Nuclear Reactor Physics Lecture Note (9)
-Nuclear Reactor Kinetics-

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8. Nuclear Reactor Kinetics

8.1 Prompt neutron and delayed neutron

There are two types of fission neutrons.

Prompt Neutrons: appear instantaneously of the fission events

Delayed Neutrons: appear with an appreciable time delay from the subsequent decay of radioactive fission products

Delayed Neutron Precursors: fission products that emit delayed neutrons by the radioactive decay. They are grouped into six classes (or groups) by the half-lives.

λ_i: decay constant of i-th precursor group

β_i: fraction of delayed fission neutrons emitted per fission that appear from i-th precursor group

$$\beta = \sum_i \beta_i$$ total fraction of fission neutron which are delayed

8.2 The point reactor kinetics equation

The point reactor kinetics equations are often used for the analysis of reactor power transient.

The point reactor kinetics equations consist of two types of equations.

I. The equation that represents the change of number of neutrons (i.e. the reactor power)

First, we think about prompt neutrons only n neutrons + fissile material → prompt neutrons + fission products

The next generation

Prompt neutron life time *l*

n' neutrons + fissile material → prompt neutrons + fission products

$$\frac{n'}{n} = k_{eff}(1-\beta)$$

$$\frac{\Delta n}{n} = \frac{n' - n}{n} = k_{eff}(1-\beta) - 1$$

change ratio for unit time

$$\frac{1}{n}\frac{dn}{dt} = \frac{k_{eff}(1-\beta)-1}{l}$$

$$\frac{dn}{dt} = \frac{k_{eff}(1-\beta)-1}{l} n$$

Number of delayed neutrons emitted by the decay of precursors per unit time

$$\sum_{i=1}^{6} \lambda_i C_i \qquad C_i : i-\text{th precursor concentration}$$

$$\frac{dn}{dt} = \frac{k_{eff}(1-\beta)-1}{l} n + \sum_{i=1}^{6} \lambda_i C_i$$

$$\rho \equiv \frac{k_{eff}-1}{k_{eff}} \quad \text{(reactivity)}$$

$$\Lambda \equiv \frac{l}{k} \quad \text{(neutron generation time)}$$

$$\frac{dn(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \qquad \cdots (1)$$

II. Change of i-th precursors condition

$$\frac{dC_i}{dt} = \frac{k_{eff}\beta_i}{l} n(t) - \lambda_i C_i(t)$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \quad (i = 1, \cdots, 6) \qquad \cdots (2)$$

From Eq.(1) and Eq.(2)

The point reactor kinetics equations

$$\begin{cases} \dfrac{dn(t)}{dt} = \left(\dfrac{\rho(t) - \beta}{\Lambda}\right) n(t) + \displaystyle\sum_{i=1}^{6} \lambda_i C_i \\ \dfrac{dC_i(t)}{dt} = \dfrac{\beta}{\Lambda} n(t) - \lambda_i C_i(t) \qquad (i = 1, \cdots, 6) \end{cases}$$

The change of number of neutrons can be analyzed when the initial condition and the reactivity ρ are given.

8.3 Solution of the point reactor kinetics equations

In general, the equations are solved by numerical calculations.

An example of analytical solution

A reactor operating at power P_0 prior to time $t = 0$, at which point the reactivity is changed to a nonzero value ρ_0.

$$n(t) = \sum_{j=1}^{7} n_j \exp(s_j t)$$

n_j : constants decided by the initial condition

s_j : solutions of inhour equation(or reactivity equation)

Inhour equation(reactivity equation)

$$\rho_0 = \frac{sl}{sl+1} + \frac{1}{sl+1} \sum_{i=1}^{6} \left(\frac{s\beta_i}{s+\lambda_i}\right) \equiv \rho(s)$$

In $t \gg 0$

$$n(t) \cong n_1 \exp(s_1 t) \qquad (\text{where } S_1 > S_2 > \cdots > S_7)$$

Stable period

$$T = \frac{1}{s_1} \qquad (\text{the time that the power becames } e \text{ times})$$

8.4 Prompt critical

$$\frac{dn}{dt} = \left(\frac{\rho(t) - \beta}{\Lambda}\right) n + \sum_{i=1}^{6} \lambda_i C_i$$

If $\rho = \beta$, the reactor can be critical only by prompt neutrons (prompt critical)

$\rho = \beta$ is referred as one dollar ($) of reactivity

If $\rho > \beta$, the reactor power increases rapidly (prompt super critical)

The effect of delayed neutrons is small in prompt super critical, so it can be ignored.

8.5 Effective delayed neutron fraction β_{eff}

The energy of emitted delayed neutron is smaller than the energy of prompt neutron. The probability of inducing fission is different.

This effect is included in the effective delayed neutron fraction β_{eff}

(In thermal reactors, $\beta_{eff} > \beta$)