低雑音増幅回路

通信機器システムの要

雑音の評価

雑音の定式化

瞬時値での評価→困難 (不可)

↓

統計的な評価

2乗平均値：$\overline{v_n{}^2} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} v_n{}^2(t)dt$

複数の雑音源の取り扱い

$$\lim_{T\to\infty}\left[\frac{1}{T}\int_{-T/2}^{T/2}\{v_{n1}(t)+v_{n2}(t)\}^2 dt\right]$$

$$=\lim_{T\to\infty}\left[\frac{1}{T}\left\{\int_{-T/2}^{T/2}v_{n1}{}^2(t)+2v_{n1}(t)v_{n2}(t)+v_{n2}{}^2(t)dt\right\}\right]$$

$$=\overline{v_{n1}{}^2}+\lim_{T\to\infty}\frac{2}{T}\int_{-T/2}^{T/2}v_{n1}(t)v_{n2}(t)dt+\overline{v_{n2}{}^2}$$

$v_{n1}(t)$と$v_{n2}(t)$が全く独立：無相関

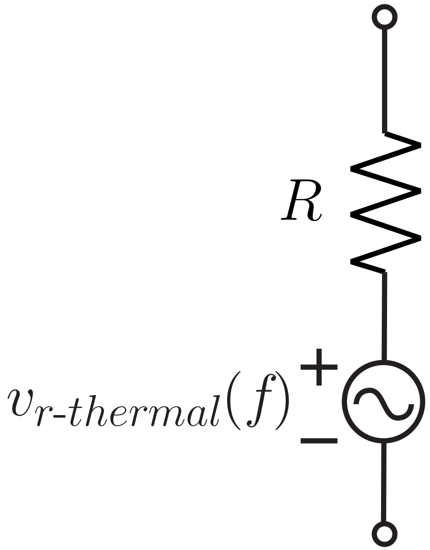
$$\int_0^{\tau}v_{n1}(t)v_{n2}(t)dt=0$$

雑音源を考慮した回路素子モデル

熱雑音, 1/f雑音, ショット雑音

熱雑音：キャリアが移動する際のランダムな動きによって生じる雑音

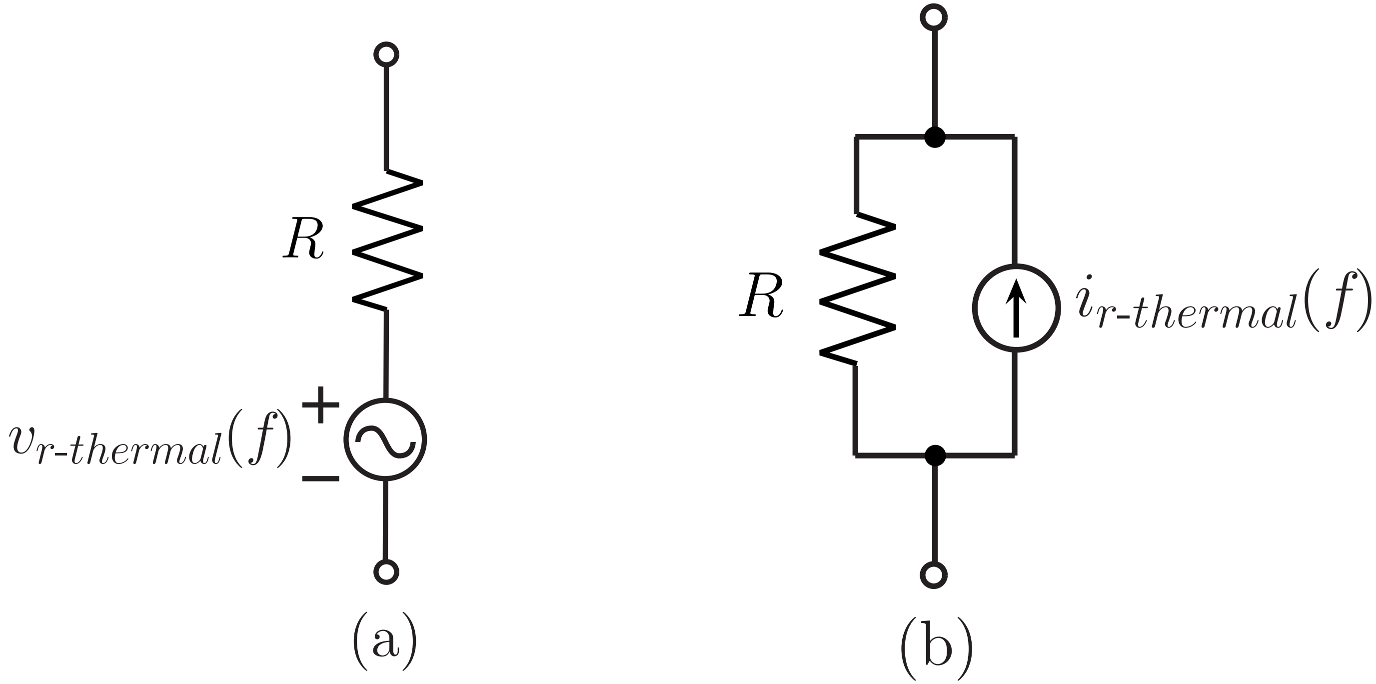
抵抗の熱雑音



(a)

$$\overline{v_{r-thermal}{}^{2}(f)} = 4kTR\,[V^2/Hz]$$

問　「電源の等価性」の定理を用いて
下記に示す抵抗の等価回路の雑音を
表している電流源の値を求めよ.



$$\overline{v_{r-thermal}^{2}(f)} = 4kTR\,[V^2/Hz]$$

MOSトランジスタの熱雑音

キャリアがチャネルを通過する際に発生

$$\overline{v_{mos-thermal}^{\,2}} = 4kT\gamma_n \frac{1}{g_m}$$

$$\overline{i_{mos-thermal}^{\,2}} = 4kT\gamma_n g_m$$

$\gamma_n \approx \dfrac{2}{3}$　（最小線幅が短くなると増加）

バイポーラトランジスタの熱雑音

ベース広がり抵抗：$\overline{v_{b-thermal}(f)^2}=4kTr_b$

エミッタ抵抗, コレクタ・エミッタ間抵抗：仮想の抵抗

↓

熱雑音の発生無し

スペクトラム強度が一定：白色雑音

1/f雑音：シリコンの汚れや結晶欠陥によりキャリアが捕らわれたり，捕らわれたキャリアが離されたりを繰り返すというランダムな過程によって生じる雑音

直流電流がないと発生しない

$$\overline{v_{mos-1/f}^{\;2}(f)} = \frac{\alpha_{1/f}}{C_{OX} W L f}$$

$$\overline{i_{b-1/f}^{\;2}(f)} = \frac{K_{1/f} I_B^{\;a}}{f}$$

ショット雑音：

電流＝キャリアによる電流パルスの和の平均値

電流の平均値からの揺らぎ

$$\overline{i_{b-shot}^{2}(f)}=2qI_{B}$$

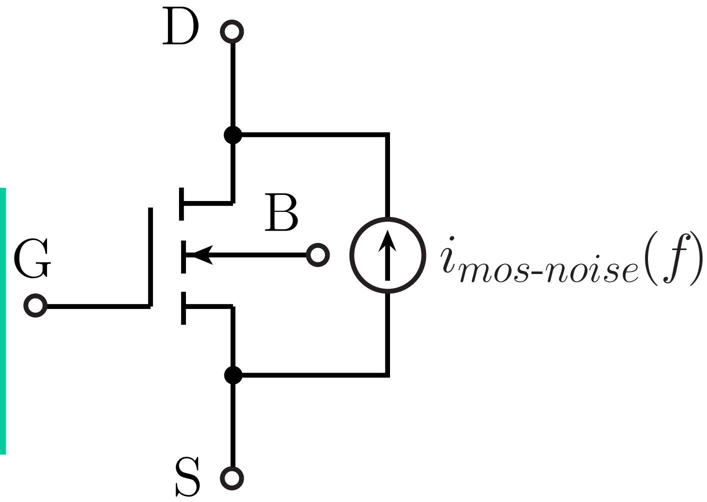
$$\overline{i_{c-shot}^{2}(f)}=2qI_{C}$$

1nA@1GHz→1.6×10^{-19}C×6.25個

雑音源を含むトランジスタモデル

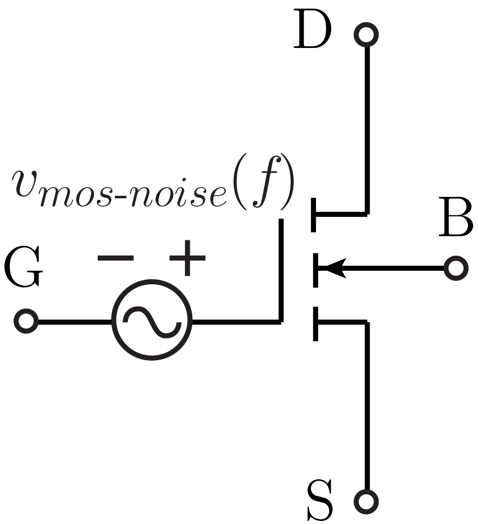
MOSトランジスタモデル

$$\overline{i_{mos\text{–}noise}{}^2(f)} = 4kT\gamma_n g_m + \frac{\alpha_{1/f}\, g_m{}^2}{C_{OX} W L f}$$



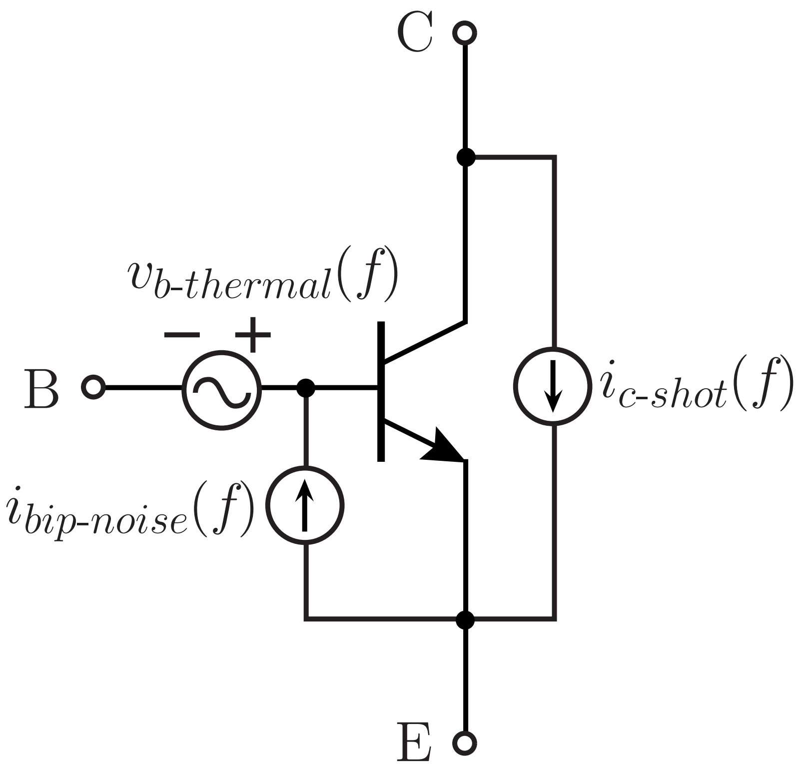
(a)

$$\overline{v_{mos\text{–}noise}{}^2(f)} = 4kT\gamma_n \frac{1}{g_m} + \frac{\alpha_{1/f}}{C_{OX} W L f}$$



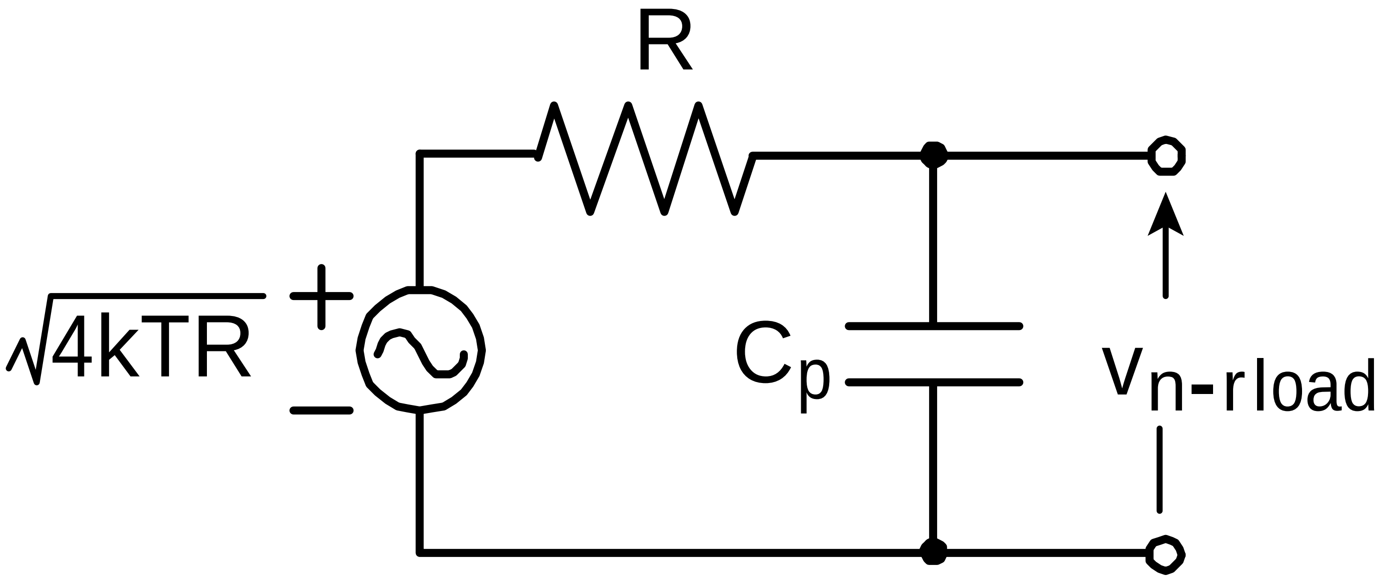
(b)

バイポーラトランジスタモデル



$$\overline{i_{bip-noise}{}^{2}(f)}=\frac{K_{1/f}\,I_B{}^{a}}{f}+2qI_B$$

雑音解析の例(抵抗が発生する雑音)

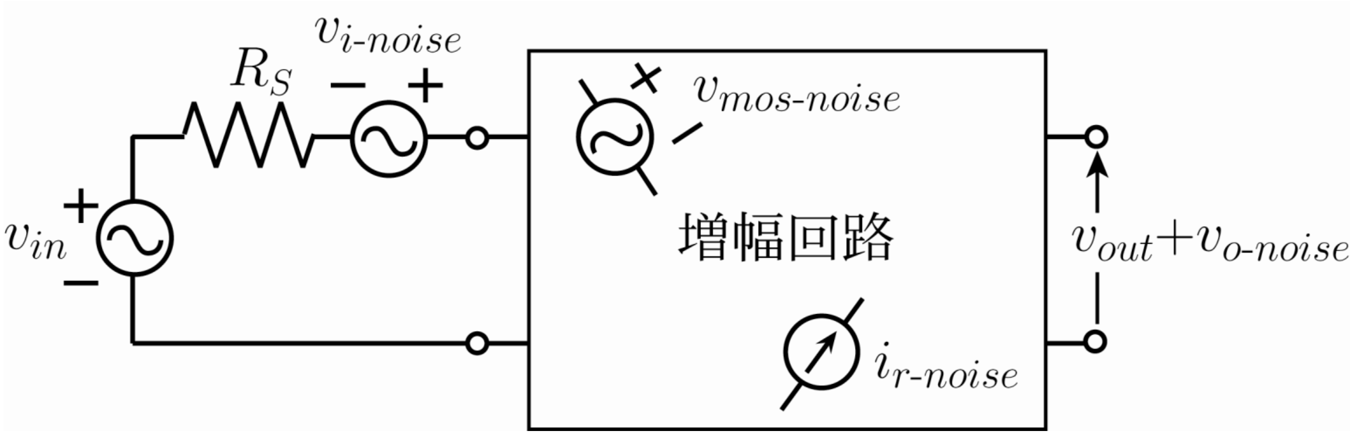


問１　上の回路の伝達関数$T_{rload}(s)=\dfrac{v_{n-rload}}{\sqrt{4kTR}}$を求めよ.

問２　$\overline{v_{n-rload}}^{2}=\int_0^{\infty}\left|T_{rload}(j\omega)\right|^2\dfrac{4kTR}{2\pi}d\omega$を求めよ.

増幅回路と雑音

雑音係数と雑音指数

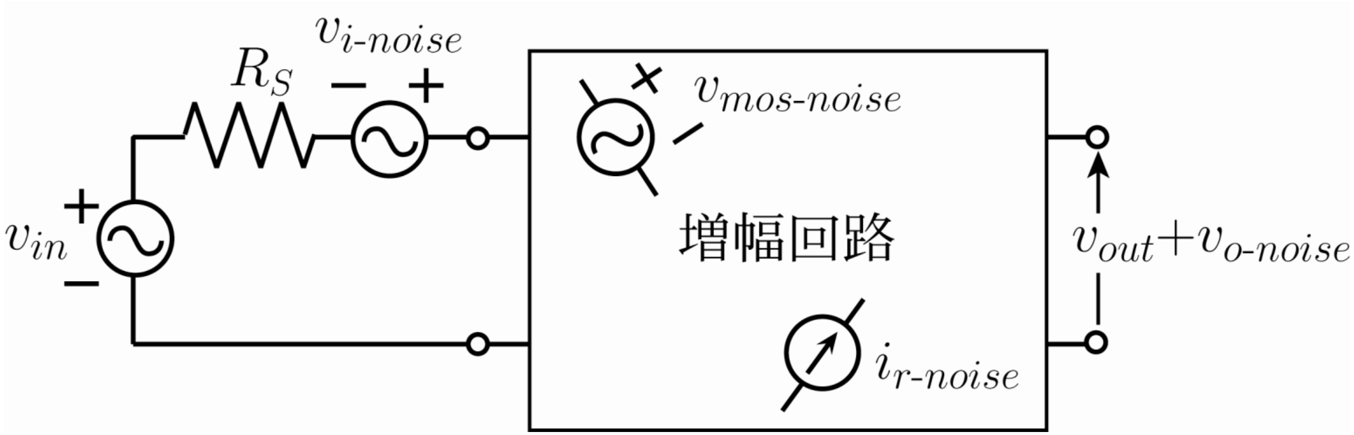


$$SNR_{in}=\frac{\overline{v_{in}^{2}}}{v_{i-noise}^{2}}$$

$$SNR_{out}=\frac{A^{2}\overline{v_{in}^{2}}}{v_{o-noise}^{2}}$$

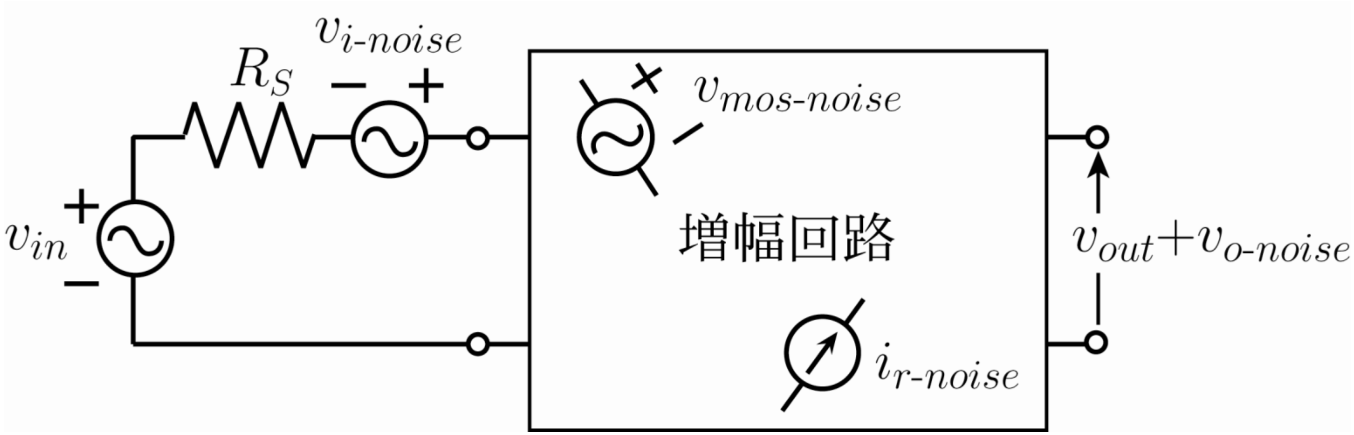
$$F=\frac{SNR_{in}}{SNR_{out}}$$: 雑音係数

$NF=10\log F$ ：雑音指数



$$\overline{v_{o-noise}^{\,2}} = A^2\,\overline{v_{i-noise}^{\,2}} + \overline{v_{inner-noise}^{\,2}}$$

$v_{mos-noise}$や$i_{r-noise}$などの増幅回路内部の雑音に起因する成分



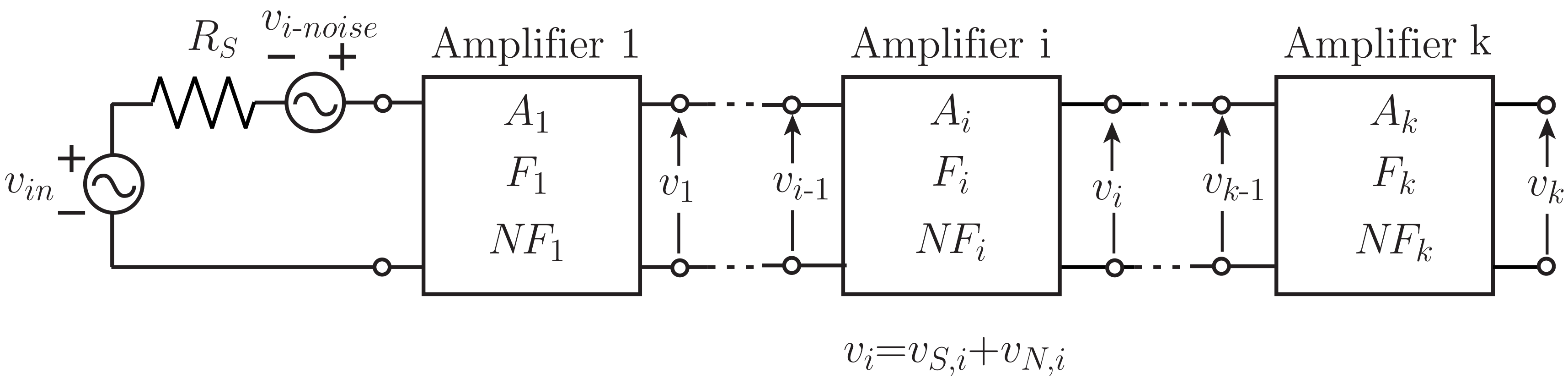
$$\overline{v_{o-noise}^{2}} = A^{2}\overline{v_{i-noise}^{2}} + \overline{v_{inner-noise}^{2}}$$

$$SNR_{out} = \frac{A^{2}\overline{v_{in}^{2}}}{\overline{v_{o-noise}^{2}}} = \frac{\overline{v_{in}^{2}}}{\overline{v_{i-noise}^{2}} + \overline{v_{inner-noise}^{2}}/A^{2}}$$

$$SNR_{in} = \frac{\overline{v_{in}^{2}}}{\overline{v_{i-noise}^{2}}} \geq SNR_{out}$$

$$F = 1 + \frac{\overline{v_{inner-noise}^{2}}}{A^{2}\overline{v_{i-noise}^{2}}}$$

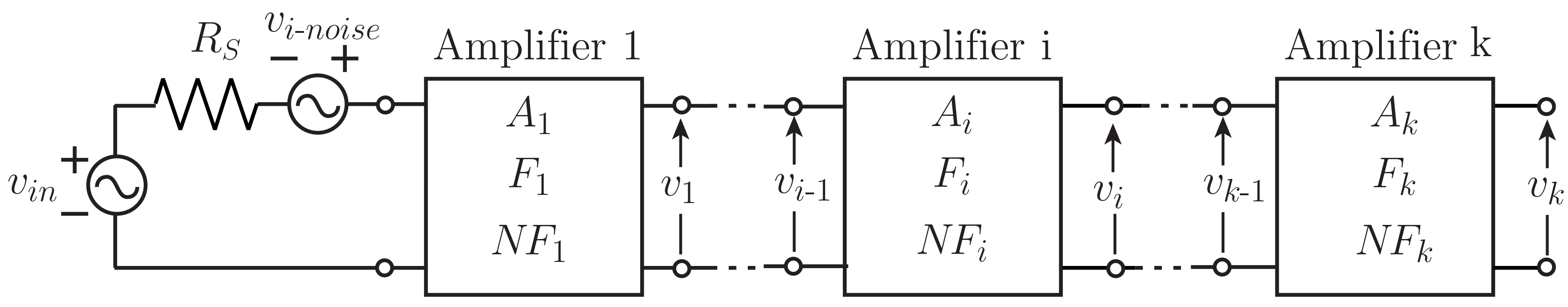
縦続接続型システムの評価



$$\overline{v_{o-noise}^{\,2}} = A^2\,\overline{v_{i-noise}^{\,2}} + \overline{v_{inner-noise}^{\,2}}$$

$$\overline{v_{N,i}^{\,2}} = A_i^{\,2} F_i\,\overline{v_{N,i-1}^{\,2}}$$

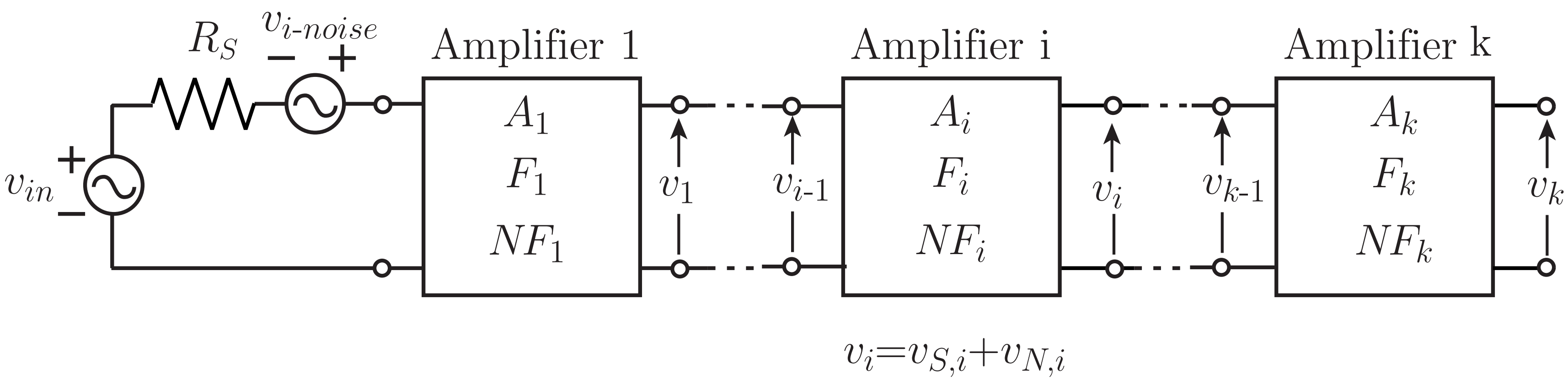
$$F = 1 + \frac{\overline{v_{inner-noise}^{\,2}}}{A^2\,\overline{v_{i-noise}^{\,2}}}$$



$$v_i = v_{S,i} + v_{N,i}$$

$$\overline{v_{N,i}^2} = A_i^2 F_i \overline{v_{N,i-1}^2} \qquad \overline{v_{S,i}^2} = A_i^2 \overline{v_{S,i-1}^2}$$

$$F_{total} = \frac{\dfrac{\overline{v_{in}^2}}{\overline{v_{i-noise}^2}}}{\dfrac{\overline{v_{S,k}^2}}{\overline{v_{N,k}^2}}} = \frac{\dfrac{\overline{v_{in}^2}}{\overline{v_{i-noise}^2}}}{\dfrac{A_1^2 A_2^2 \cdots A_k^2 \overline{v_{in}^2}}{A_1^2 F_1 A_2^2 F_2 \cdots A_k^2 F_k \overline{v_{i-noise}^2}}} = F_1 F_2 \cdots F_k$$



$$\overline{v_{N,i}^{\,2}} = A_i^{\,2}\,\overline{v_{N,i-1}^{\,2}} + \overline{v_{inner-noise,i}^{\,2}}$$

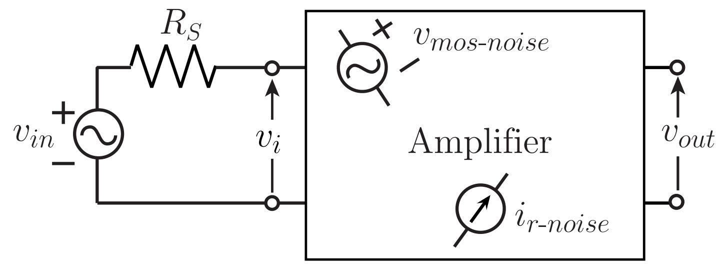
$$\begin{aligned}\overline{v_{N,k}^{\,2}} = {} & A_k^{\,2} A_{k-1}^{\,2} \cdots A_1^{\,2}\,\overline{v_{i-noise}^{\,2}} \\ & + A_k^{\,2} A_{k-1}^{\,2} \cdots A_2^{\,2}\,\overline{v_{inner-noise,1}^{\,2}} \\ & + A_k^{\,2} A_{k-1}^{\,2} \cdots A_3^{\,2}\,\overline{v_{inner-noise,2}^{\,2}} + \cdots \\ & + A_k^{\,2}\,\overline{v_{inner-noise,k-1}^{\,2}} + \overline{v_{inner-noise,k}^{\,2}}\end{aligned}$$

$$\overline{v_{N,k}^2} = A_k^2 A_{k-1}^2 \cdots A_1^2 \overline{v_{i-noise}^2}$$
$$+ A_k^2 A_{k-1}^2 \cdots A_2^2 \overline{v_{inner-noise,1}^2}$$
$$+ A_k^2 A_{k-1}^2 \cdots A_3^2 \overline{v_{inner-noise,2}^2} + \cdots$$
$$+ A_k^2 \overline{v_{inner-noise,k-1}^2} + \overline{v_{inner-noise,k}^2}$$

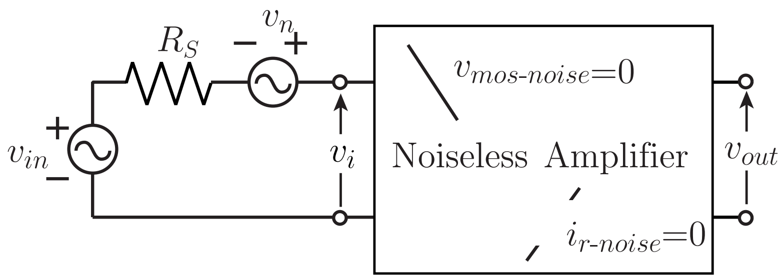
$$F_{total} = \frac{\overline{v_{in}^2}}{\overline{v_{i-noise}^2}} \cdot \frac{\overline{v_{N,k}^2}}{\overline{v_{S,k}^2}} = 1 + \frac{\overline{v_{inner-noise,1}^2}}{A_1^2 \overline{v_{i-noise}^2}} + \frac{\overline{v_{inner-noise,2}^2}}{A_1^2 A_2^2 \overline{v_{i-noise}^2}}$$
$$+ \cdots + \frac{\overline{v_{inner-noise,k-1}^2}}{A_1^2 A_2^2 \cdots A_{k-1}^2 \overline{v_{i-noise}^2}} + \frac{\overline{v_{inner-noise,k}^2}}{A_1^2 A_2^2 \cdots A_k^2 \overline{v_{i-noise}^2}}$$

低雑音増幅回路の設計

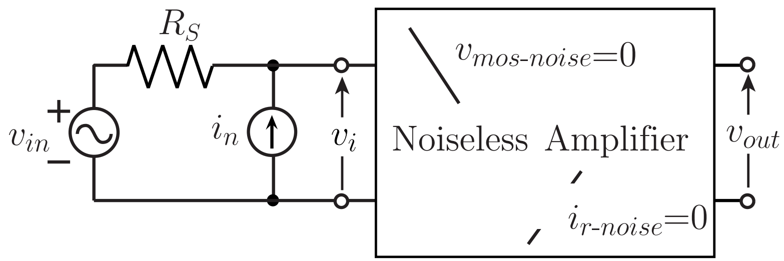
増幅回路内部で発生する雑音の等価表現



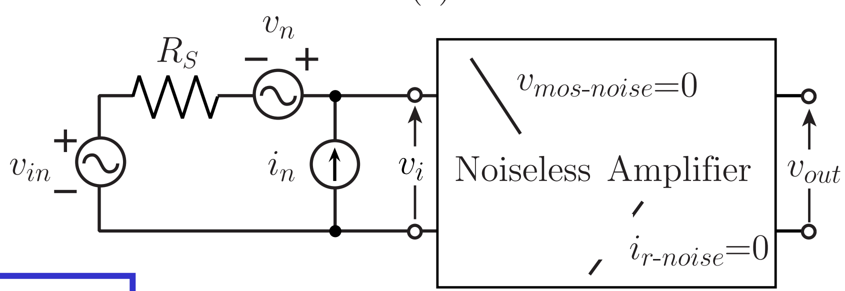
(a)



(b)



(c)



(d)

(b): R_Sが無限大のとき矛盾

(c): R_Sが零のとき矛盾

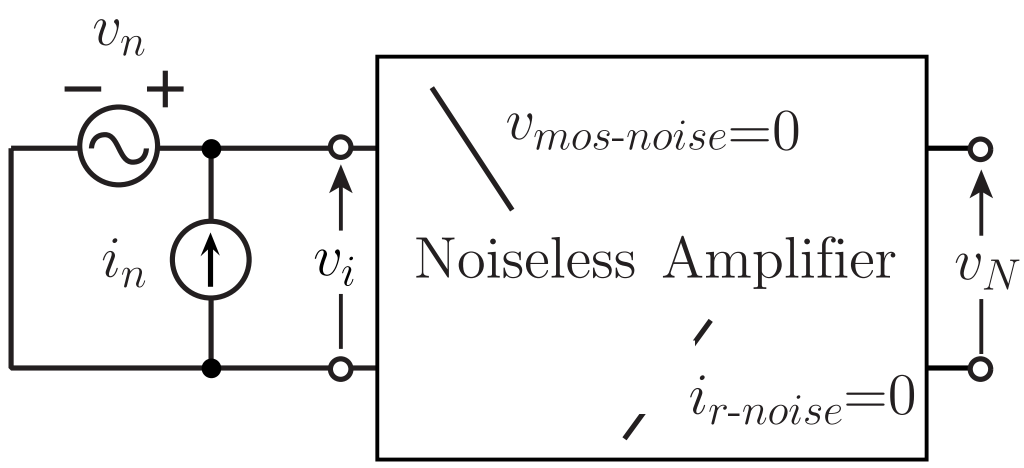
(d): 任意のR_Sについて矛盾無し

増幅回路の出力雑音のみ考慮

v_nの求め方

$$v_n = \frac{v_N}{A}$$

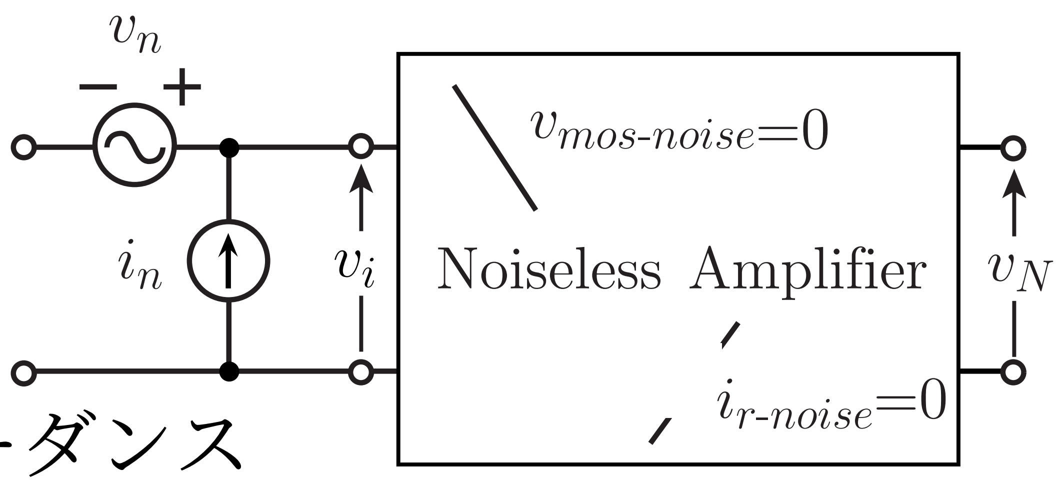
A：電圧利得



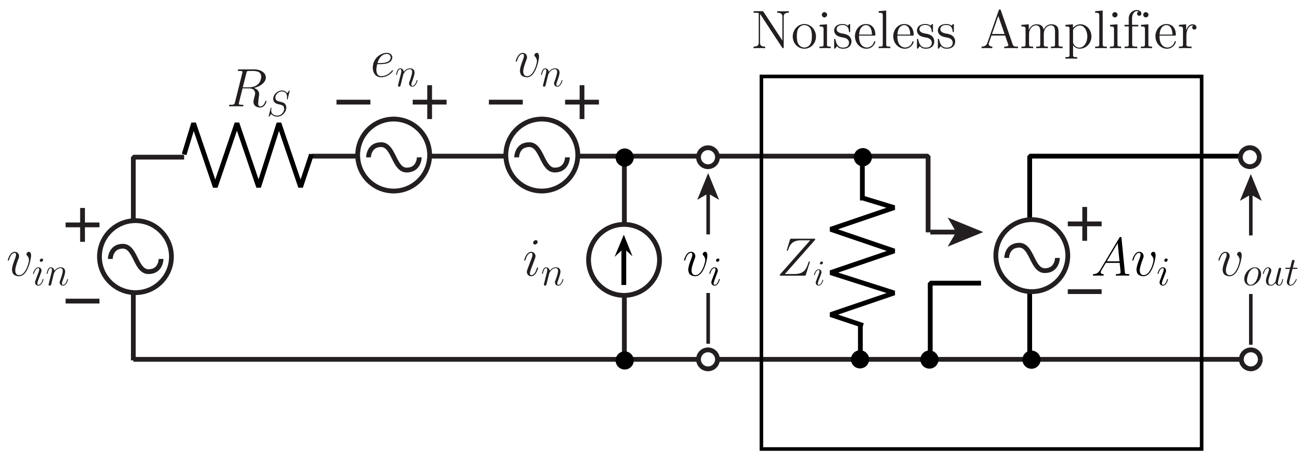
i_nの求め方

$$i_n = \frac{v_N}{Z_T}$$

Z_T: 伝達インピーダンス

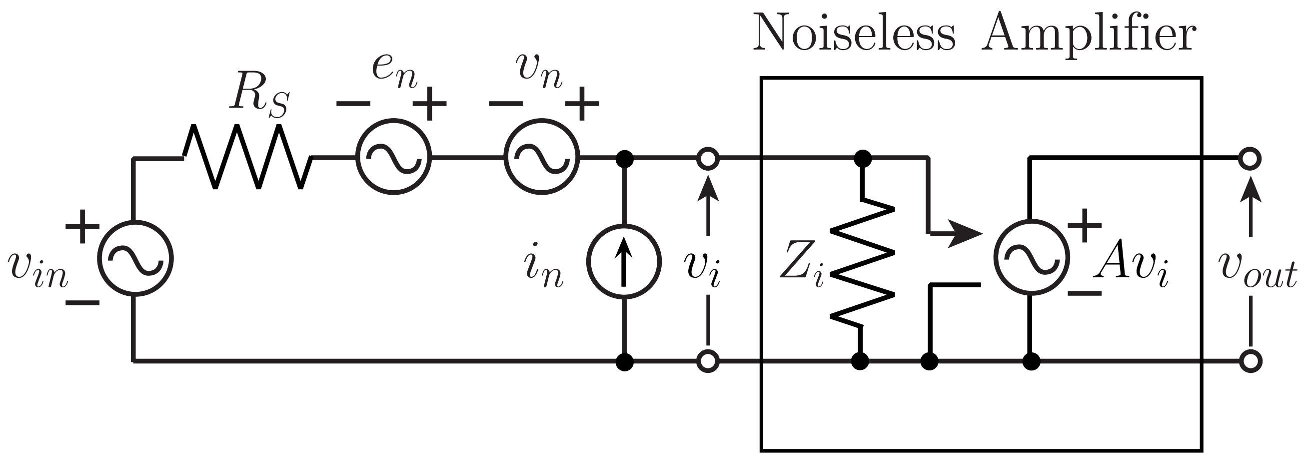


雑音整合



$$\overline{e_n^{\,2}} = 4kTR_S$$

$$v_S = \frac{AZ_i}{R_S + Z_i} v_{in}$$



e_nとv_n，　e_nとi_nは無相関

v_nとi_nも無相関と仮定

$$\overline{v_N{}^2}=\frac{A^2 Z_i{}^2}{|R_S+Z_i|^2}(\overline{e_n{}^2}+\overline{v_n{}^2}+R_S{}^2\overline{i_n{}^2})$$

$$\overline{e_n^2} = 4kTR_S \qquad v_S = \frac{AZ_{in}}{R_S + Z_{in}} v_{in}$$

$$\overline{v_N^2} = \frac{A^2 Z_{in}^2}{|R_S + Z_{in}|^2} \left(\overline{e_n^2} + \overline{v_n^2} + R_S^2 \overline{i_n^2} \right)$$

$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{e_n^2}}$$

$$SNR_{out} = \frac{\overline{v_S^2}}{\overline{v_N^2}} = \frac{\overline{v_{in}^2}}{\overline{e_n^2} + \overline{v_n^2} + R_S^2 \overline{i_n^2}}$$

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{\overline{e_n^2} + \overline{v_n^2} + R_S^2 \overline{i_n^2}}{\overline{e_n^2}} = 1 + \frac{\overline{v_n^2} + R_S^2 \overline{i_n^2}}{4kTR_S}$$

$$F = 1 + \frac{\overline{v_n^2} + R_S^2 \overline{i_n^2}}{4kTR_S}$$

相加・相乗平均の定理

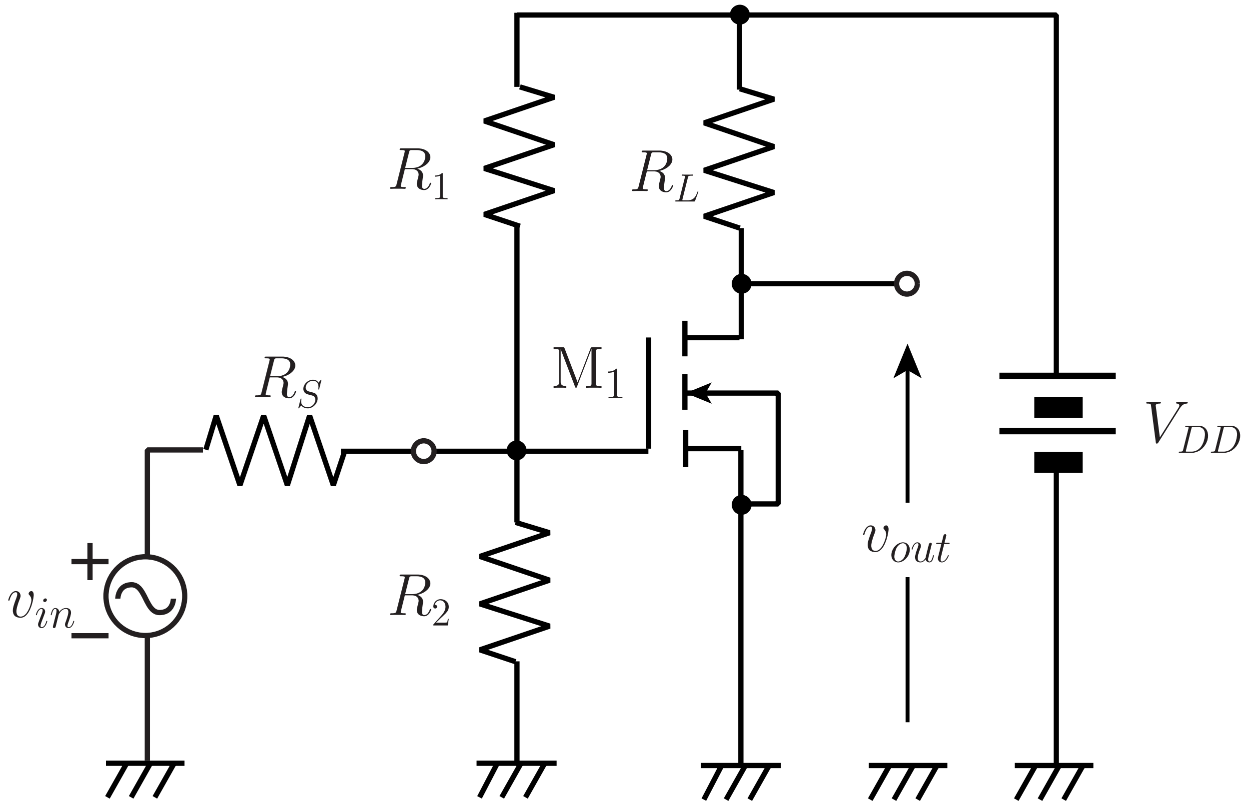
$$a + b \geq 2\sqrt{ab}$$

$$\frac{\overline{v_n^2}}{R_S} = R_S \overline{i_n^2}$$ のときFが最小

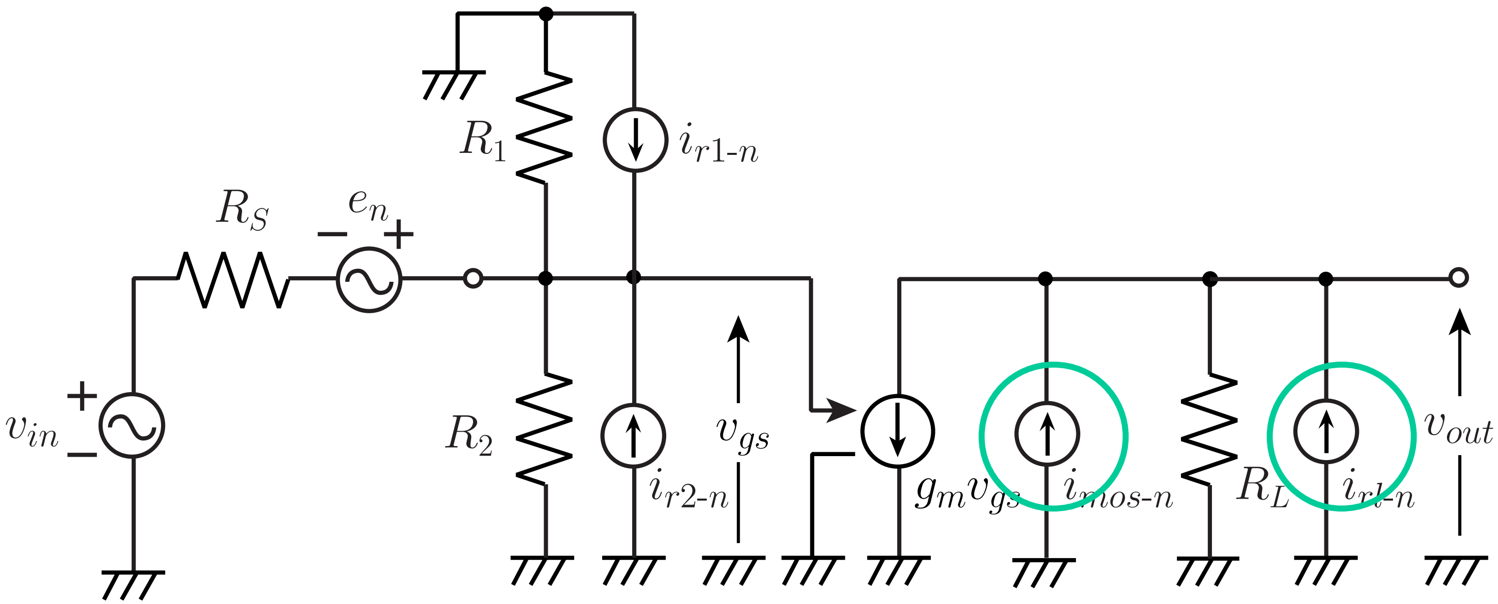
$$R_S = \sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}}$$

実際の増幅回路の雑音解析例

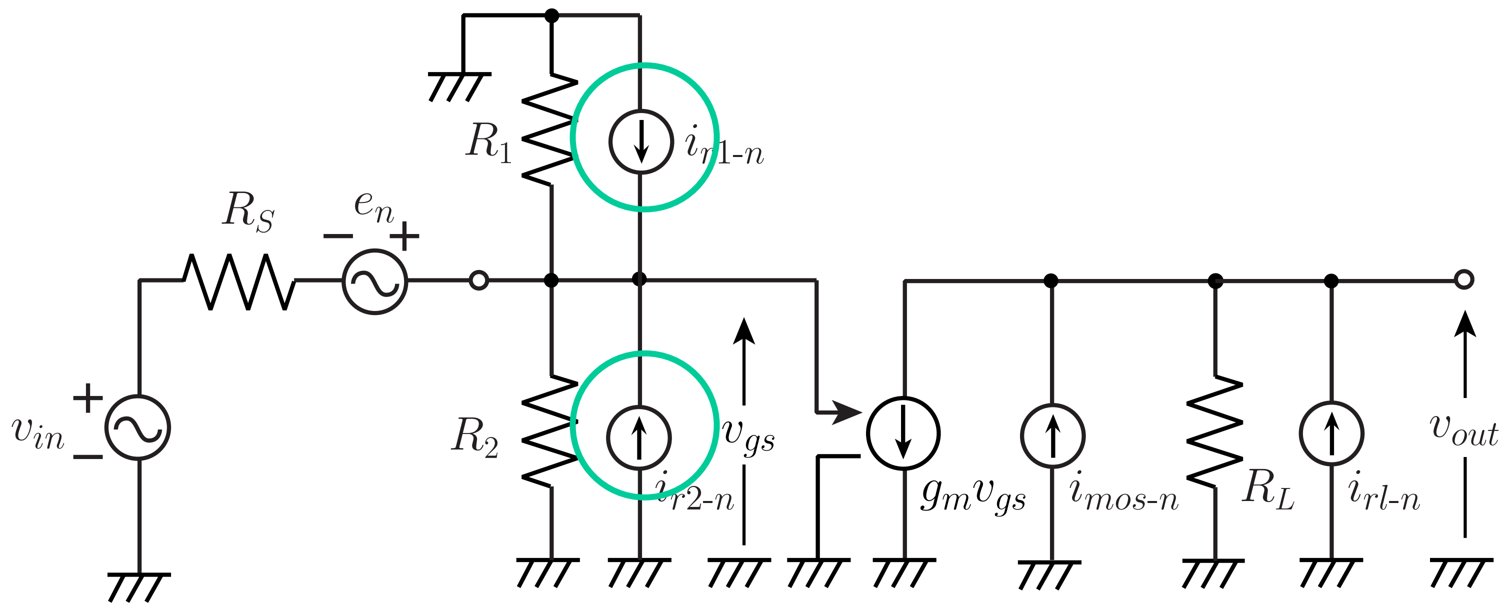
ソース接地増幅回路



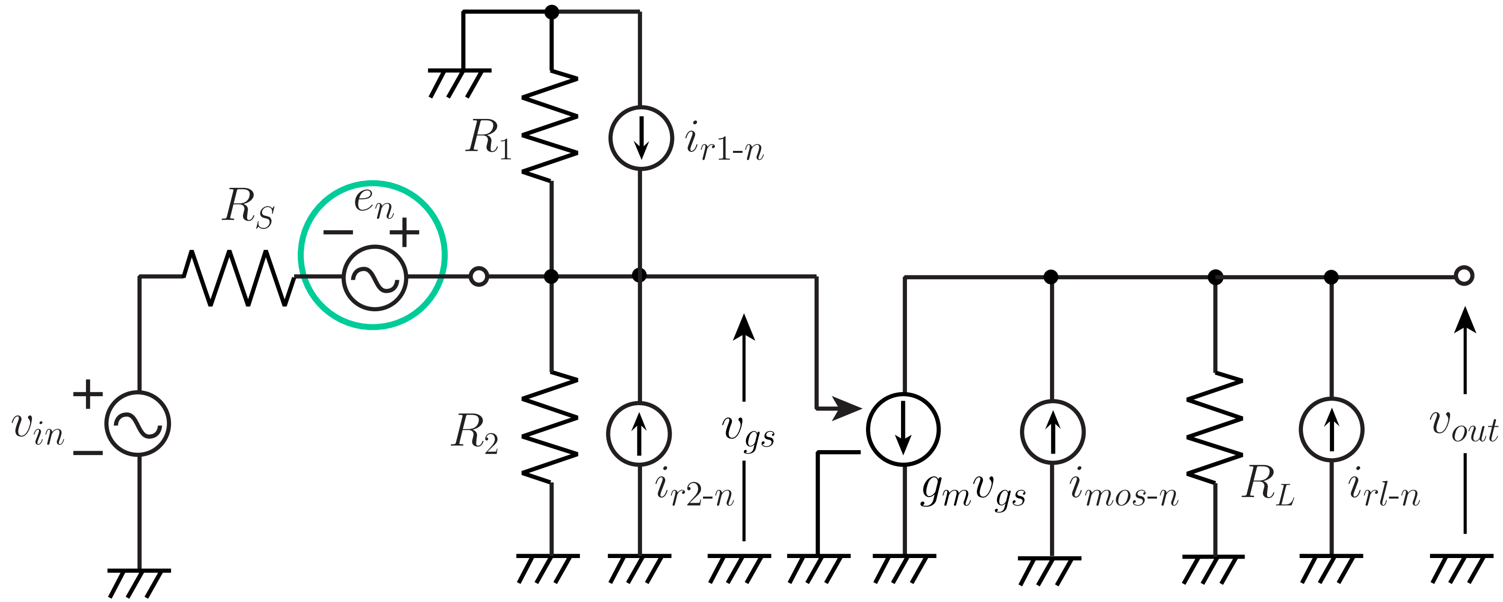
ソース接地増幅回路の小信号モデル



$$\overline{v_{o\text{-}n1}^2} = R_L^2\left(\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2}\right)$$



$$\overline{v_{o\text{-}n2}^{2}} = \left| g_m R_L \frac{R_1 R_2 R_S}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \left(\overline{i_{r1\text{-}n}^{2}} + \overline{i_{r2\text{-}n}^{2}}\right)$$



$$\overline{v_{o\text{-}n3}{}^2} = \left| g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \overline{e_n{}^2}$$

$$\overline{v_N^2} = \overline{v_{o\text{-}n1}^2} + \overline{v_{o\text{-}n2}^2} + \overline{v_{o\text{-}n3}^2}$$

$$= R_L^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

$$+ \left| g_m R_L \frac{R_1 R_2 R_S}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 (\overline{i_{r1\text{-}n}^2} + \overline{i_{r2\text{-}n}^2})$$

$$+ \left| g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \overline{e_n^2}$$

$$\overline{v_S^2} = \left| g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \overline{v_{in}^2}$$

$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{e_n^2}}$$

$$\overline{v_N^2}=\overline{v_{o\text{-}n1}^2}+\overline{v_{o\text{-}n2}^2}+\overline{v_{o\text{-}n3}^2}$$

$$\overline{v_S^2}=\left|g_m R_L \frac{R_1 R_2}{R_1 R_2+R_2 R_S+R_S R_1}\right|^2 \overline{v_{in}^2}$$

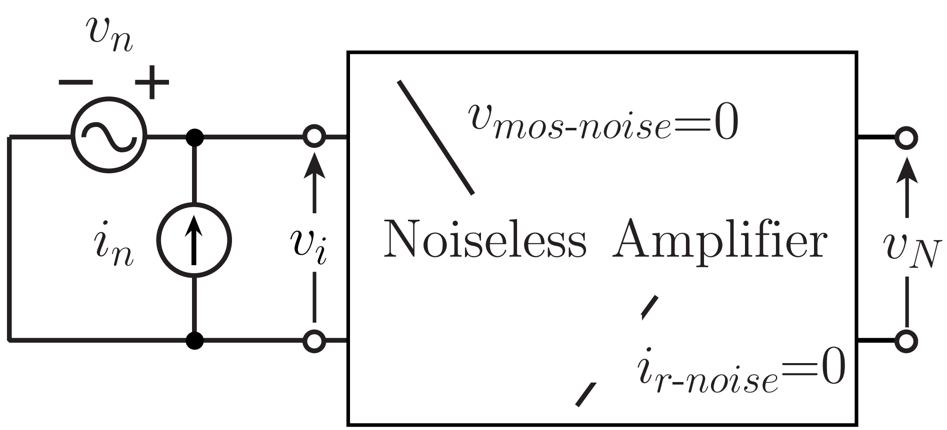
$$SNR_{in}=\frac{\overline{v_{in}^2}}{\overline{e_n^2}} \qquad e_n^2=4kTR_S$$

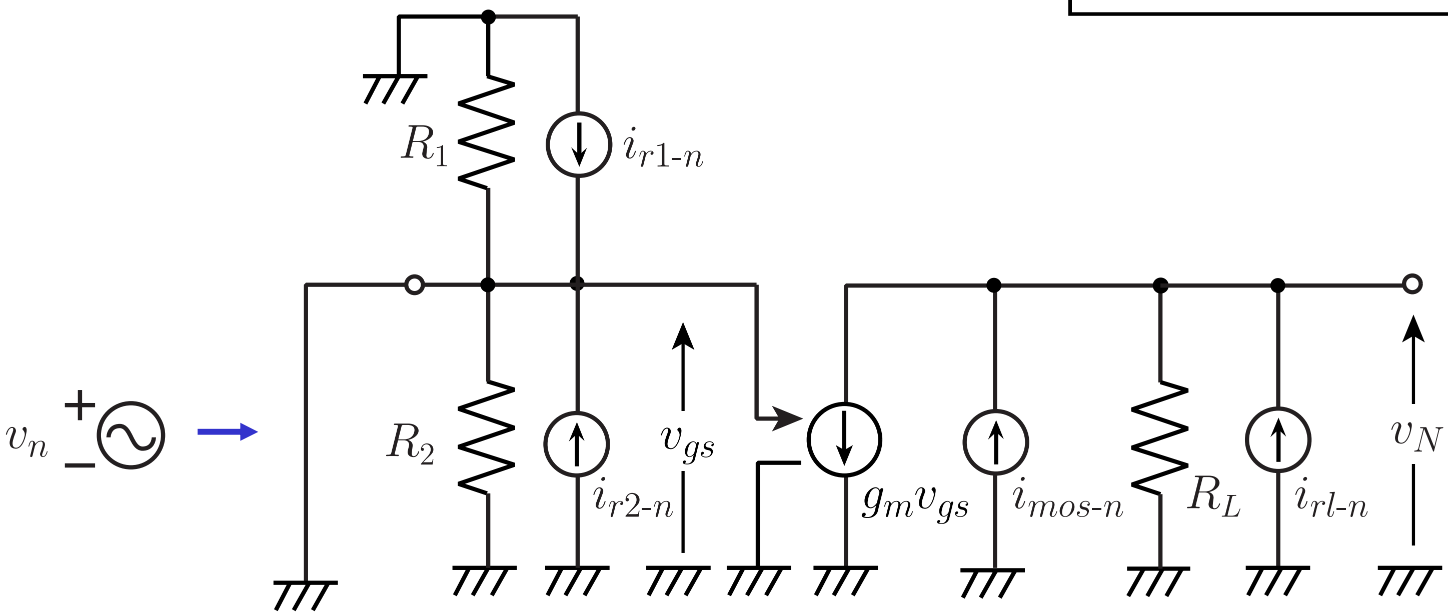
$$F=\frac{SNR_{in}}{SNR_{out}}$$

$$=1+\frac{1}{4kTR_S}\left\{\left|\frac{R_1 R_2+R_2 R_S+R_S R_1}{g_m R_1 R_2}\right|^2 (\overline{i_{rl\text{-}n}^2}+\overline{i_{mos\text{-}n}^2})\right.$$

$$\left.+R_S^2(\overline{i_{r1\text{-}n}^2}+\overline{i_{r2\text{-}n}^2})\right\}$$

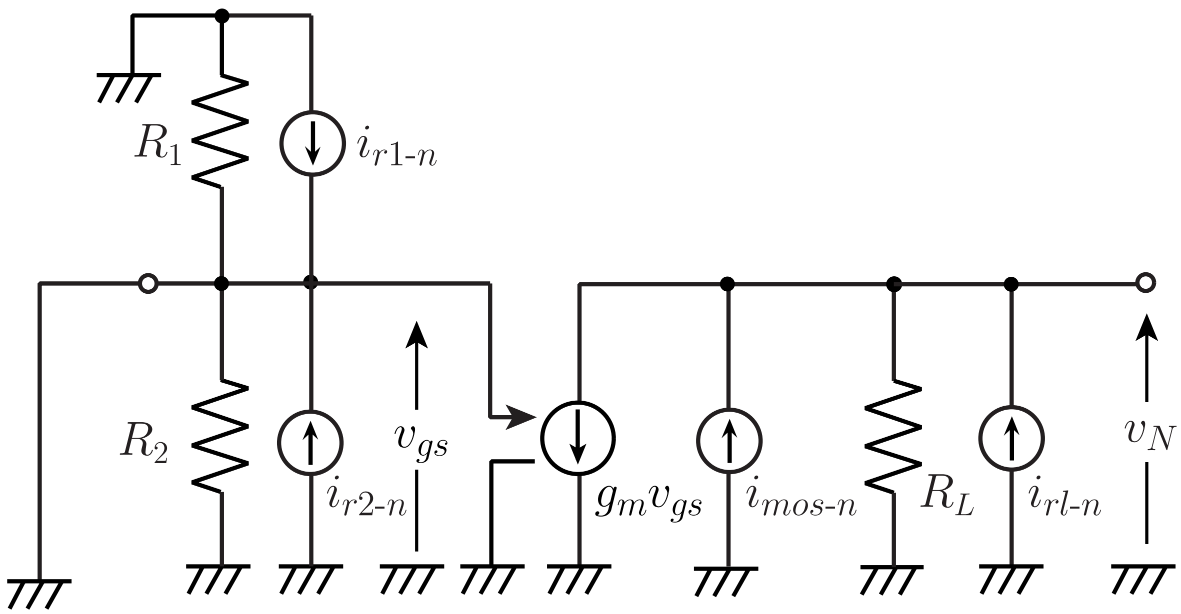
v_nの求め方





$$A=\frac{v_{out}}{v_n}=-g_m R_L$$

$$\overline{v_n{}^2}=\frac{\overline{v_N{}^2}}{A^2}$$

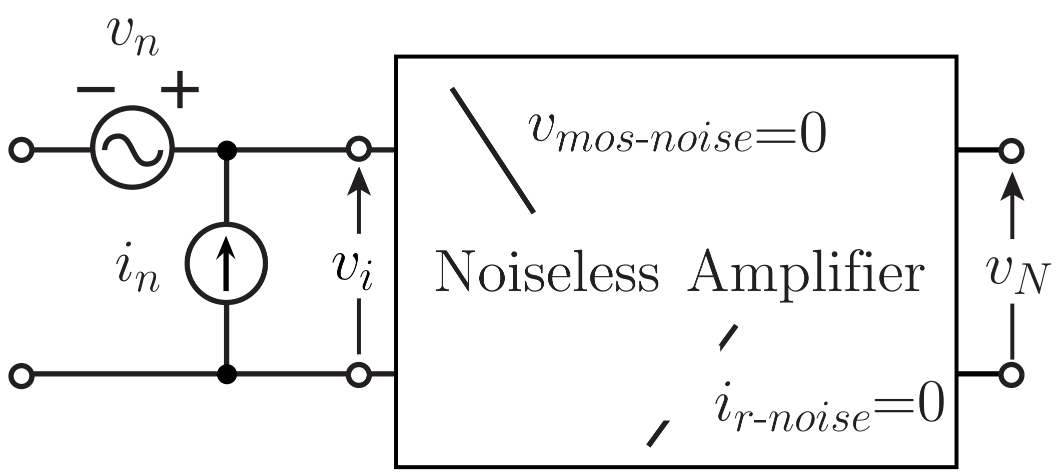


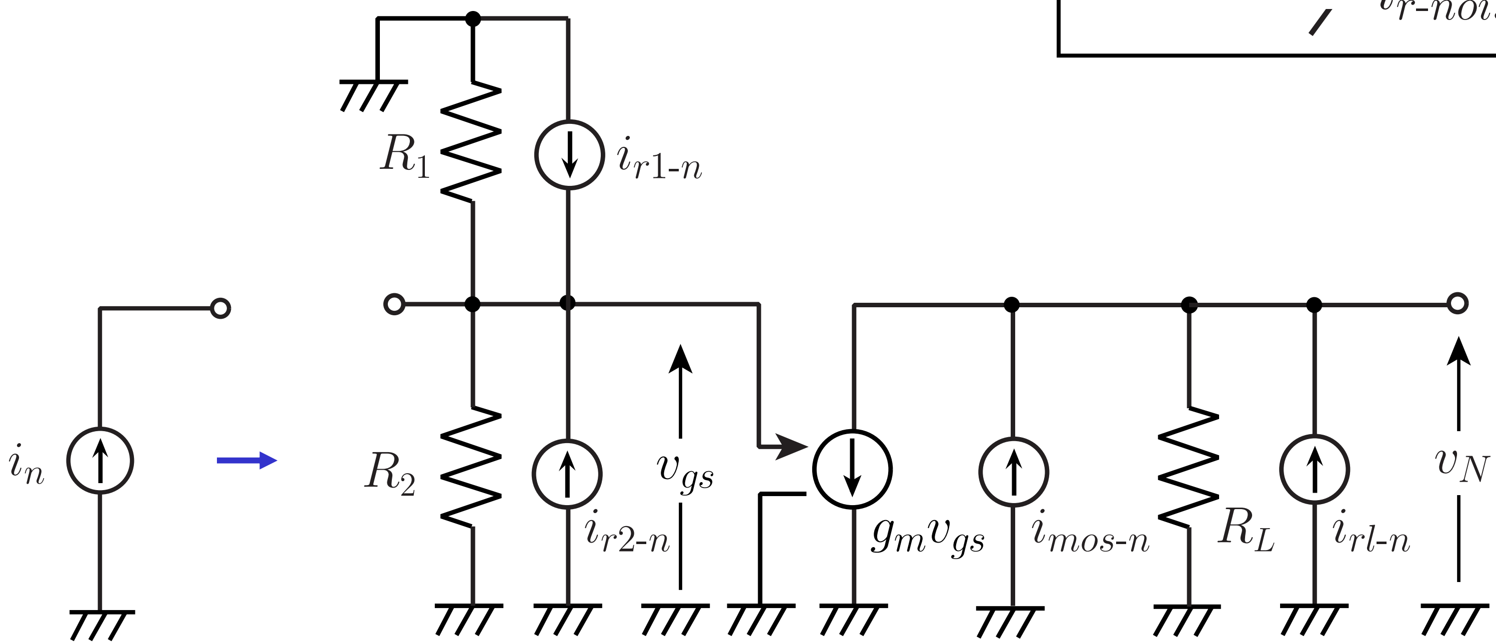
$$\overline{v_N{}^2} = R_L{}^2\left(\overline{i_{rl\text{-}n}{}^2} + \overline{i_{mos\text{-}n}{}^2}\right)$$

$$A = \frac{v_{out}}{v_n} = -g_m R_L$$

$$\boxed{\overline{v_n{}^2} = \frac{1}{g_m{}^2}\left(\overline{i_{rl\text{-}n}{}^2} + \overline{i_{mos\text{-}n}{}^2}\right)}$$

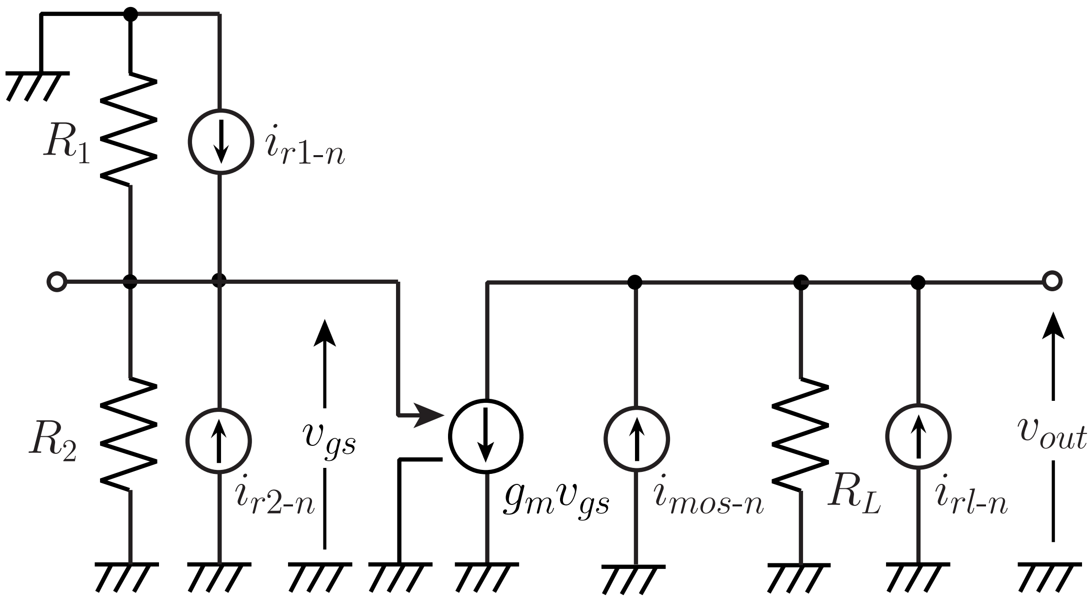
i_nの求め方





$$Z_T = -g_m R_L \frac{R_1 R_2}{R_2 + R_1}$$

$$\overline{i_n^{\,2}} = \frac{\overline{v_N^{\,2}}}{Z_T^{\,2}}$$

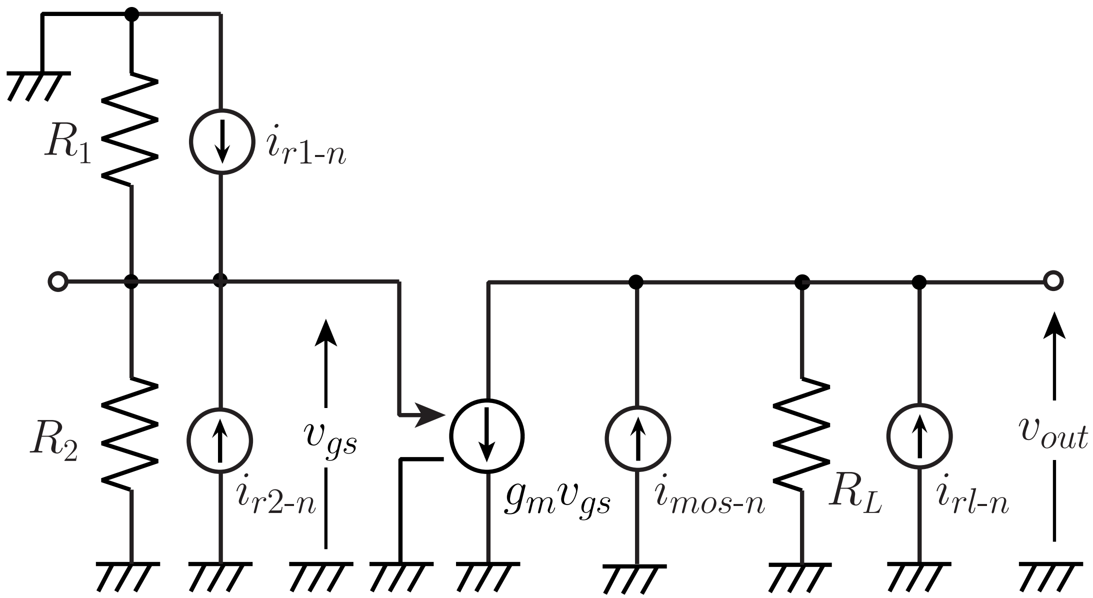


$$\overline{v_{o\text{-}n1}^2} = R_L^2 \left(\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2} \right)$$

$$\overline{v_{o\text{-}n2}^2} = \left| g_m R_L \frac{R_1 R_2}{R_2 + R_1} \right|^2 \left(\overline{i_{r1\text{-}n}^2} + \overline{i_{r2\text{-}n}^2} \right)$$

$$\overline{v_{out}^2} = \overline{v_{o\text{-}n1}^2} + \overline{v_{o\text{-}n2}^2}$$

$$\overline{v_N^{\,2}} = R_L^{\,2}\left(\overline{i_{rl-n}^{\,2}} + \overline{i_{mos-n}^{\,2}}\right) + \left| g_m R_L \frac{R_1 R_2}{R_2 + R_1} \right|^2 \left(\overline{i_{r1\text{-}n}^{\,2}} + \overline{i_{r2\text{-}n}^{\,2}}\right)$$



$$i_n = \frac{v_N}{Z_T}$$

$$Z_T = -g_m R_L \frac{R_1 R_2}{R_2 + R_1}$$

$$\overline{i_n^{\,2}} = \left| \frac{R_1 + R_2}{g_m R_2 R_1} \right|^2 \left(\overline{i_{rl\text{-}n}^{\,2}} + \overline{i_{mos\text{-}n}^{\,2}}\right) + \left(\overline{i_{r1\text{-}n}^{\,2}} + \overline{i_{r2\text{-}n}^{\,2}}\right)$$

$$\overline{v_n^2} = \frac{1}{g_m^2}\left(\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2}\right)$$

$$\overline{i_n^2} = \left|\frac{R_1+R_2}{g_m R_2 R_1}\right|^2 \left(\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2}\right) + \left(\overline{i_{r1\text{-}n}^2} + \overline{i_{r2\text{-}n}^2}\right)$$

$$F = 1 + \frac{\overline{v_n^2} + R_S^2\,\overline{i_n^2}}{4kTR_S}$$ より

$$F = 1 + \frac{1}{4kTR_S}\left[\left\{\frac{1}{g_m^2} + \left|\frac{(R_1+R_2)R_S}{g_m R_1 R_2}\right|^2\right\}\left(\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2}\right) + R_S^2\left(\overline{i_{r1\text{-}n}^2} + \overline{i_{r2\text{-}n}^2}\right)\right]$$

v_nとi_nの間の相関も考慮

$$F = 1 + \frac{1}{4kTR_S}\left\{\left|\frac{R_1R_2 + R_2R_S + R_SR_1}{g_mR_1R_2}\right|^2 (\overline{i_{rl\text{-}n}^{\ 2}} + \overline{i_{mos\text{-}n}^{\ 2}}) + R_S^{\ 2}(\overline{i_{r1\text{-}n}^{\ 2}} + \overline{i_{r2\text{-}n}^{\ 2}})\right\}$$

v_nとi_nは無相関と仮定

$$F = 1 + \frac{1}{4kTR_S}\left[\left\{\frac{1}{g_m^{\ 2}} + \left|\frac{(R_1 + R_2)R_S}{g_mR_1R_2}\right|^2\right\}(\overline{i_{rl\text{-}n}^{\ 2}} + \overline{i_{mos\text{-}n}^{\ 2}}) + R_S^{\ 2}(\overline{i_{r1\text{-}n}^{\ 2}} + \overline{i_{r2\text{-}n}^{\ 2}})\right]$$

低雑音増幅回路の設計

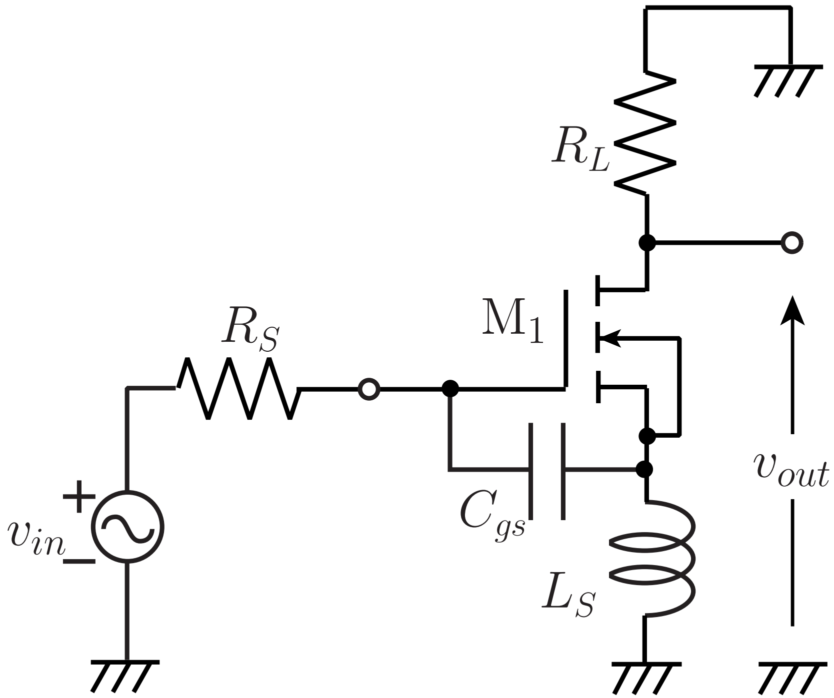
通信機器用低雑音増幅回路

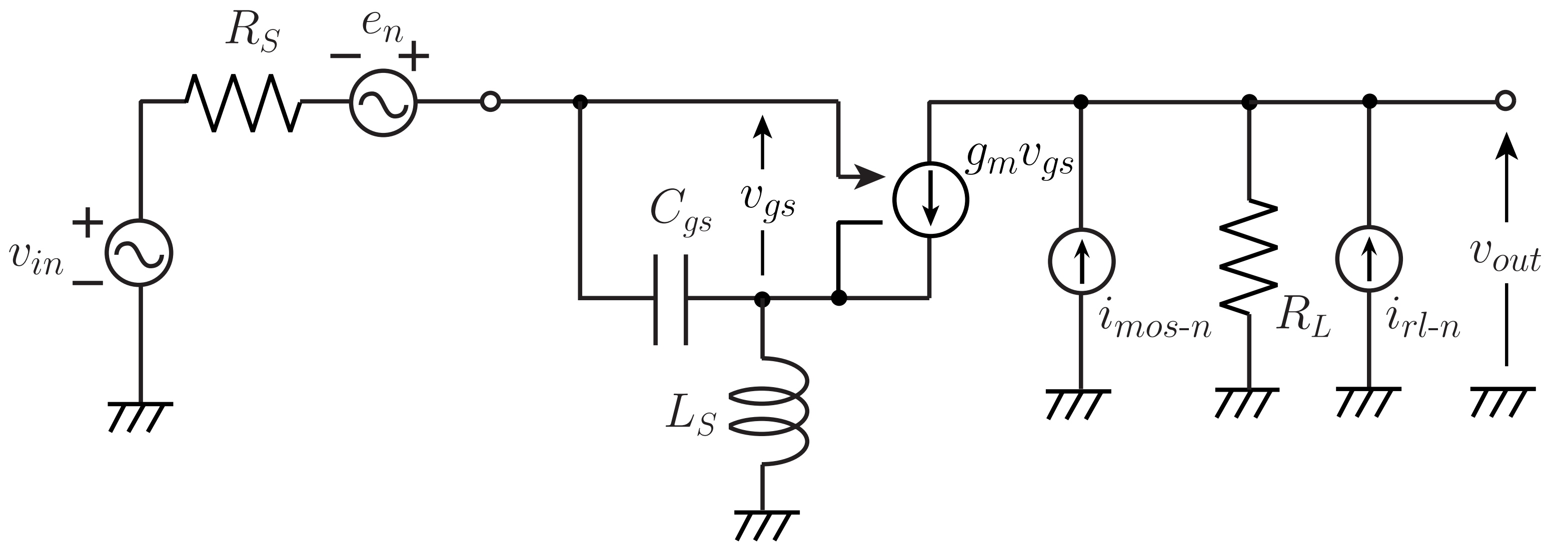
高周波(数GHz)

狭帯域

LC共振特性の応用

LC共振特性の応用





$$\overline{v_N^{\,2}} = R_L^{\,2}\left(\overline{i_{rl\text{-}n}^{\,2}} + \overline{i_{mos\text{-}n}^{\,2}}\right)$$

$$A = \frac{-g_m R_L}{1 + sL_S(g_m + sC_{gs})}$$

$$Z_T = \frac{g_m}{sC_{gs}} R_L$$

$$\overline{v_N^2} = R_L^2 (\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2})$$

$$A = \frac{-g_m R_L}{1 + sL_S(g_m + sC_{gs})} \qquad Z_T = \frac{g_m}{sC_{gs}} R_L$$

$$\overline{v_n^2} = \left| \frac{1 + sL_S(g_m + sC_{gs})}{g_m} \right|^2 (\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2})$$

$$\overline{i_n^2} = \left| \frac{sC_{gs}}{g_m} \right|^2 (\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2})$$

$$\overline{v_n^2}=\left|\frac{1+sL_S(g_m+sC_{gs})}{g_m}\right|^2(\overline{i_{rl\text{-}n}^2}+\overline{i_{mos\text{-}n}^2})$$

$$\overline{i_n^2}=\left|\frac{sC_{gs}}{g_m}\right|^2(\overline{i_{rl\text{-}n}^2}+\overline{i_{mos\text{-}n}^2})$$

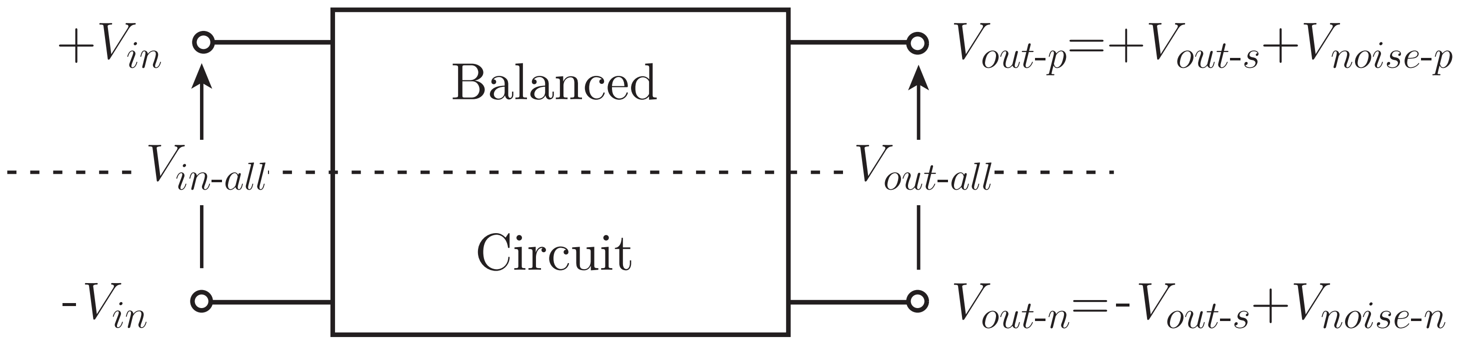
$$\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}}=\left|\frac{1+sL_S(g_m+sC_{gs})}{sC_{gs}}\right|$$

$$\left|\frac{1+sL_S(g_m+sC_{gs})}{sC_{gs}}\right|=R_S$$

↑

$f=\frac{1}{2\pi}\sqrt{\frac{1}{L_SC_{gs}}}$のとき$\frac{L_Sg_m}{C_{gs}}$の純抵抗

平衡型構成による雑音の低減？



Symmetry Structure

$$V_{noise\text{-}d}=\frac{V_{noise\text{-}p}-V_{noise\text{-}n}}{2}$$

$$V_{noise\text{-}p}=+V_{noise\text{-}d}+V_{noise\text{-}c}$$

$$V_{noise\text{-}c}=\frac{V_{noise\text{-}p}+V_{noise\text{-}n}}{2}$$

$$V_{noise\text{-}n}=-V_{noise\text{-}d}+V_{noise\text{-}c}$$

$$V_{out\text{-}all}=V_{out\text{-}p}-V_{out\text{-}n}=2V_{out\text{-}s}+2V_{noise\text{-}d}$$

不平衡型回路のSNR

$$V_{out} = V_{out\text{-}s} + V_{noise}$$

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T {V_{out\text{-}s}}^2 dt = \overline{V_{out\text{-}s}}^2 \qquad \lim_{T\to\infty}\frac{1}{T}\int_0^T {V_{noise}}^2 dt = \overline{V_{noise}}^2$$

$$SNR_{imbal} = \frac{\overline{V_{out\text{-}s}}^2}{\overline{V_{noise}}^2}$$

平衡型回路のSNR

$$V_{out\text{-}all} = V_{out\text{-}p} - V_{out\text{-}n} = 2(V_{out\text{-}s} + V_{noise\text{-}d})$$

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T V_{out\text{-}s}{}^2 dt = \overline{V_{out\text{-}s}}^2$$

$$\overline{V_{noise\text{-}d}}^2 = \lim_{T\to\infty}\frac{1}{T}\int_0^T V_{noise\text{-}d}{}^2 dt = \lim_{T\to\infty}\frac{1}{T}\int_0^T V_{noise\text{-}c}{}^2 dt = \frac{1}{2}\overline{V_{noise}}^2$$

SNRの改善： $SNR_{bal} = \dfrac{\overline{V_{out\text{-}s}}^2}{\overline{V_{noise\text{-}d}}^2} = 2\dfrac{\overline{V_{out\text{-}s}}^2}{\overline{V_{noise}}^2}$

ただし，回路規模は2倍