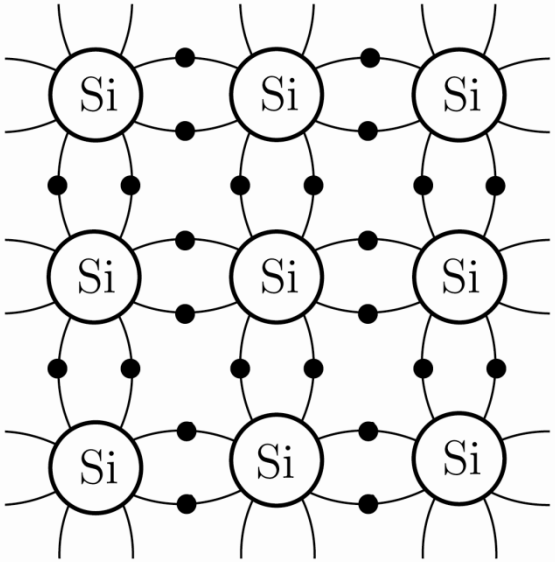
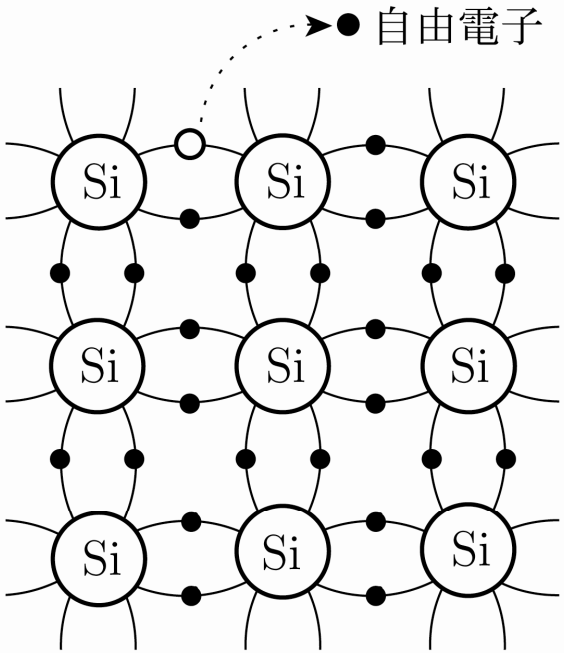
トランジスタのモデリング

半導体の構造



Si シリコンの原子核

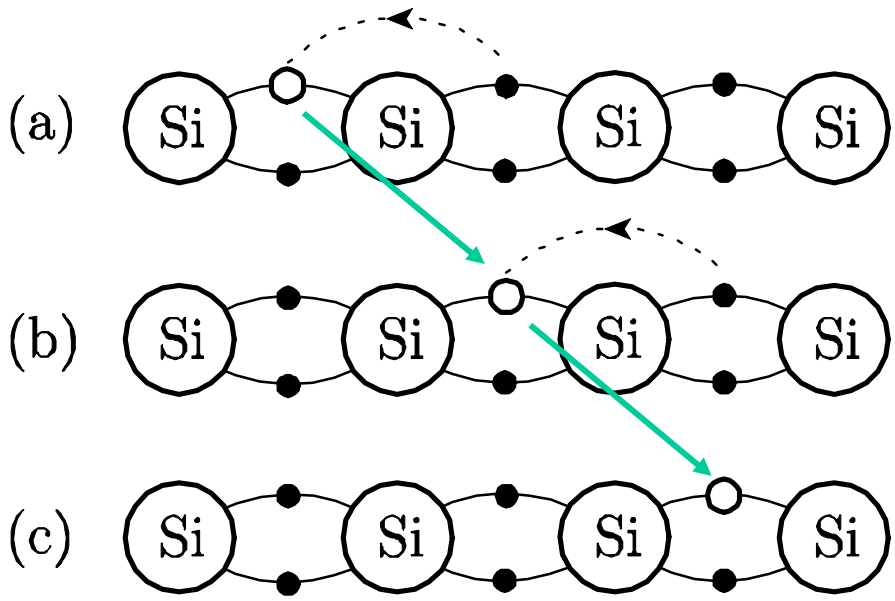
● 電子



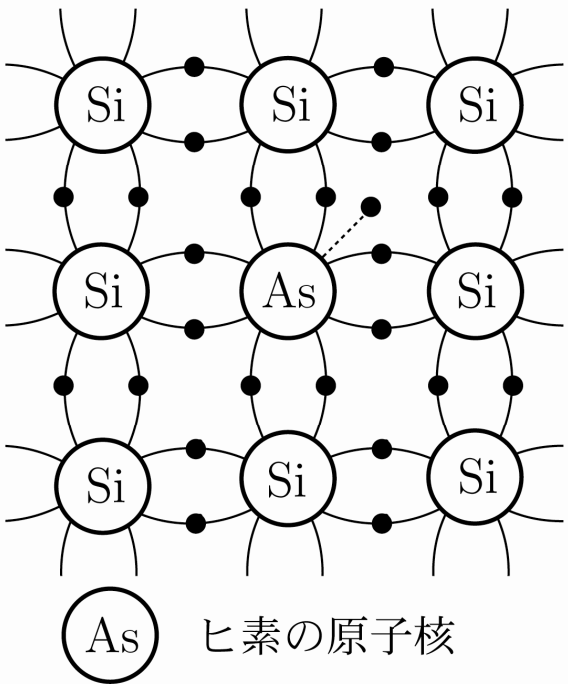
○ ホール

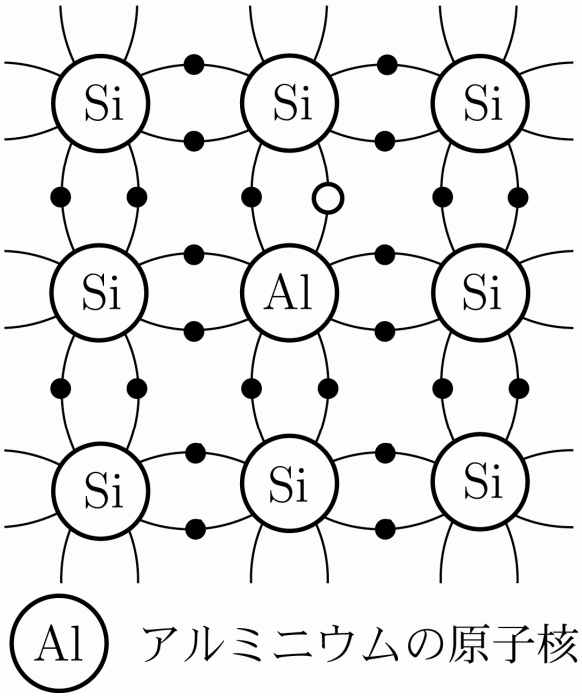
自由電子とホールの発生

ホールの移動



不純物半導体

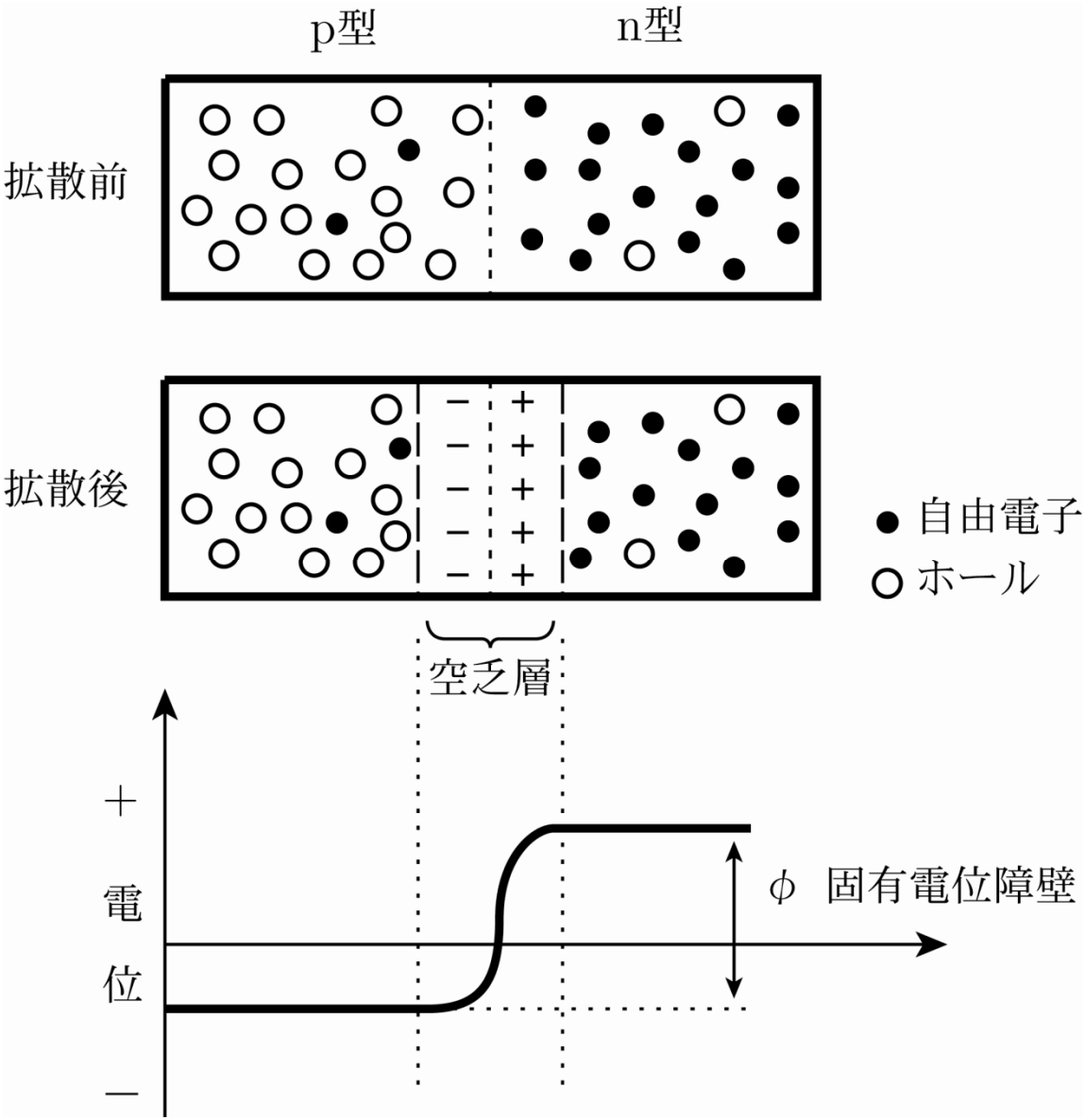




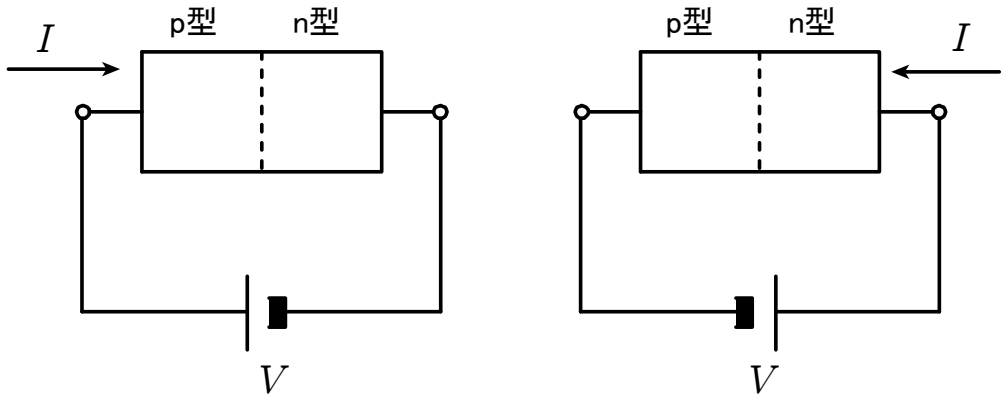
n型半導体

p型半導体

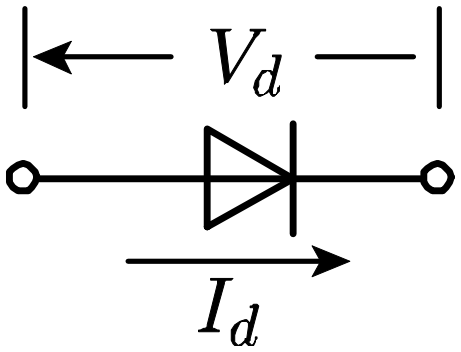
拡散と再結合



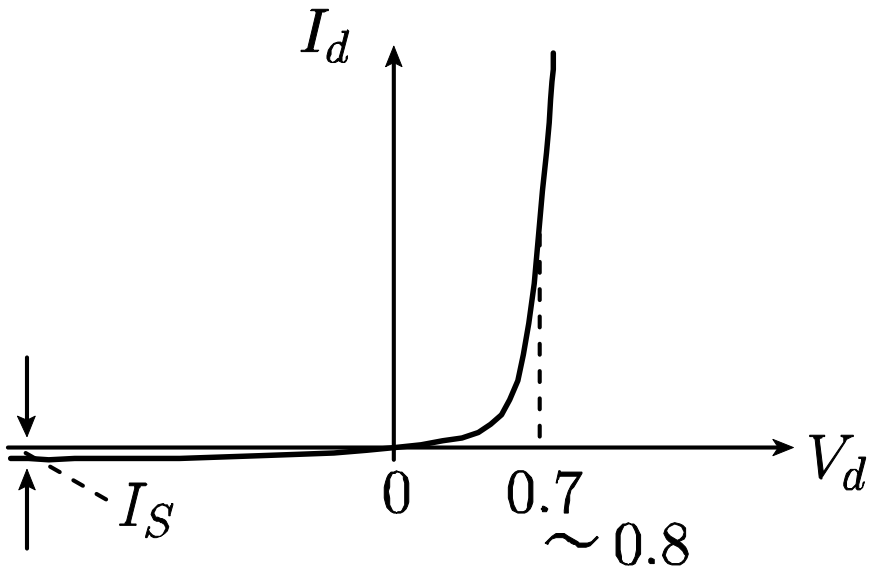
pn接合ダイオードの特性



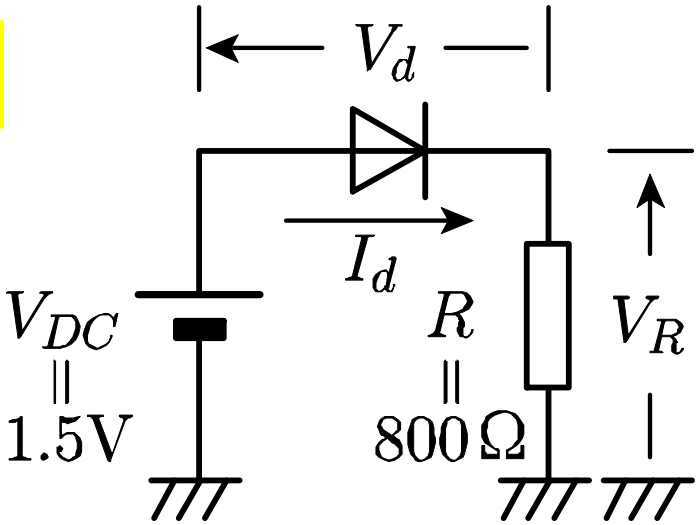
(a) 順方向バイアス　　(b) 逆方向バイアス



$$I_d = I_S\left\{\exp\left(\frac{qV_d}{kT}\right)-1\right\}$$



計算ができない？

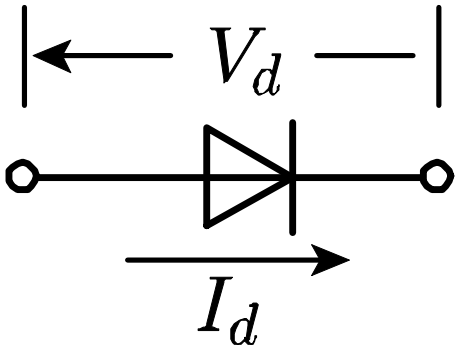


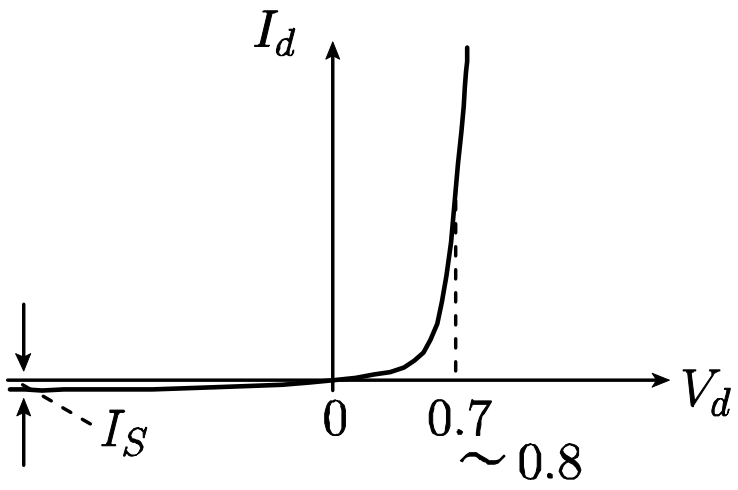
R=800Ω，I_S=10^{-15}A，k=1.38×10^{-23}J·K^{-1}，

q=1.6×10^{-19}C，T=300K，V_{DC}=1.5V

$$I_d = I_S\left\{\exp\left(\frac{qV_d}{kT}\right)-1\right\} \qquad I_d = \frac{V_{DC}-V_d}{R}$$

未知の変数はV_dとI_dの2個で，方程式も2個だが，
V_dやI_dについて解くことができない．





実際のpn接合ダイオードの特性

I_S=10^{-15}A，k=1.38×10^{-23}J·K^{-1}，q=1.6×10^{-19}C，T=300K

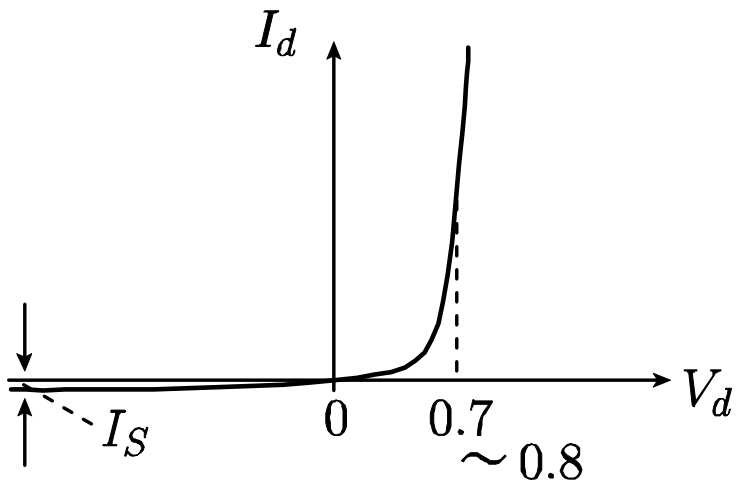
$$V_D=\frac{kT}{q}\ln\left(\frac{I_D}{I_S}+1\right)$$ より

I_D=0.10mAのときV_D=0.655V

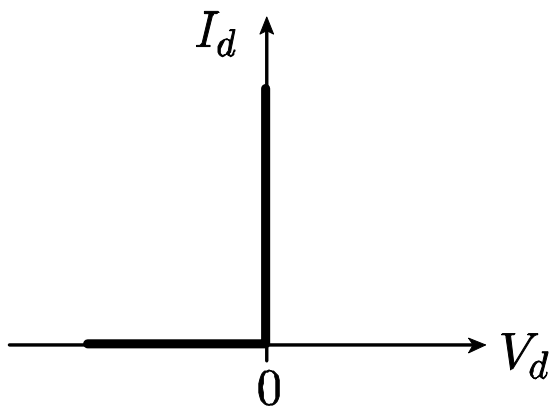
I_D=1.0mAのときV_D=0.715V

I_D=10mAのときV_D=0.775V

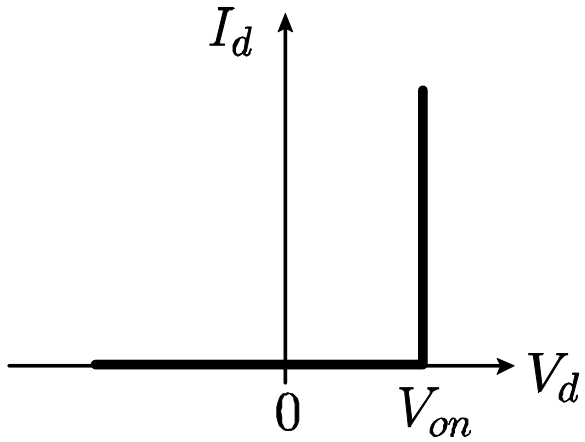
I_Dが大きく変化してもV_Dはほぼ一定.



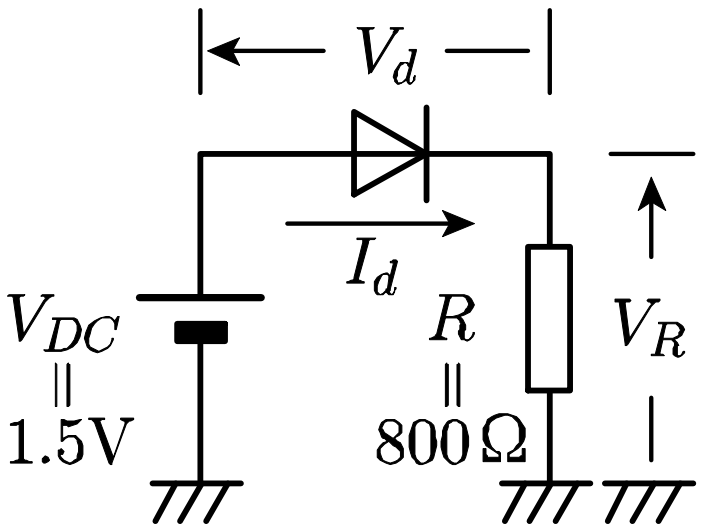
実際のpn接合ダイオードの特性



理想ダイオード特性



近似ダイオード特性



R=800Ω, I_S=10^{-15}A, k=1.38×10^{-23}J·K^{-1},

q=1.6×10^{-19}C, T=300K, V_{DC}=1.5V, V_D=0.70V

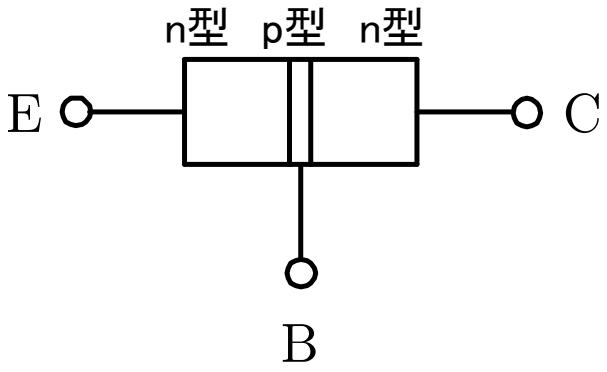
近似計算: V_D=0.70V $I_D=\dfrac{V_{DC}-V_D}{R}$=1.0mA

問：

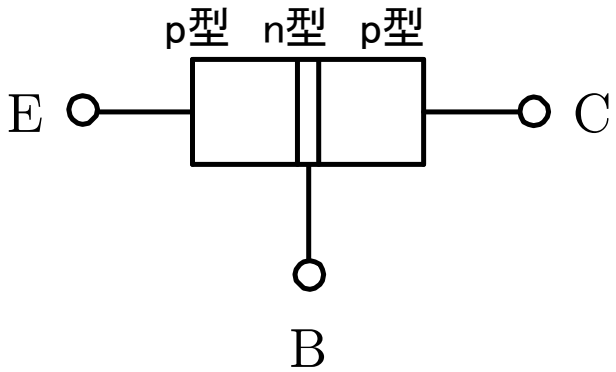
$$I_D^{(n+1)} = \frac{V_{DC} - \frac{kT}{q}\ln\left(\frac{I_D^{(n)}}{I_S} + 1\right)}{R}$$ という

漸化式を用いて真のI_D及びV_Dを求めよ.

バイポーラトランジスタのモデリング



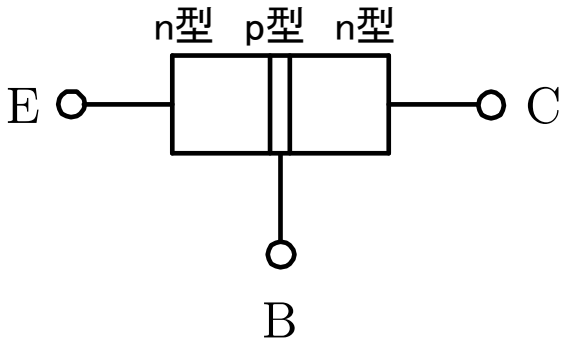
(a) npnトランジスタ



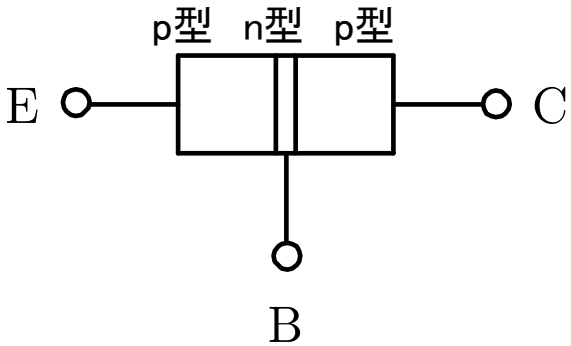
(b) pnpトランジスタ

E: Emitter, エミッタ
B: Base, ベース
C: Collector, コレクタ

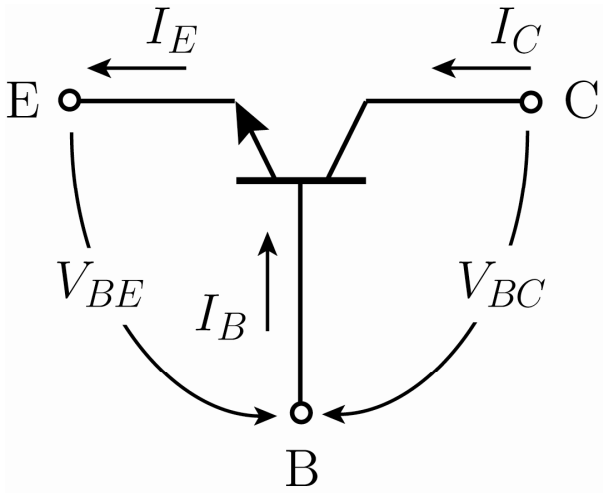
バイポーラトランジスタの記号



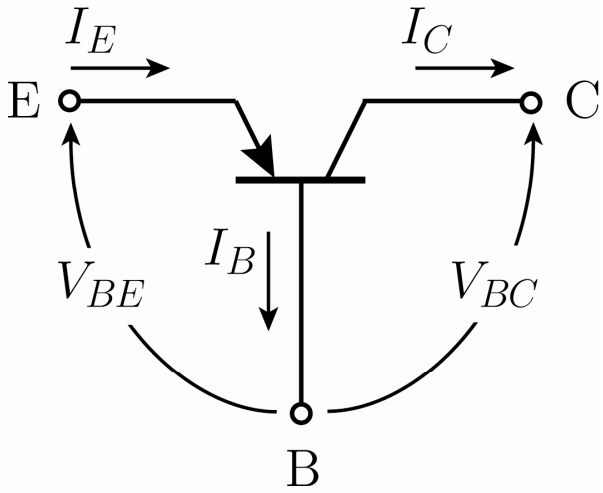
(a) npnトランジスタ



(b) pnpトランジスタ

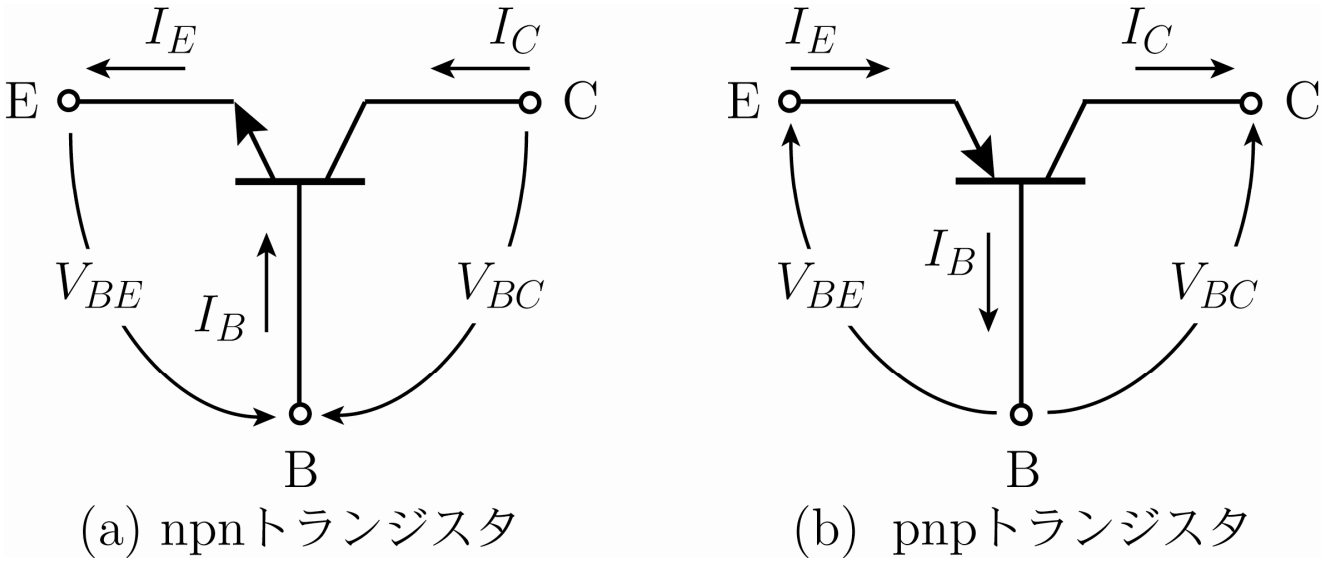


(a) npnトランジスタ



(b) pnpトランジスタ

バイポーラトランジスタの動作領域

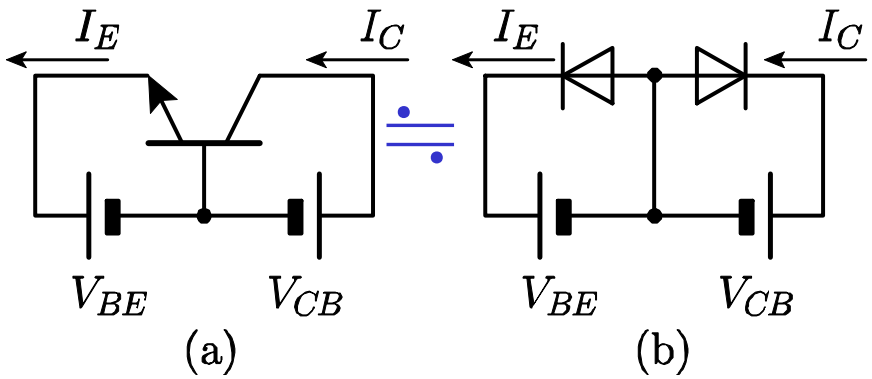


遮断領域： $V_{BE}<0,\ V_{BC}<0$

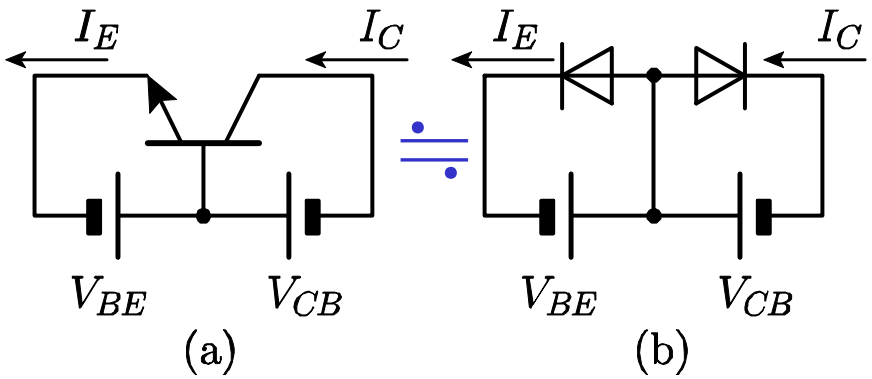
能動活性領域： $V_{BE}>0,\ V_{BC}<0$

飽和領域： $V_{BE}>0,\ V_{BC}>0$

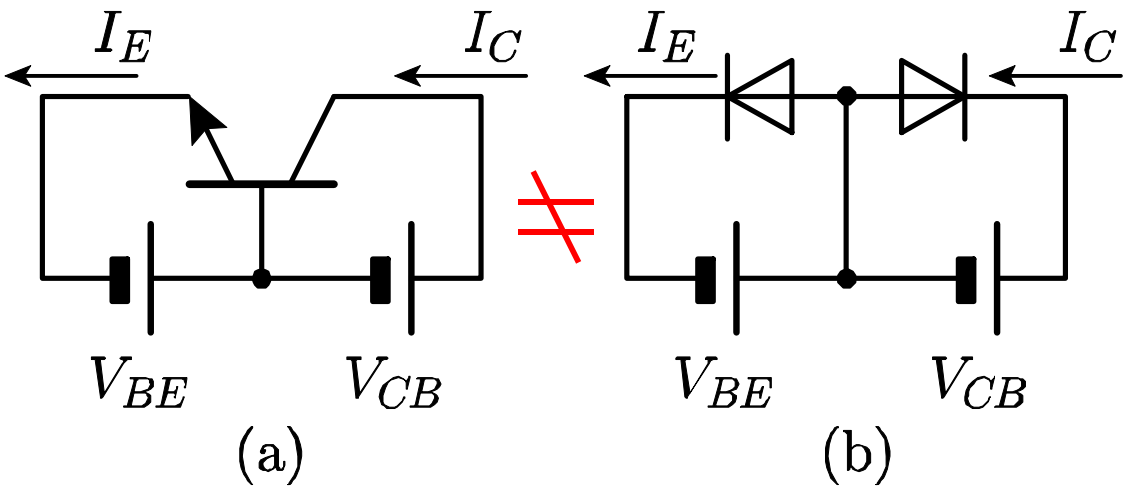
逆方向能動活性領域：$V_{BE}<0,\ V_{BC}>0$



遮断領域： $V_{BE}<0, V_{BC}<0$

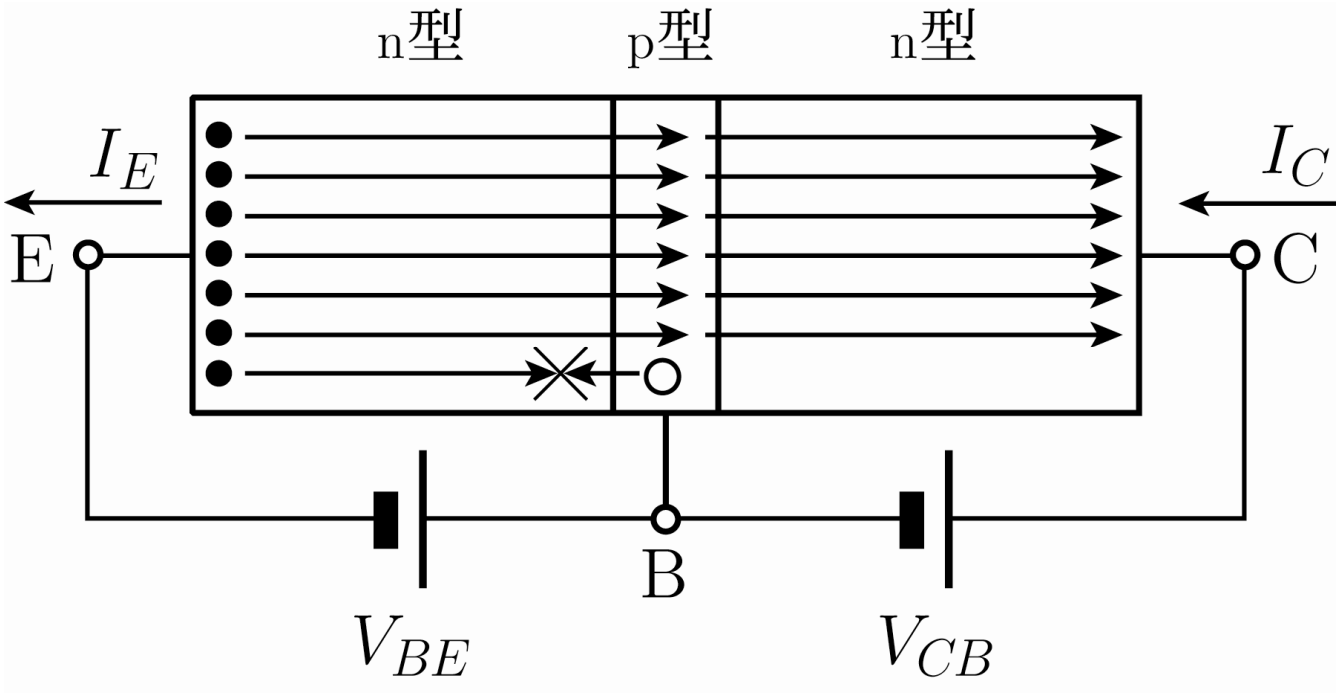


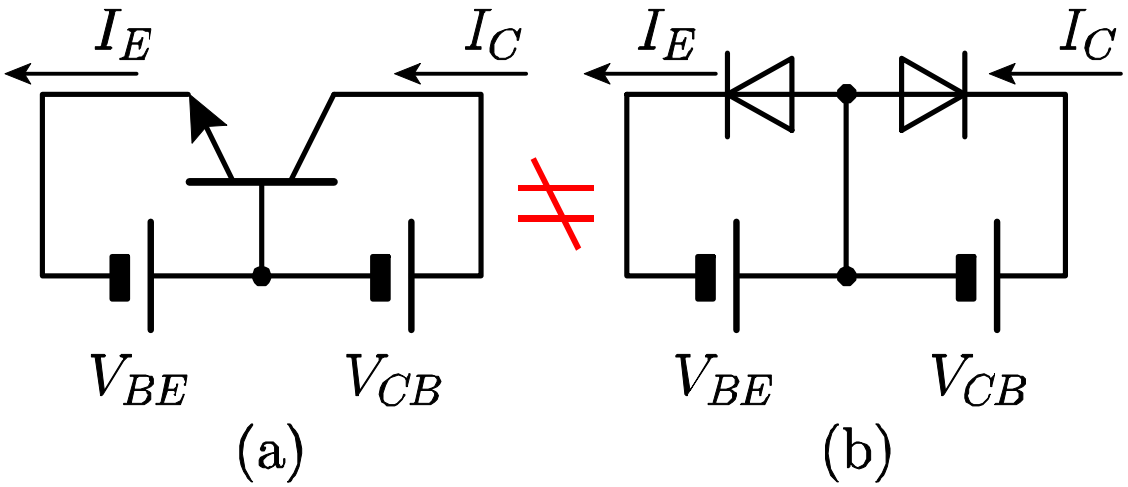
飽和領域：$V_{BE}>0, V_{BC}>0$



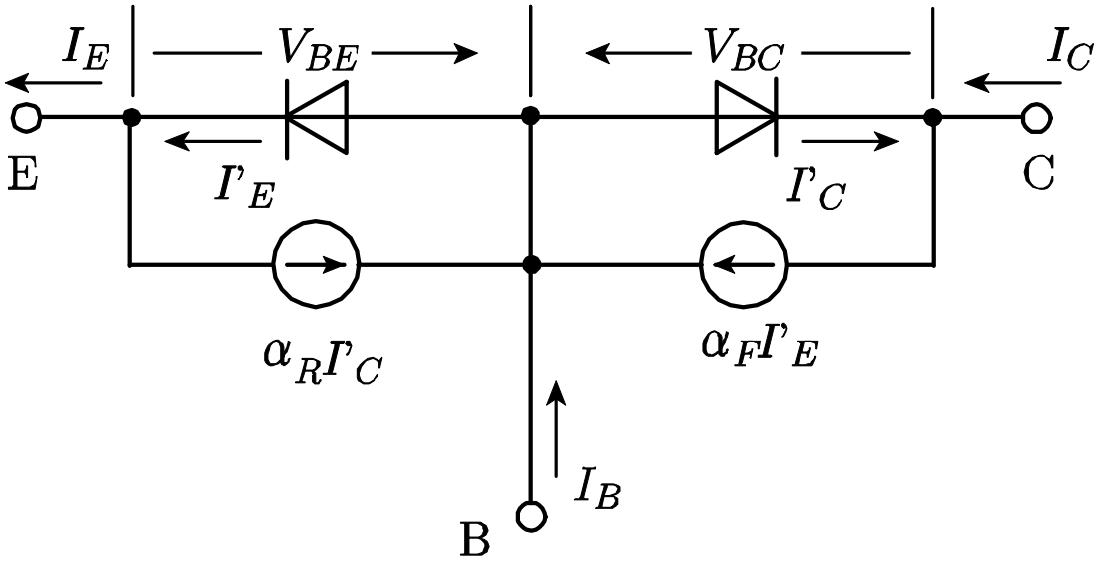
能動活性領域：$V_{BE}>0, V_{BC}<0$

能動活性領域





Ebers-Mollモデル



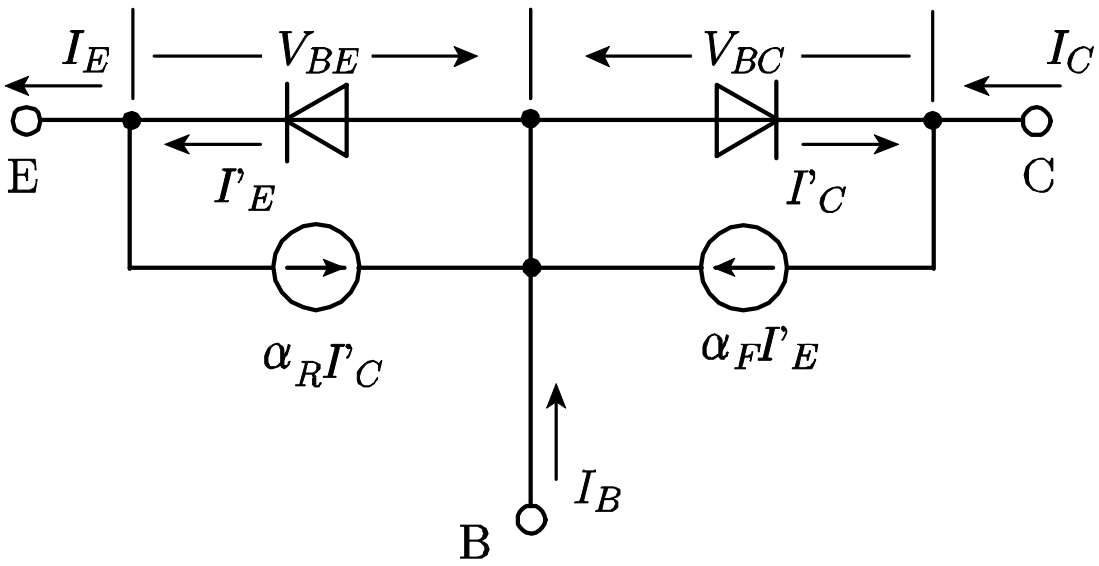
遮断領域：　$V_{BE}<0,\ V_{BC}<0$

能動活性領域：　$V_{BE}>0,\ V_{BC}<0$

飽和領域：　$V_{BE}>0,\ V_{BC}>0$

逆方向能動活性領域：$V_{BE}<0,\ V_{BC}>0$

Ebers-Mollの方程式



$$I_C = \alpha_F I_E' - I_C'$$
$$I_E = I_E' - \alpha_R I_C'$$

$$I_C' = I_{CS}\left\{\exp\left(\frac{qV_{BC}}{kT}\right) - 1\right\}$$

$$I_E' = I_{ES}\left\{\exp\left(\frac{qV_{BE}}{kT}\right) - 1\right\}$$

$$I_C=\alpha_F I_{ES}\left\{\exp\left(\frac{qV_{BE}}{kT}\right)-1\right\}-I_{CS}\left\{\exp\left(\frac{qV_{BC}}{kT}\right)-1\right\}$$

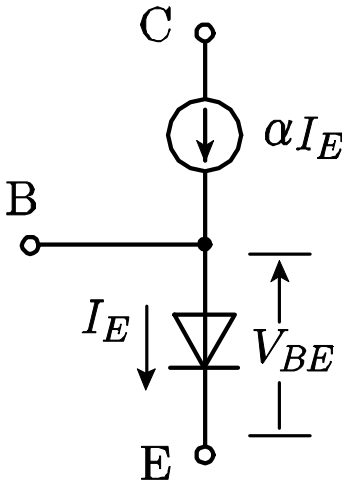
$$I_E=I_{ES}\left\{\exp\left(\frac{qV_{BE}}{kT}\right)-1\right\}-\alpha_R I_{CS}\left\{\exp\left(\frac{qV_{BC}}{kT}\right)-1\right\}$$

能動活性領域では, $\exp\left(\frac{qV_{BE}}{kT}\right)>>1$, $\exp\left(\frac{qV_{BC}}{kT}\right)<<1$と近似

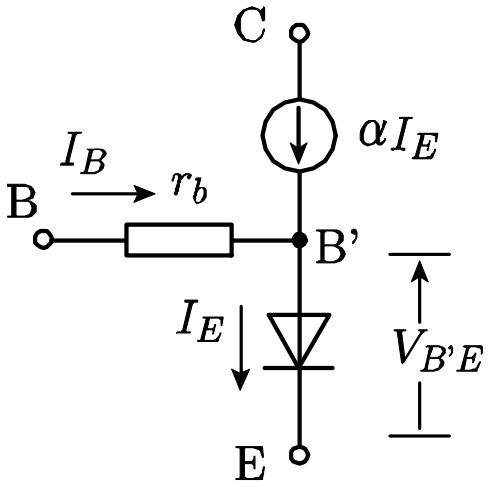
$$I_C=\alpha_F I_{ES}\exp\left(\frac{qV_{BE}}{kT}\right)+I_{CS}$$

$$I_E=I_{ES}\exp\left(\frac{qV_{BE}}{kT}\right)+\alpha_R I_{CS}$$

$$I_C=\alpha_F I_E+(1-\alpha_F\alpha_R)I_{CS}=\alpha_F I_E+I_{CO}$$



ベース広がり抵抗



精度が劣る

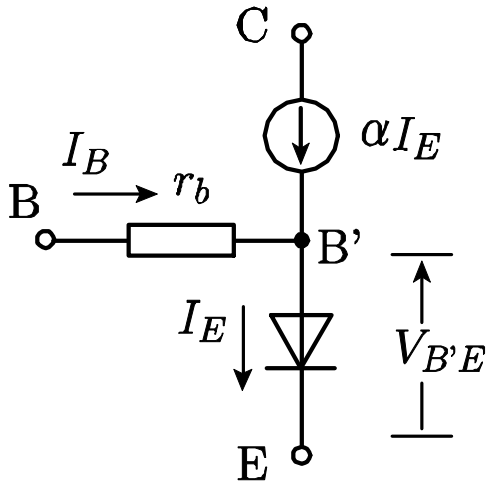
精度の向上

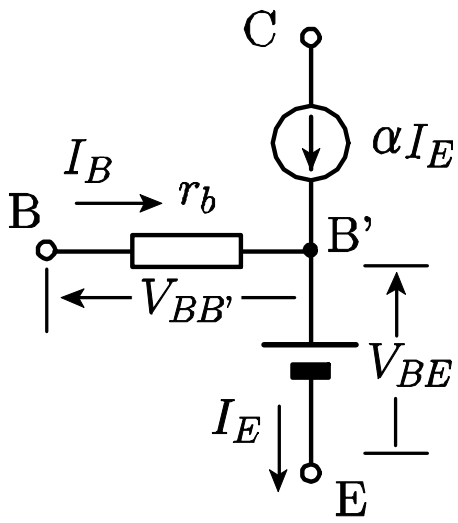
簡単

複雑

見通しの良さ

モデルの簡単化＝見通しの良い解析





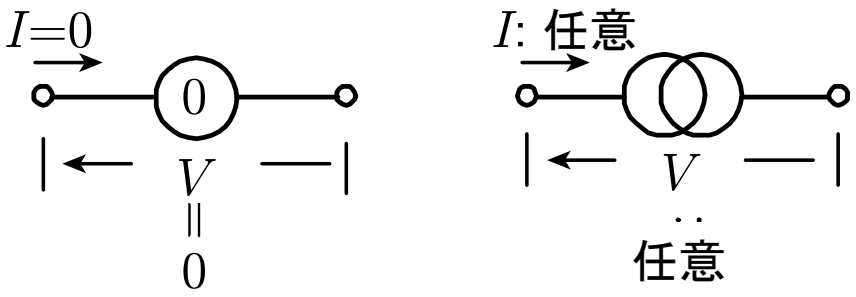
V_{BE}=0.70V

さらなる簡単化

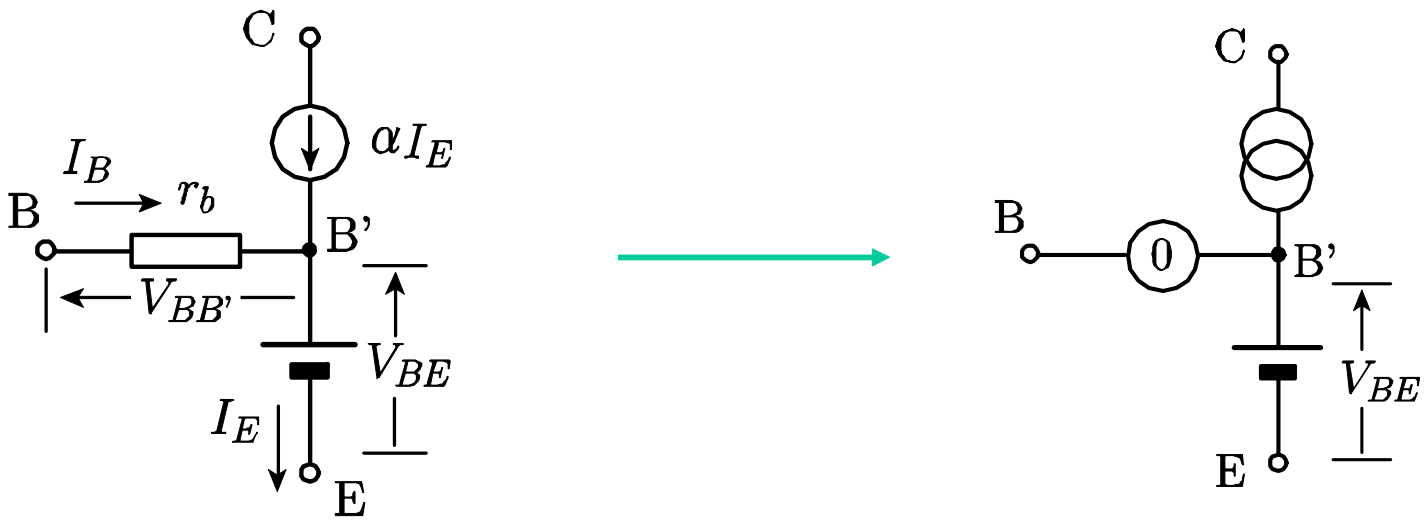
$\alpha \to 1$

$I_B = I_E - I_C = (1-\alpha) I_E$ → $I_B = 0$かつ$V_{BB'} = r_b I_B = 0$

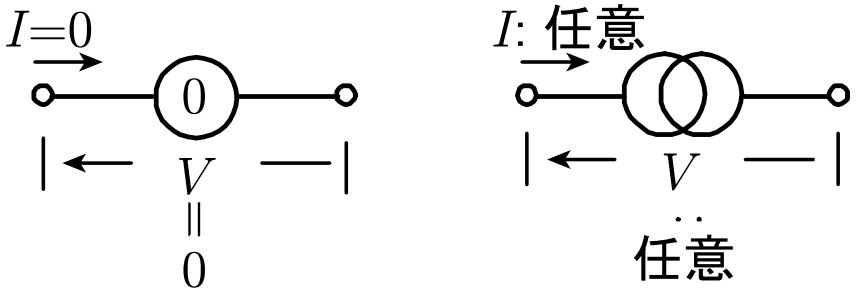
I_B=0かつ$V_{BB'}$=$r_b I_B$=0　(α=1)



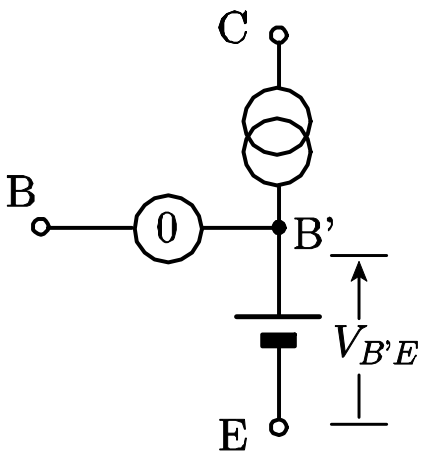
(a) ナレータ　　(b) ノレータ



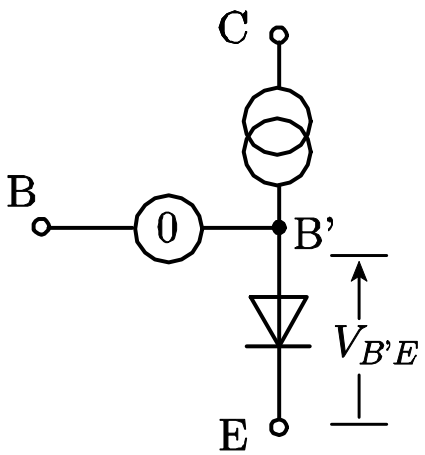
ナレータノ・レータによるトランジスタのモデリング



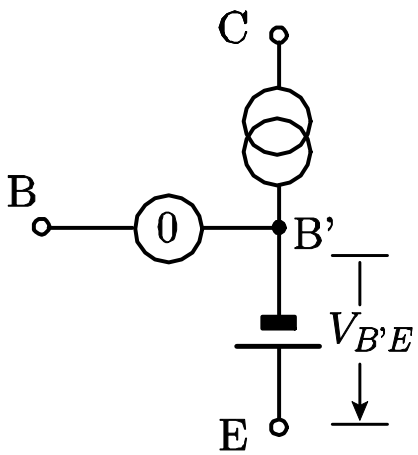
(a) ナレータ　　(b) ノレータ



npnトランジスタ
モデル（1）

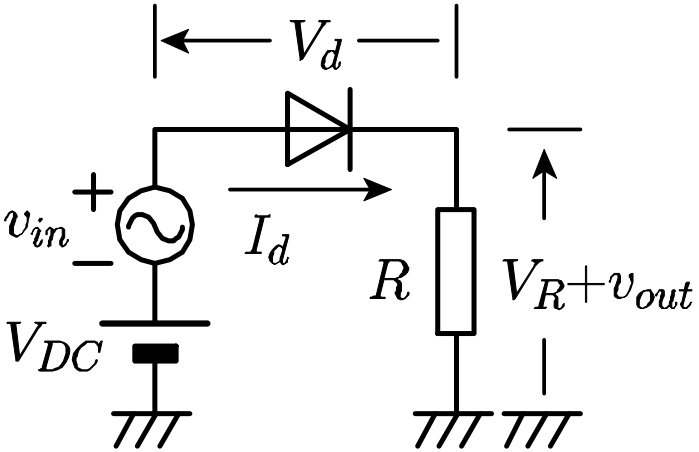


npnトランジスタ
モデル（2）

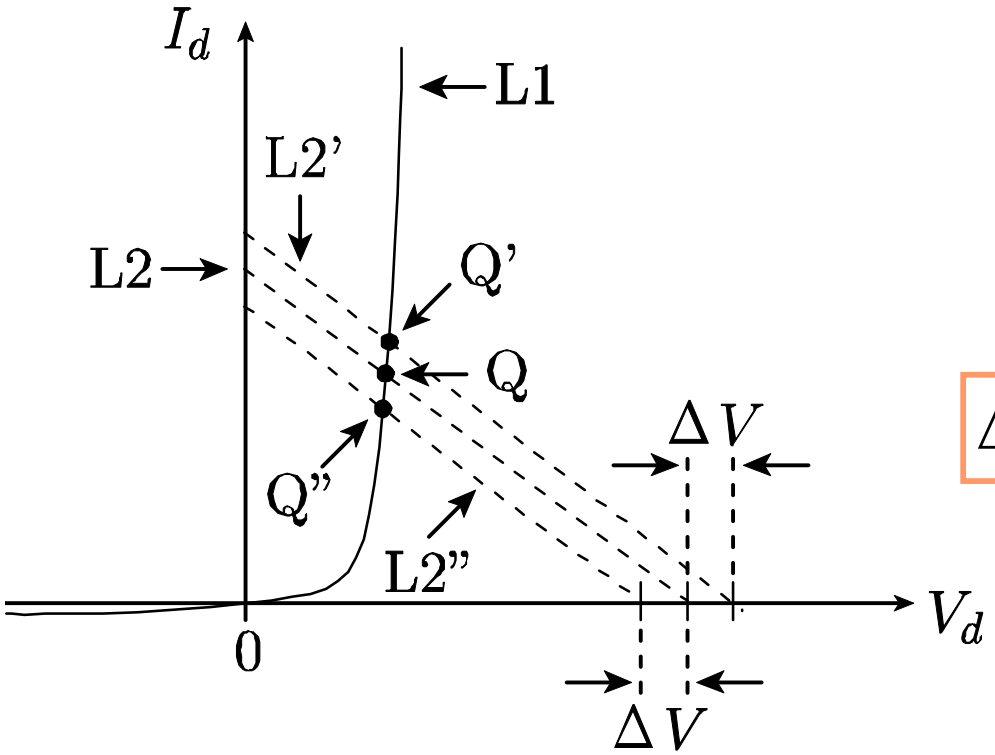


pnpトランジスタ
モデル

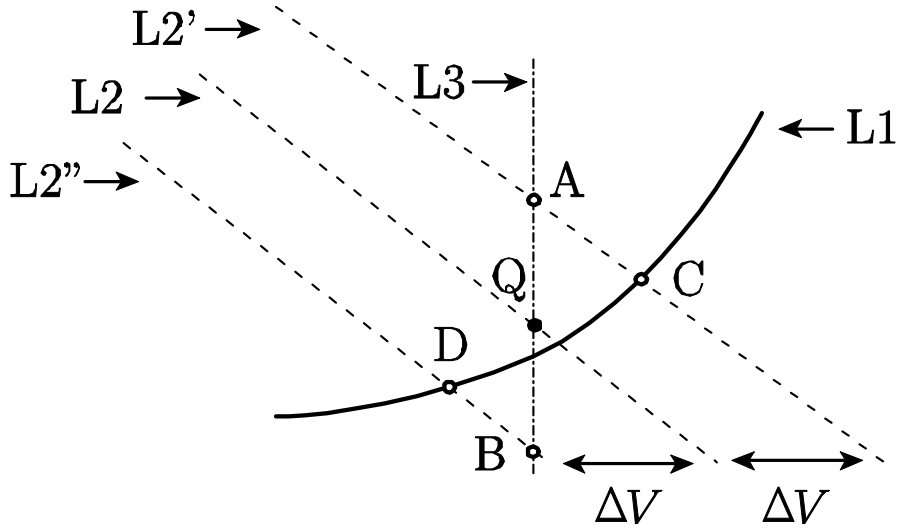
信号を加えた場合の解析

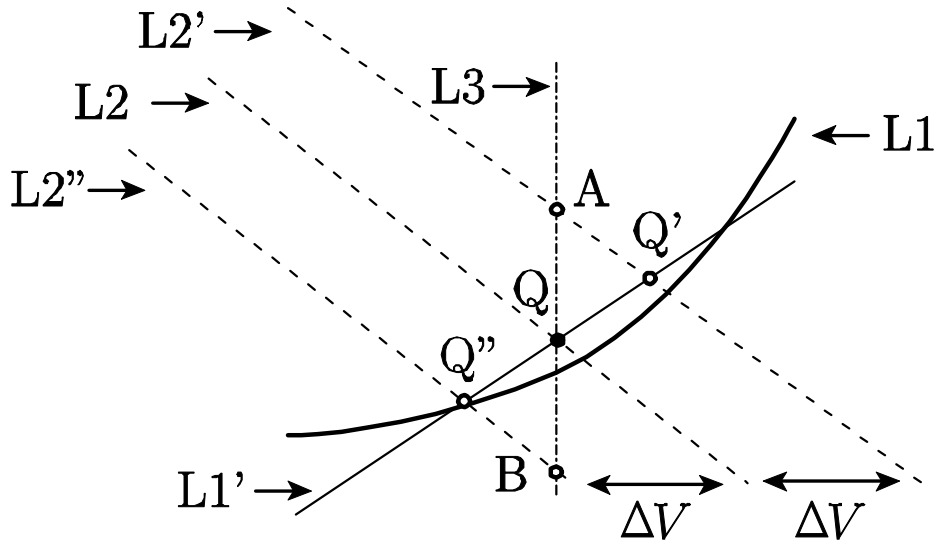


$$I_d=\frac{V_{DC}+v_{in}-V_d}{R}=-\frac{1}{R}V_d+\frac{V_{DC}+v_{in}}{R}$$



ΔV：v_{in}の振幅



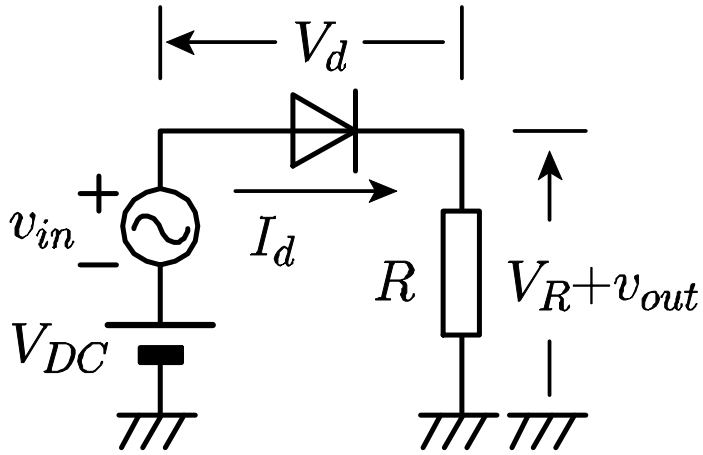


$$I_d = I_S\left\{\exp\left(\frac{qV_d}{kT}\right) - 1\right\}$$

$$\boxed{G = \left.\frac{\partial I_d}{\partial V_d}\right|_{V_d = V_Q} = \frac{qI_S}{kT}\exp\left(\frac{qV_d}{kT}\right) \doteqdot \frac{qI_Q}{kT}}$$

$$I_Q \doteqdot I_S \exp\left(\frac{qV_Q}{kT}\right)$$

直流分と信号分を分けた解析

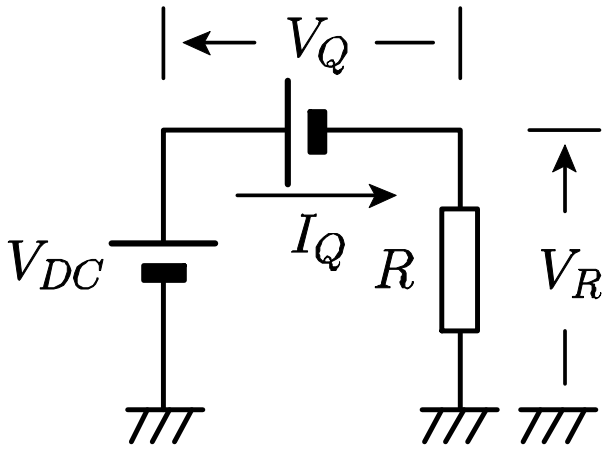


$$V_{DC}+v_{in}=V_Q+v_d+V_R+v_{out}$$ （全体）　　$V_d=V_Q+v_d$

−）$$V_{DC}=V_Q+V_R$$ （直流分）

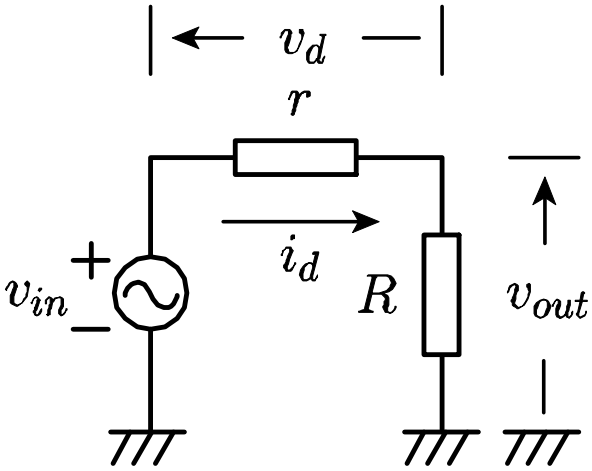
$$v_{in}=v_d+v_{out}$$ （信号分）

$$V_{DC}=V_Q+V_R$$



直流成分

$$v_{in}=v_d+v_{out}$$



小信号成分

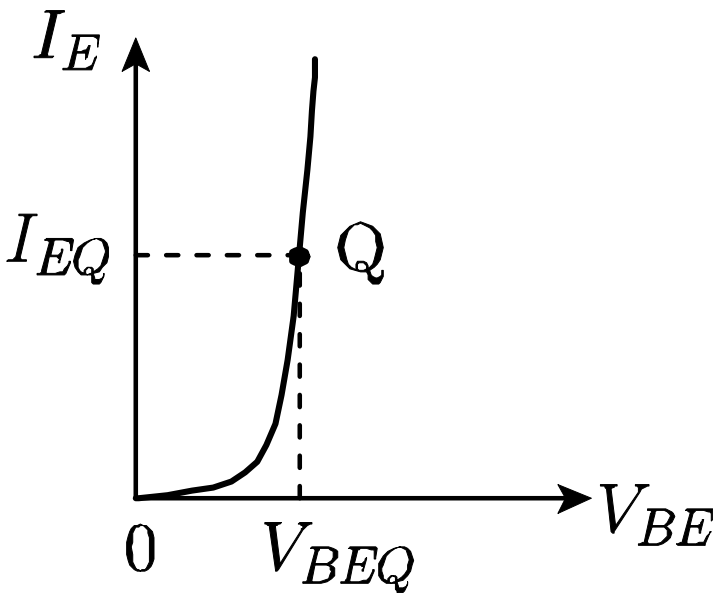
$$V_{DC}=V_Q+RI_Q$$

$$v_{in}=v_d+Ri_d$$

$$i_d=Gv_d$$

$$r=\frac{1}{G}=\frac{kT}{qI_Q}$$

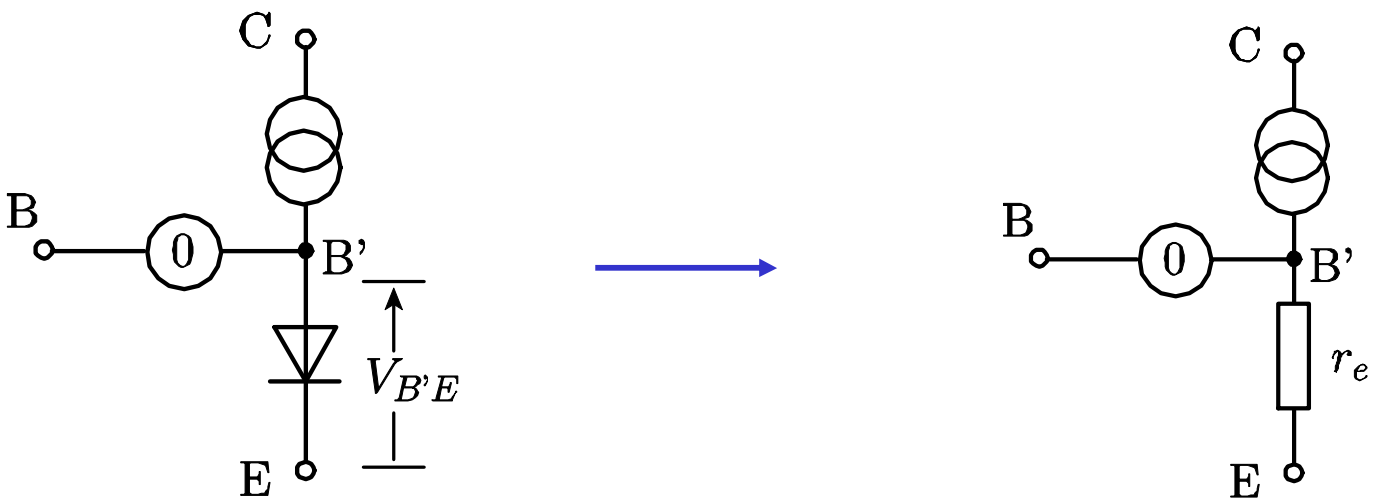
バイポーラトランジスタの小信号モデル



$$I_E = I_{ES}\left\{\exp\left(\frac{qV_{BE}}{kT}\right) - 1\right\}$$

$$I_E = \left.\frac{\partial I_E}{\partial V_{BE}}\right|_{V_{BE}=V_{BEQ}} \cdot \Delta V_{BE} + I_{EQ}$$

$$r_e = \left[\left.\frac{\partial I_E}{\partial V_{BE}}\right|_{V_{BE}=V_{BEQ}}\right]^{-1} \approx \frac{kT}{qI_{EQ}} = \frac{0.026[\mathrm{V}]}{I_{EQ}[\mathrm{A}]}$$

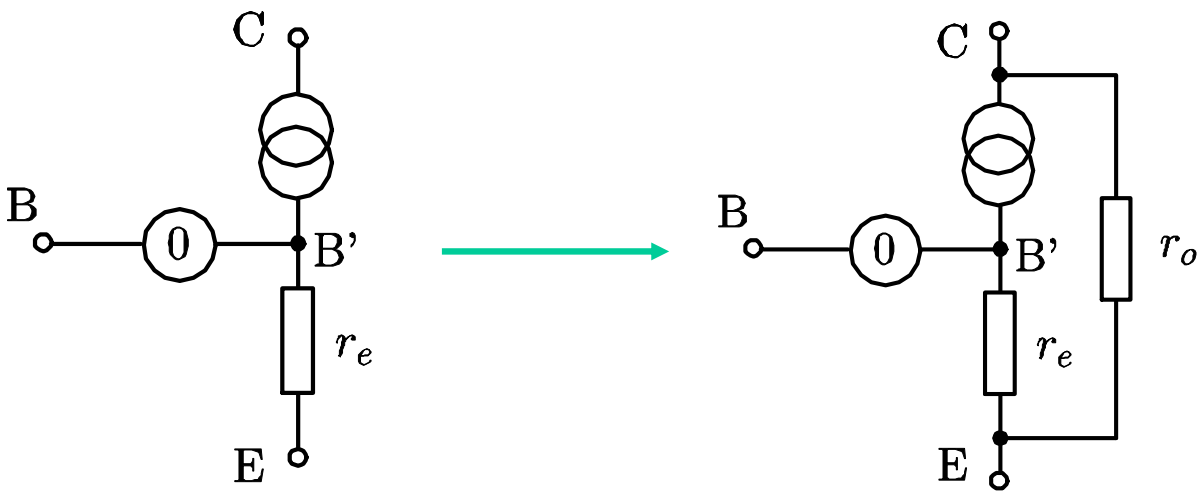


$$r_e = \left[\left. \frac{\partial I_E}{\partial V_{BE}} \right|_{V_{BE}=V_{BEQ}} \right]^{-1} \approx \frac{kT}{qI_{EQ}} = \frac{0.026[\mathrm{V}]}{I_{EQ}[\mathrm{A}]}$$

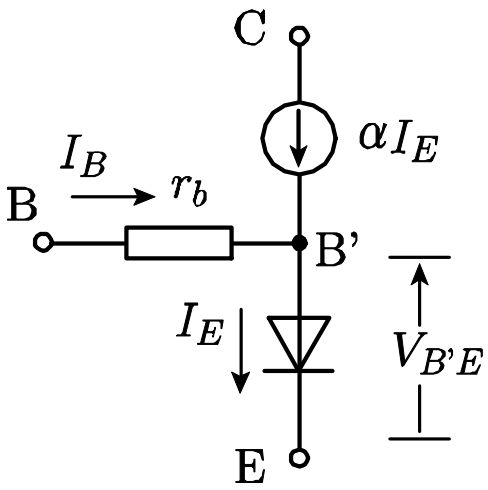
アーリー効果

コレクタ電流がコレクタ･エミッタ間電圧によって変化する現象

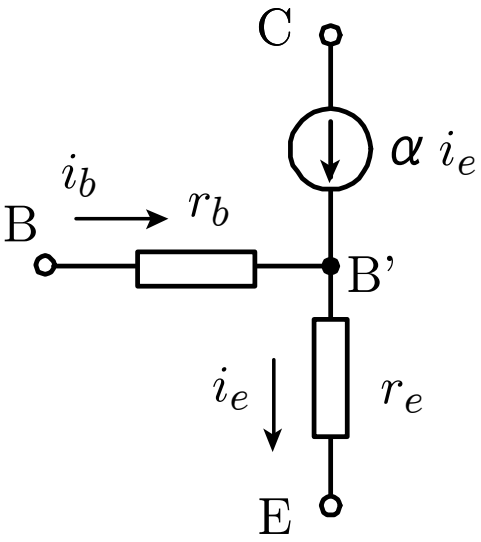
$$I_C = \alpha_F I_E \left(1 + \frac{V_{CE}}{V_A}\right) \qquad r_o = \frac{1}{\left.\dfrac{\partial I_C}{\partial V_{CE}}\right|_{V_{CE}=V_{CEQ}}} = \frac{V_A}{\alpha_F I_{EQ}}$$



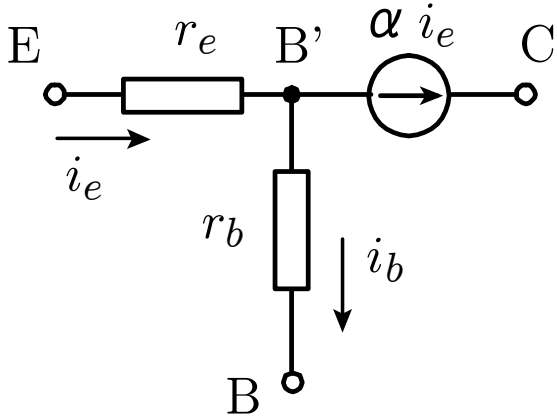
その他のバイポーラトランジスタの小信号モデル



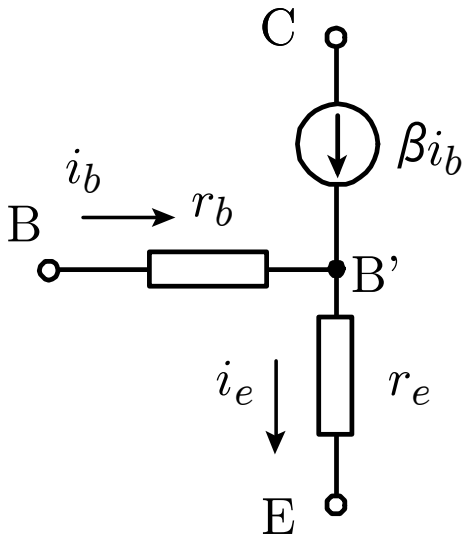
問：αとβとの関係を求めよ.



(a)

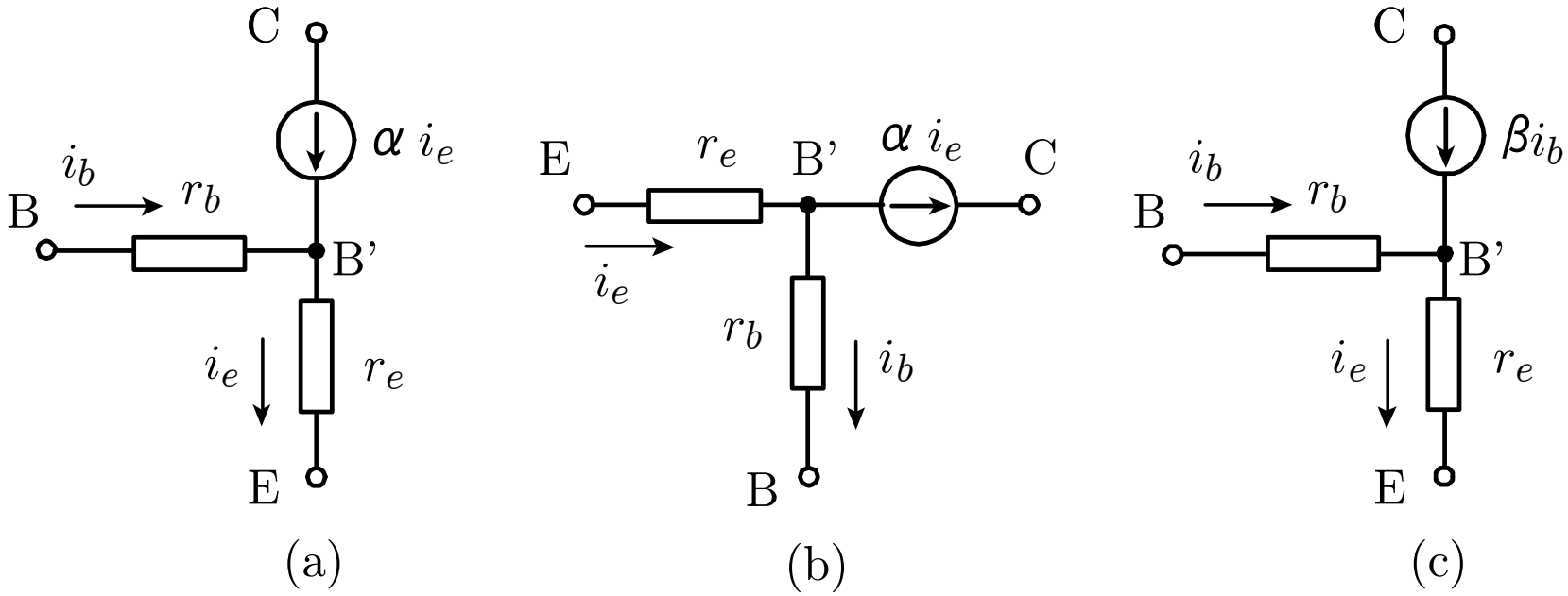


(b)



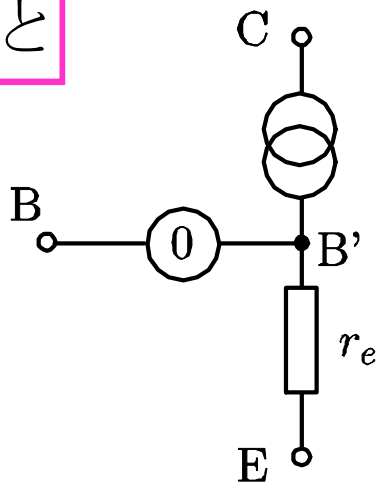
(c)

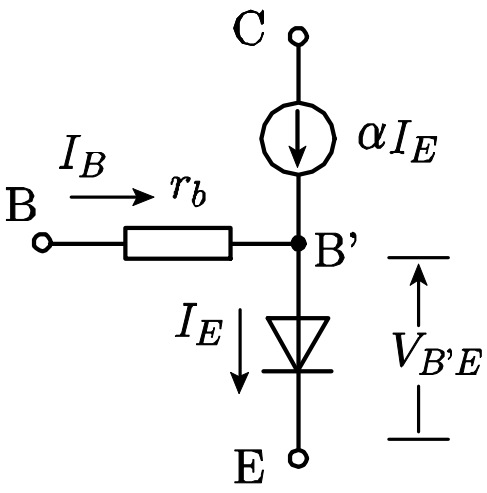
$$r_e = \frac{0.026\mathrm{V}}{I_{EQ}[\mathrm{A}]}$$



$$r_e = \frac{0.026\mathrm{V}}{I_{EQ}[\mathrm{A}]}$$

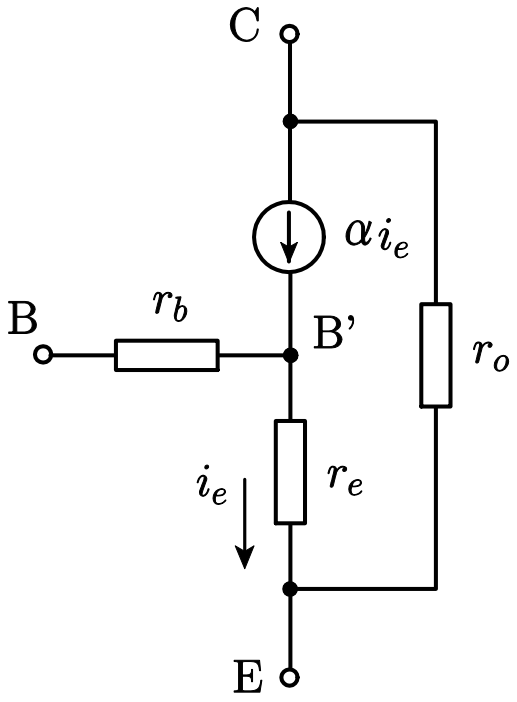
$\alpha \to 1 (\beta \to \infty)$とすると



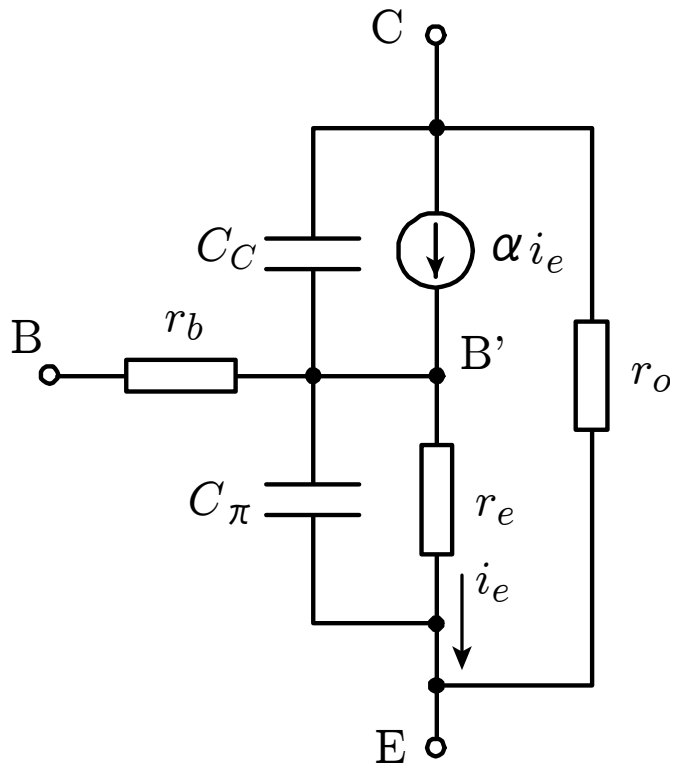


問：左図に相当するpnpトランジスタの大信号モデルを示せ．また，求めた大信号モデルから小信号モデルを求めよ．

バイポーラトランジスタの高周波小信号モデル



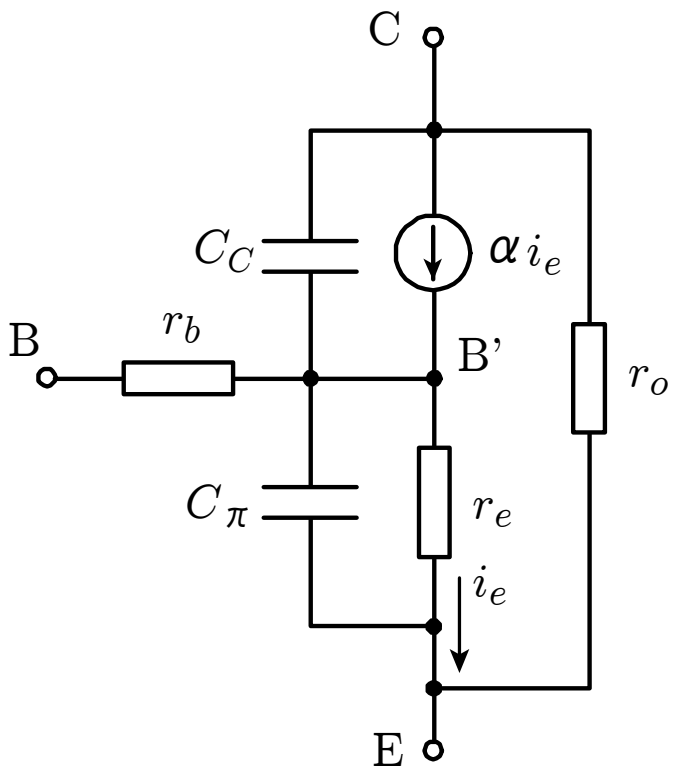
寄生容量を考慮

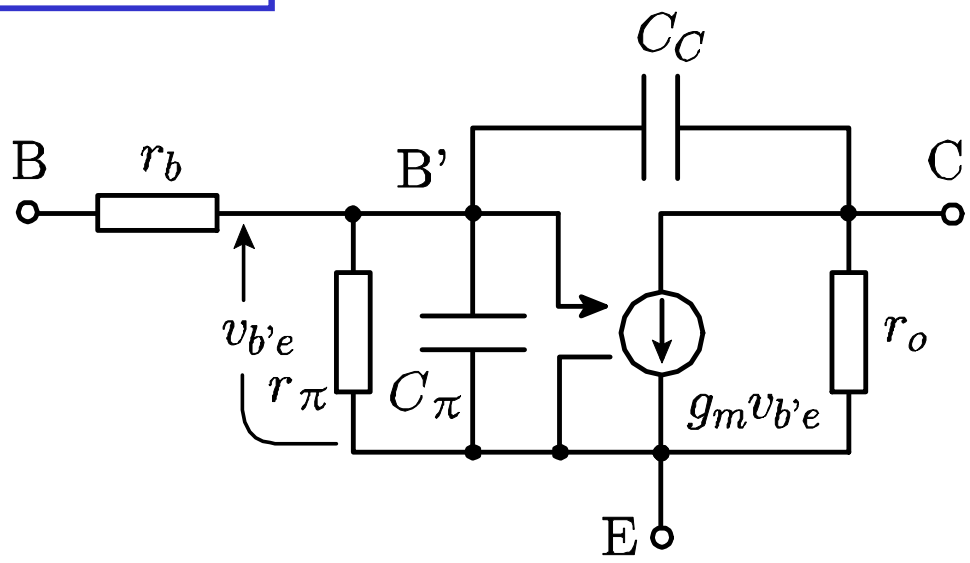


$$\alpha_F = \frac{\alpha_0}{1 + j\dfrac{\omega}{\omega_\alpha}}$$

$\omega_\alpha = \dfrac{1}{C_\pi r_e}$: 電流増幅率遮断角周波数

等価変換

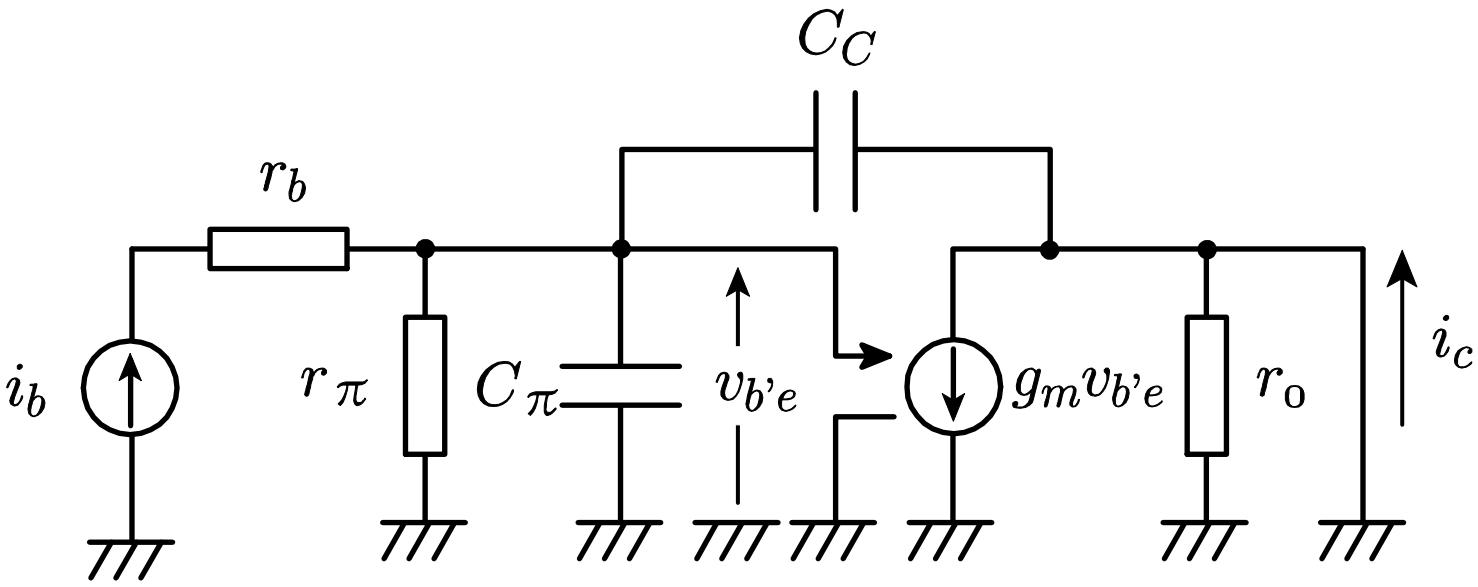


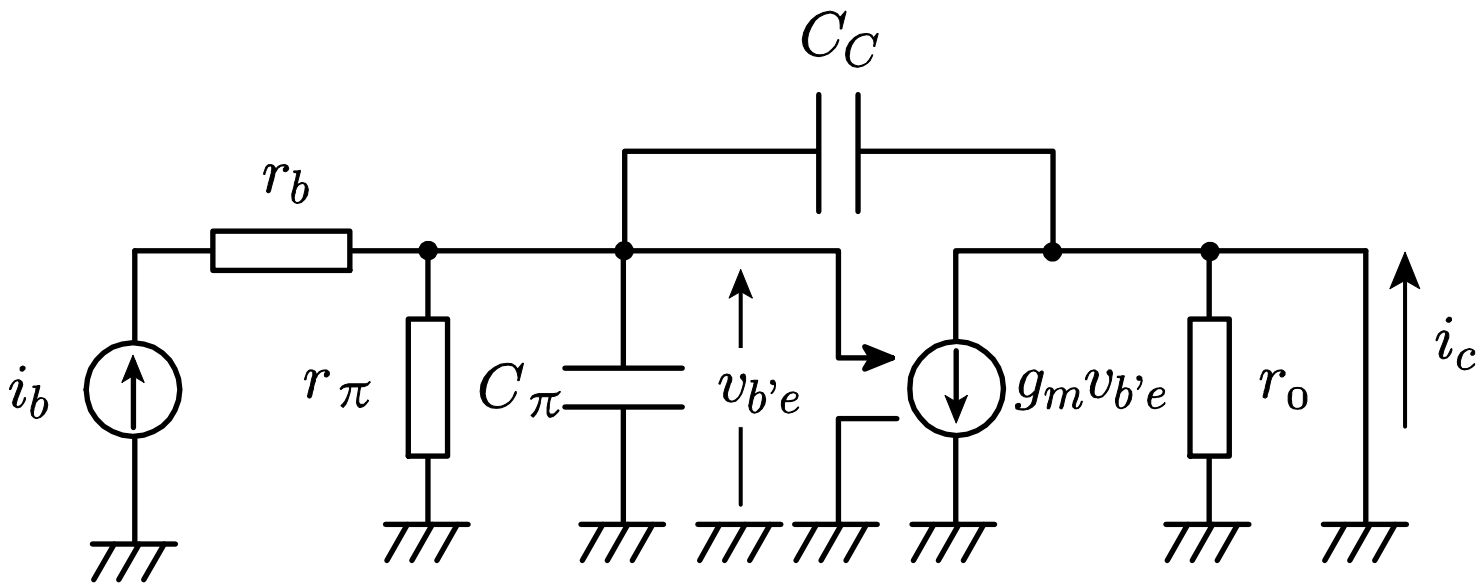


$$r_\pi = \frac{r_e}{1-\alpha_0} \qquad g_m = \frac{\alpha_0}{r_e}$$

バイポーラトランジスタのf_T

f_T: $|i_c|=|i_b|$となる周波数





$$v_{b'e}=\frac{r_\pi}{1+j\omega(C_\pi+C_C)r_\pi}i_b$$

$$\frac{i_c}{i_b}=\frac{(g_m-j\omega C_C)r_\pi}{1+j\omega(C_\pi+C_C)r_\pi}$$

$$i_c=g_m v_{b'e}-j\omega C_C v_{b'e}$$

$g_m >> \omega C_C$, $\omega(C_\pi+C_C)r_\pi >> 1$と仮定

$$\frac{i_c}{i_b}\approx\frac{g_m r_\pi}{j\omega(C_\pi+C_C)r_\pi}$$

$$f_T=\frac{g_m}{2\pi(C_\pi+C_C)}$$

$$\boxed{f_T = \frac{g_m}{2\pi(C_\pi + C_C)}}$$

$$C_\pi = \frac{\Delta Q_b}{v_{b'e}} \qquad C_\pi >> C_C$$

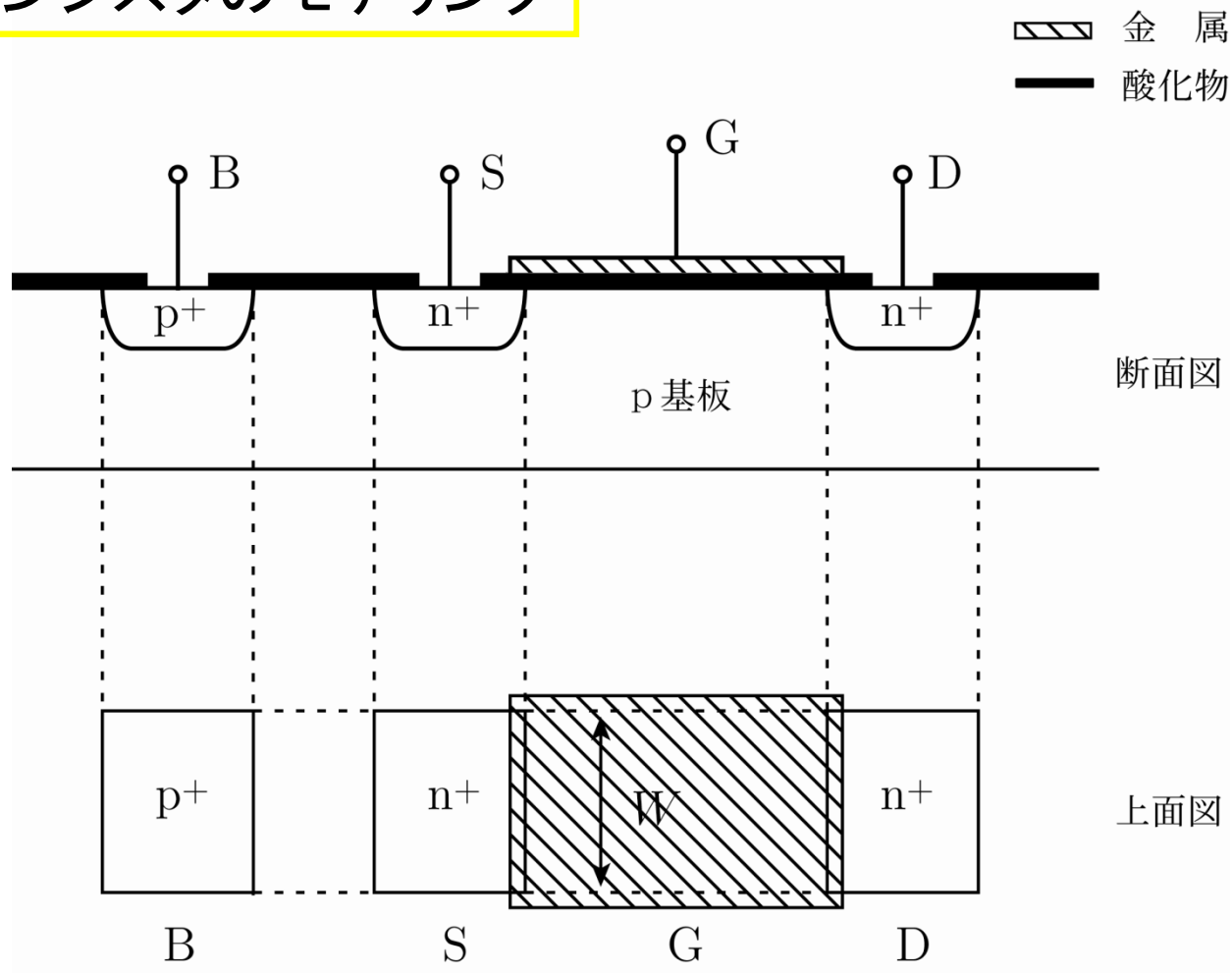
$$f_T \approx \frac{g_m}{2\pi C_\pi} = \frac{g_m v_{b'e}}{2\pi \Delta Q_b} = \frac{i_c}{2\pi \Delta Q_b}$$

$$\frac{i_c}{\Delta Q_b} = \frac{I_C}{Q_b} = \frac{qAD_n \dfrac{n_p(0)}{W_B}}{\dfrac{1}{2} n_p(0) W_B qA} = \frac{2D_n}{W_B{}^2}$$

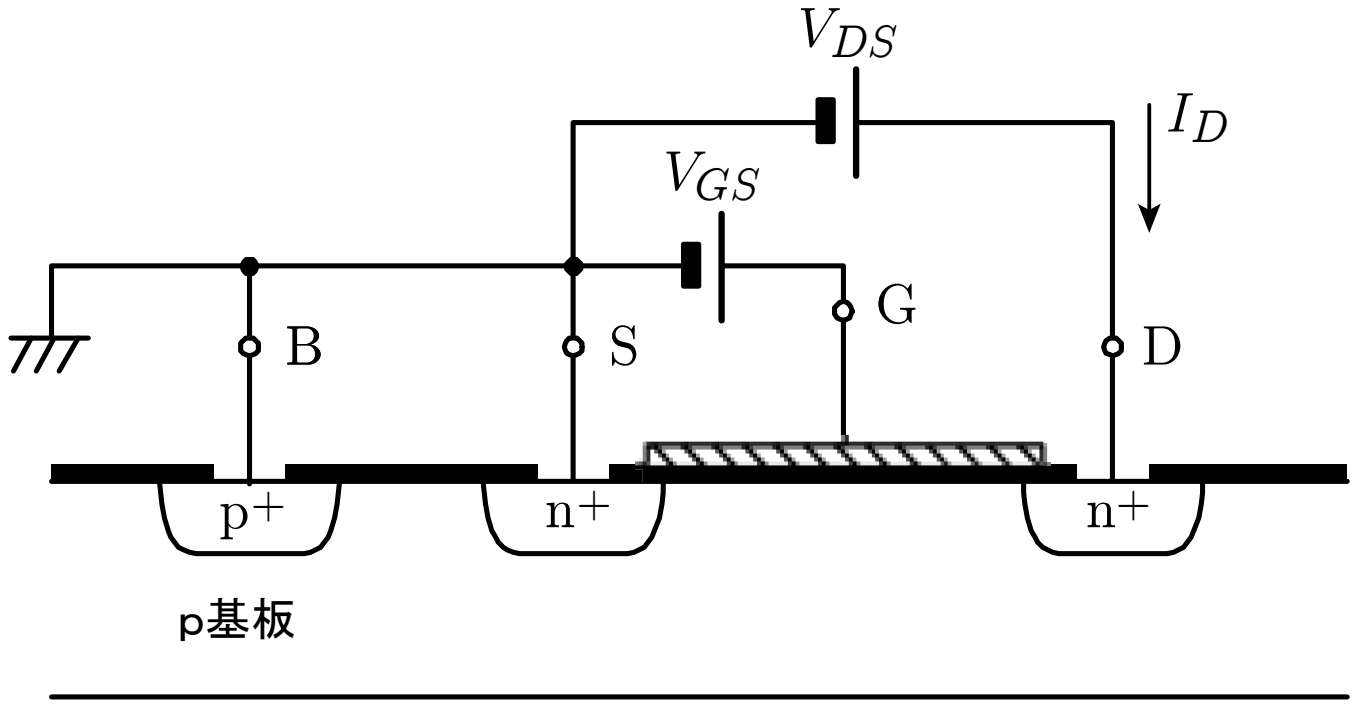
$$\frac{D_n}{\mu} = \frac{kT}{q}$$ (アインシュタインの関係式)

$$\boxed{f_T = \frac{\mu}{\pi W_B{}^2} \cdot \frac{kT}{q}}$$

MOSトランジスタのモデリング



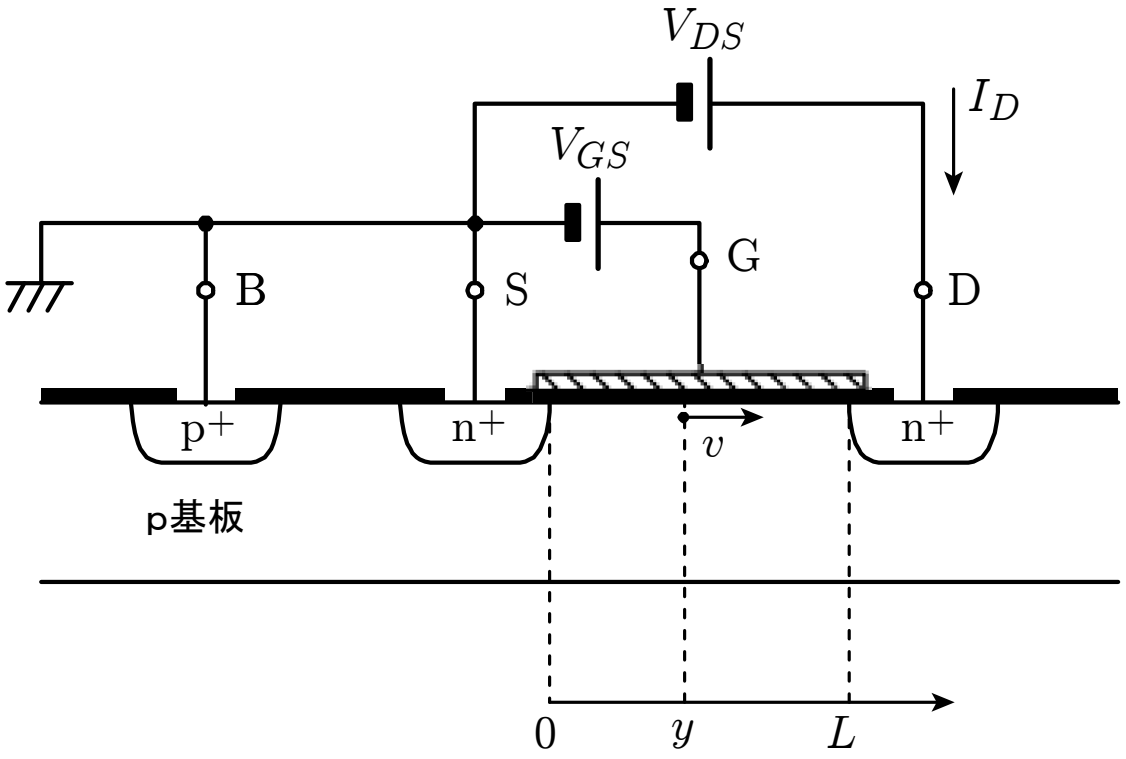
B:　Bulk, バルク　　S:　Source, ソース
G:　Gate, ゲート　　D:　Drain, ドレイン



V_{SB}=0と仮定

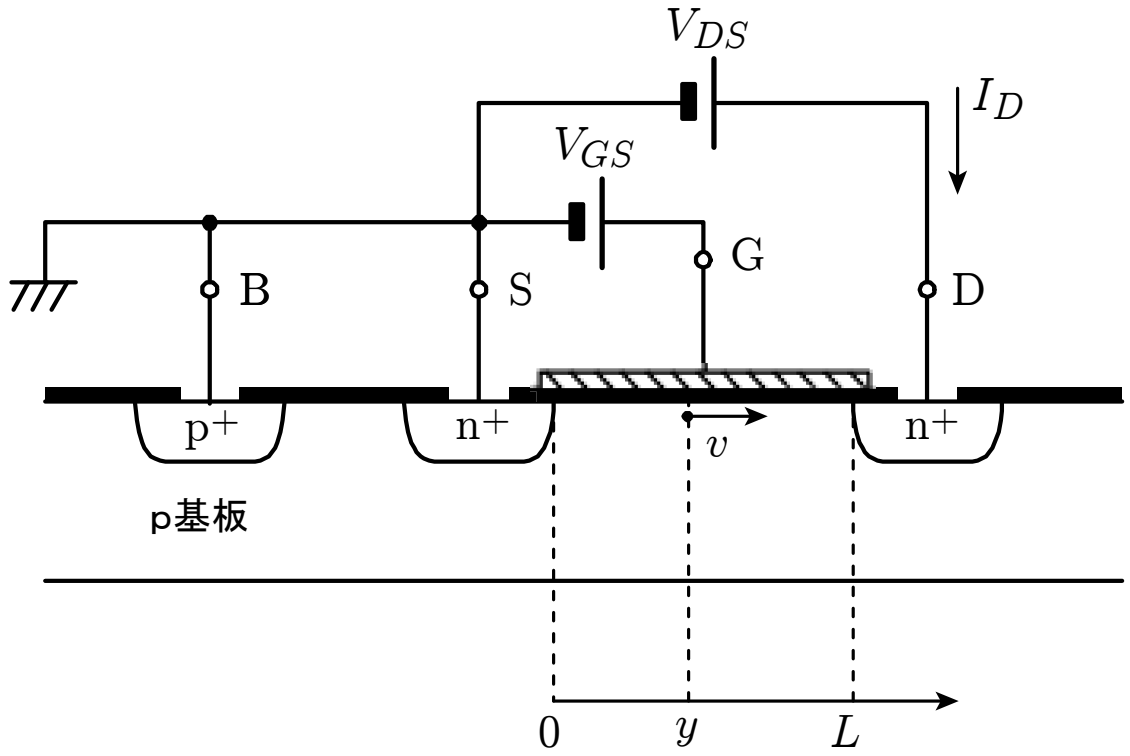
遮断領域：	$V_T \geq V_{GS}$
非飽和領域：	$V_{GS}-V_T > V_{DS} \geq 0$
飽和領域：	$V_{DS} \geq V_{GS}-V_T > 0$

非飽和領域：$V_{GS}-V_T>V_{DS}\geq 0$



地点yにおけるチャネルの電位：$V(y)$

地点yにおける単位面積当たりの電荷：$Q(y)$



μ : 移動度

C_{OX}: 単位面積当たりのゲート酸化膜容量

$$Q(y) = -C_{OX}\{V_{GS} - V_T - V(y)\}$$

$$I_D = Q(y) \times W \times (-v)$$

$$Q(y) = -C_{OX}\{V_{GS} - V_T - V(y)\}$$

$$v = -\mu E$$

$$I_D = Q(y) \times W \times (-v)$$

$$E = -\frac{dV}{dy}$$

$$\boxed{I_D = \mu C_{OX} W \{V_{GS} - V_T - V(y)\}\frac{dV}{dy}}$$

$$\int_0^L I_D dy = \int_0^L \mu C_{OX} W \{V_{GS} - V_T - V(y)\}\frac{dV}{dy} dy$$

$$= \int_0^{V_{DS}} \mu C_{OX} W \{V_{GS} - V_T - V(y)\} dV$$

$$= \left[\frac{-1}{2}\mu C_{OX} W \{V_{GS} - V_T - V(y)\}^2\right]_0^{V_{DS}}$$

$$\boxed{I_D L = \mu C_{OX} W (V_{GS} - V_T - \frac{V_{DS}}{2}) V_{DS}}$$

$$\boxed{I_D=\mu C_{OX}\frac{W}{L}\left(V_{GS}-V_T-\frac{V_{DS}}{2}\right)V_{DS}}$$

飽和領域：　　$V_{DS}\geq V_{GS}-V_T>0$

$\boxed{V_{DS}=V_{GS}-V_T\text{となったときの}I_D\text{から変化しない.}}$

2乗則

$$\boxed{I_D=\frac{1}{2}\mu C_{OX}\frac{W}{L}(V_{GS}-V_T)^2}$$

遮断領域：　$V_T \geq V_{GS}$

非飽和領域：　$V_{GS}-V_T > V_{DS} \geq 0$

飽和領域：　$V_{DS} \geq V_{GS}-V_T > 0$

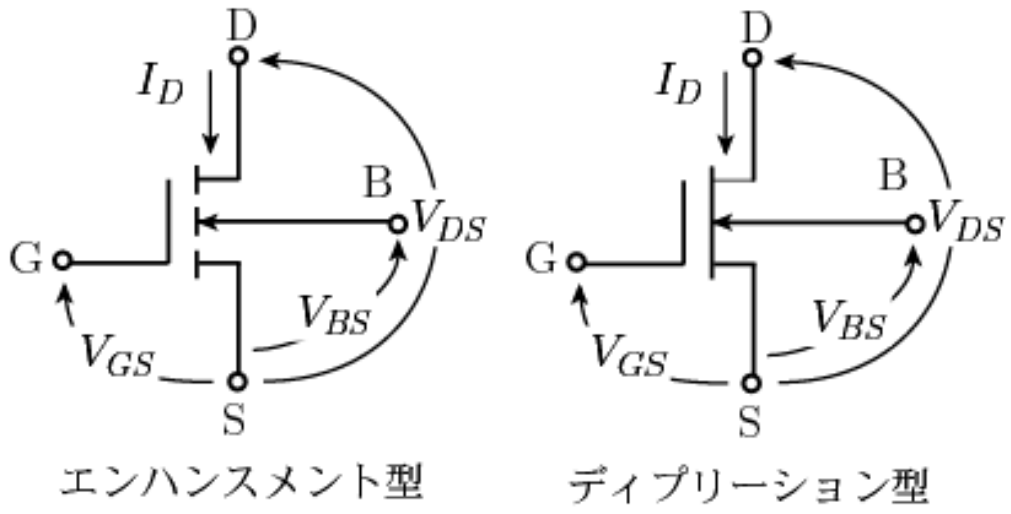
遮断領域：$I_D=0$

非飽和領域：$I_D=2K(V_{GS}-V_T-\frac{V_{DS}}{2})V_{DS}$

飽和領域：$I_D=K(V_{GS}-V_T)^2$

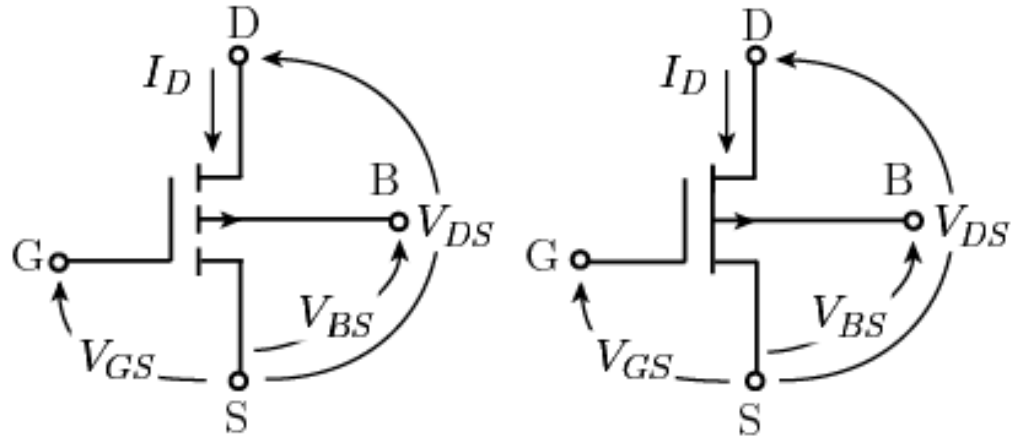
$K=\frac{1}{2}\mu C_{OX}\frac{W}{L}$: トランスコンダクタンスパラメータ

MOSトランジスタの記号



エンハンスメント型　　ディプリーション型

nチャネルMOSトランジスタ



エンハンスメント型　　ディプリーション型

pチャネルMOSトランジスタ

MOSトランジスタにおける2次的効果

基板効果

$$V_T = V_{T0} + \gamma(\sqrt{2\phi_F - V_{BS}} - \sqrt{2\phi_F})$$

チャネル長変調効果

$$I_D = K(V_{GS} - V_T)^2(1 + \lambda V_{DS})$$

MOSトランジスタの小信号モデル

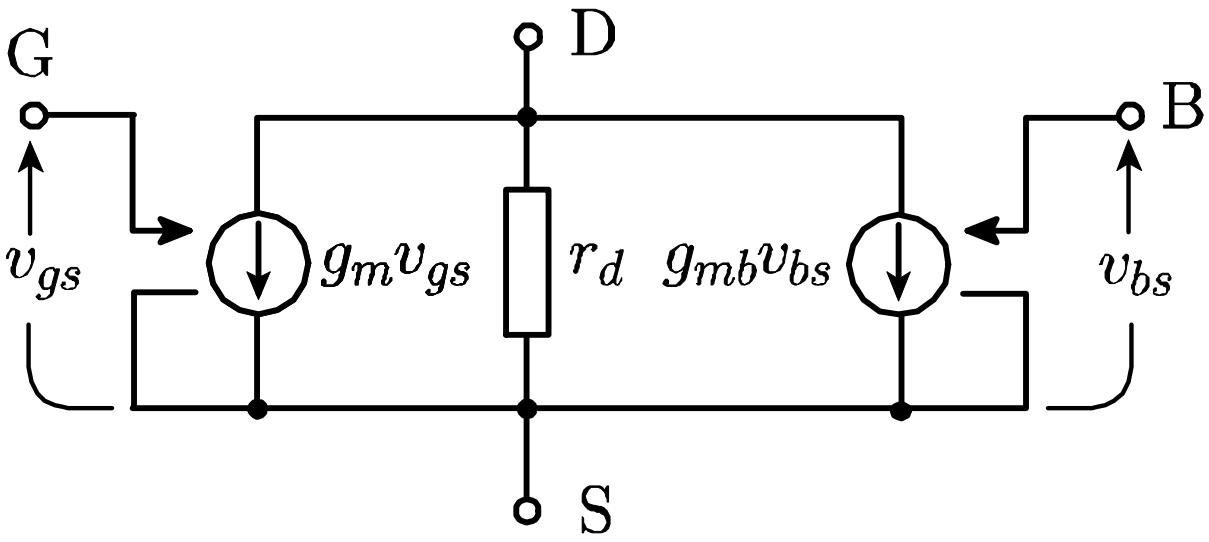
$$I_D = f(V_{GS}, V_{DS}, V_{BS})$$

$$\Delta I_D = \frac{\partial f}{\partial V_{GS}} \Delta V_{GS} + \frac{\partial f}{\partial V_{DS}} \Delta V_{DS} + \frac{\partial f}{\partial V_{BS}} \Delta V_{BS}$$

$$i_d = \frac{\partial f}{\partial V_{GS}} v_{gs} + \frac{\partial f}{\partial V_{DS}} v_{ds} + \frac{\partial f}{\partial V_{BS}} v_{bs}$$

$$\frac{\partial f}{\partial V_{GS}} = g_m, \quad \frac{\partial f}{\partial V_{DS}} = \frac{1}{r_d}, \quad \frac{\partial f}{\partial V_{BS}} = g_{mb}$$

MOSトランジスタの小信号モデル



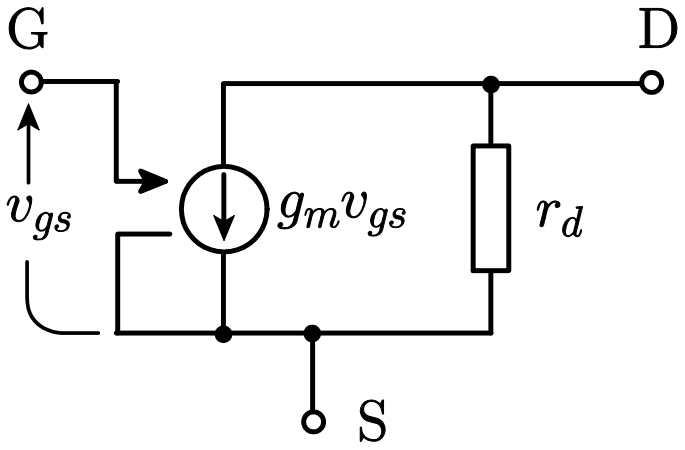
問：

$$I_D = K(V_{GS} - V_T)^2(1 + \lambda V_{DS})$$

$$V_T = V_{T0} + \gamma(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F})$$

であるとき，g_m，r_d，g_{mb}を求めよ.

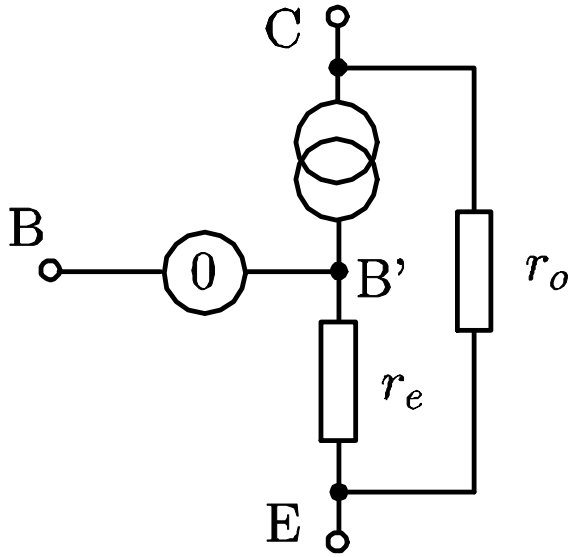
小信号モデルが等価となる条件



MOSトランジスタ

$$g_m = \frac{1}{r_e}$$

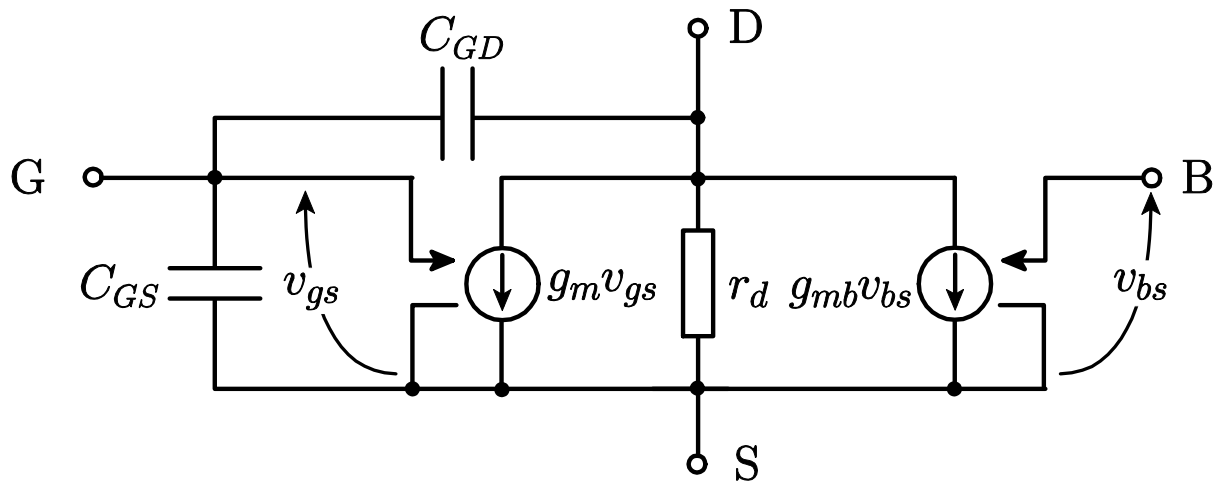
$$r_d = r_o$$



バイポーラトランジスタ

MOSトランジスタの高周波小信号モデル

基板間の寄生容量を考慮



C_{gs}は？

$Q(y)=-C_{OX}\{V_{GS}-V_T-V(y)\}$より

ゲートに蓄えられている総電荷Q_Tは

$$Q_T=-\int_0^L WQ(y)dy=\int_0^L \frac{\{WQ(y)\}^2\mu\frac{dV}{dy}}{-WQ(y)\mu\frac{dV}{dy}}dy=\int_0^L \frac{\{WQ(y)\}^2\mu\frac{dV}{dy}}{I_D}dy$$

$$=\frac{\mu C_{OX}{}^2W^2}{I_D}\int_0^{V_{DS}}\{V_{GS}-V_T-V(y)\}^2dV(y)$$

終端($y=L$)では，$V_{DS}=V_{GS}-V_T$なので

$$Q_T=\frac{\mu C_{OX}{}^2W^2}{I_D}\int_0^{V_{GS}-V_T}\{V_{GS}-V_T-V(y)\}^2\,dV(y)$$

$$=\frac{\mu C_{OX}{}^2W^2}{I_D}\left[\frac{-1}{3}\{V_{GS}-V_T-V(y)\}^3\right]_0^{V_{GS}-V_T}$$

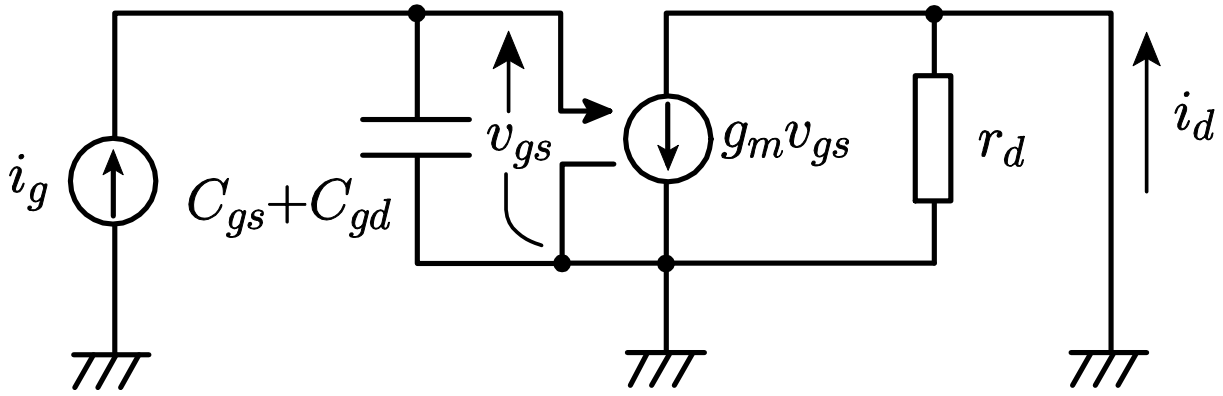
$$=\frac{\mu C_{OX}{}^2W^2}{\frac{1}{2}\mu C_{OX}\frac{W}{L}(V_{GS}-V_T)^2}\left\{0-\frac{-1}{3}(V_{GS}-V_T)^3\right\}$$

$$=\frac{2}{3}C_{OX}WL(V_{GS}-V_T)$$

$$\boxed{C_{gs}=\frac{\partial Q_T}{\partial V_{GS}}=\frac{2}{3}WLC_{OX}}$$

MOSトランジスタのf_T

f_T: $|i_d|=|i_g|$となる周波数



$$i_d=g_m v_{gs}=\frac{g_m}{j\omega\left(C_{gs}+C_{gd}\right)}i_g$$

$$f_T=\frac{g_m}{2\pi\left(C_{gs}+C_{gd}\right)}\approx\frac{g_m}{2\pi C_{gs}}$$

$$C_{gs}=\frac{\partial Q_T}{\partial V_{GS}}=\frac{2}{3}WLC_{OX}$$

$$g_m=2K(V_{GS}-V_T)=\mu C_{OX}\frac{W}{L}(V_{GS}-V_T)$$

$$\boxed{f_T=\frac{g_m}{2\pi C_{gs}}=\frac{3\mu}{4\pi L^2}(V_{GS}-V_T)}$$

参考：バイポーラトランジスタ　$f_T=\frac{\mu}{\pi W_B{}^2}\cdot\frac{kT}{q}$