis, the conclusion easily follows.

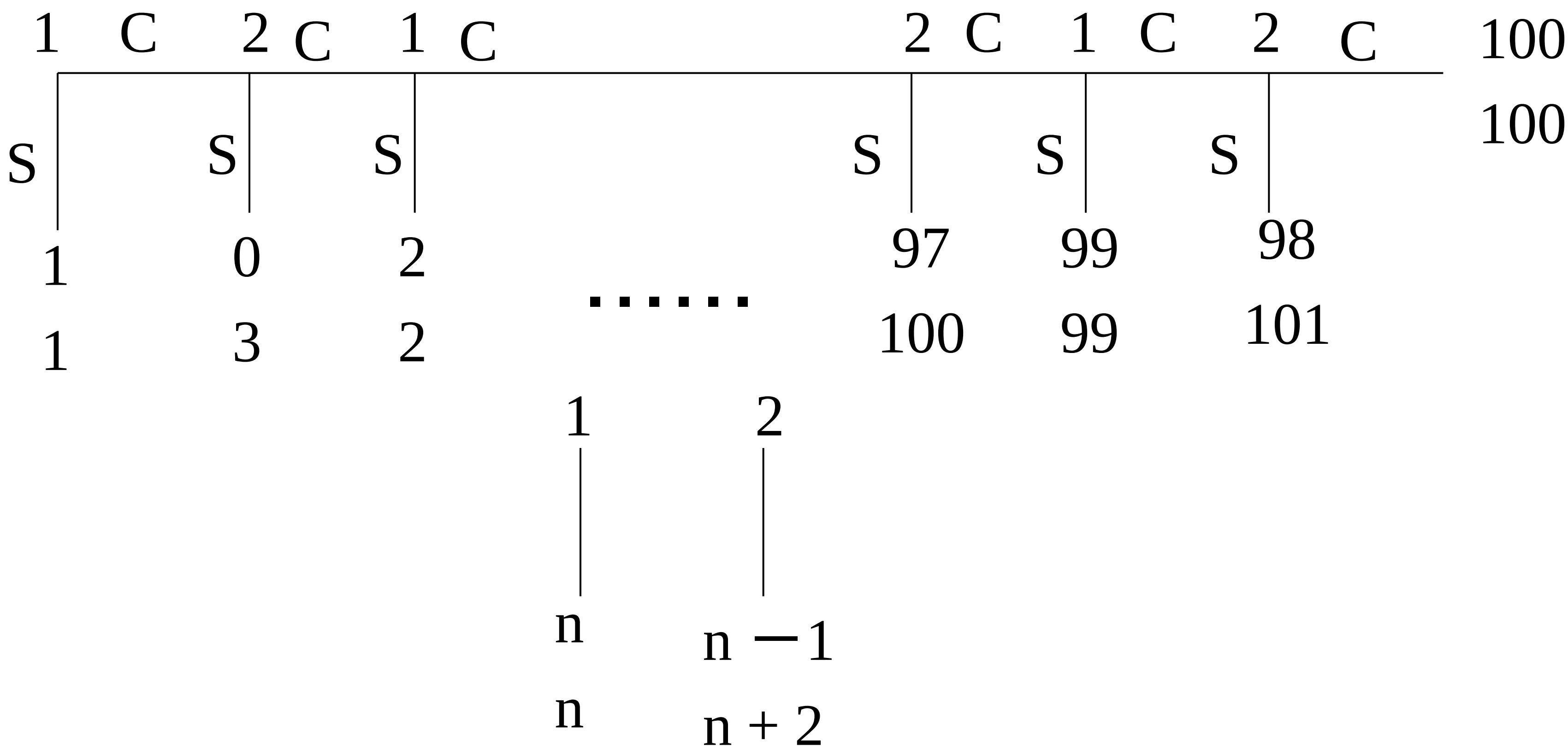
Repeated Game

$N = \{1, 2\}, \; S_1 = \{a, b\}, \; S_2 = \{c, d\}$

Two-stage game



Centipede Game



SPNE (S, S, … , S), (S, S, … , S))

Assignments

Problem Set 8 (due July 5)

Exercises (pp.301-305)

9.B.9, 9.B.10

Reading Assignment:

Text, Chapter 9, pp.282-287