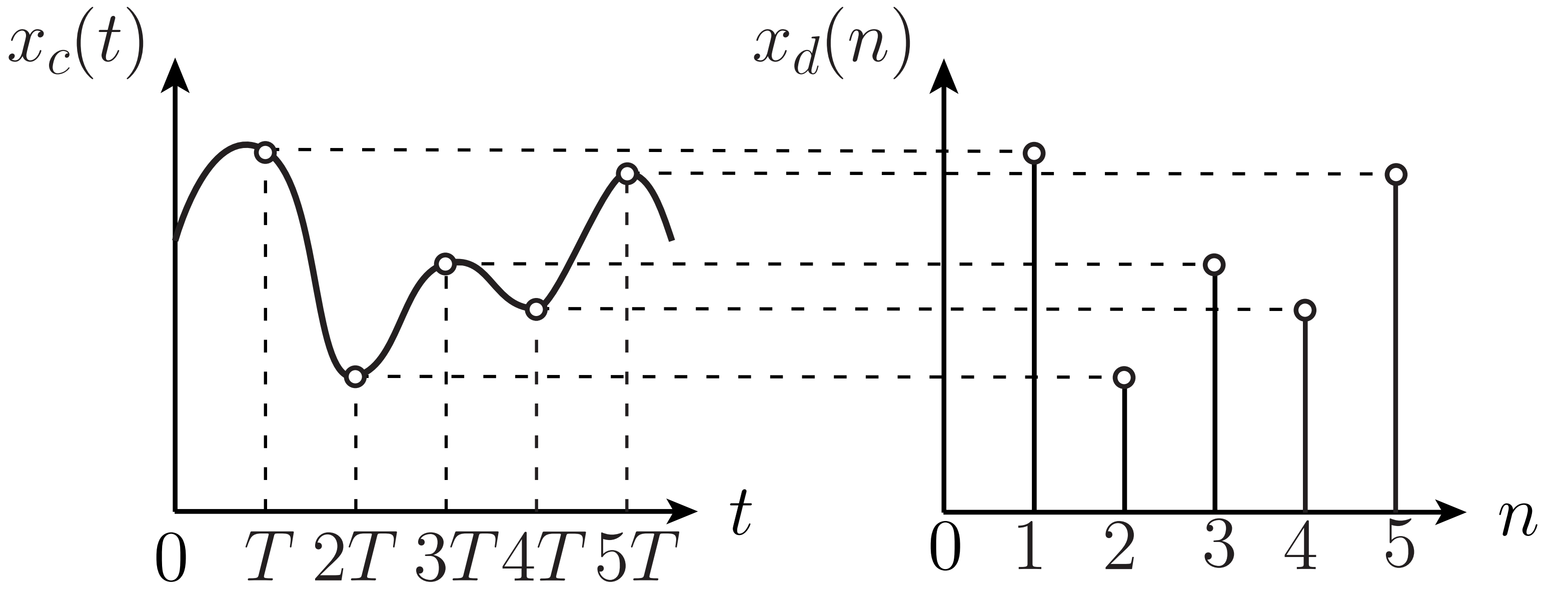
離散時間系フィルタの構成
(スイッチトキャパシタフィルタ)

z変換とその性質



$x_c(t)$: 連続時間信号

$x_d(n)$: 離散時間信号

理想サンプリング

$\delta(t)$: デルタ関数

定義：$\int_{-\infty}^{\infty}\delta(t)\varphi(t)dt=\varphi(0)$

ただし，$\varphi(t)$はテスト関数と呼ばれ,

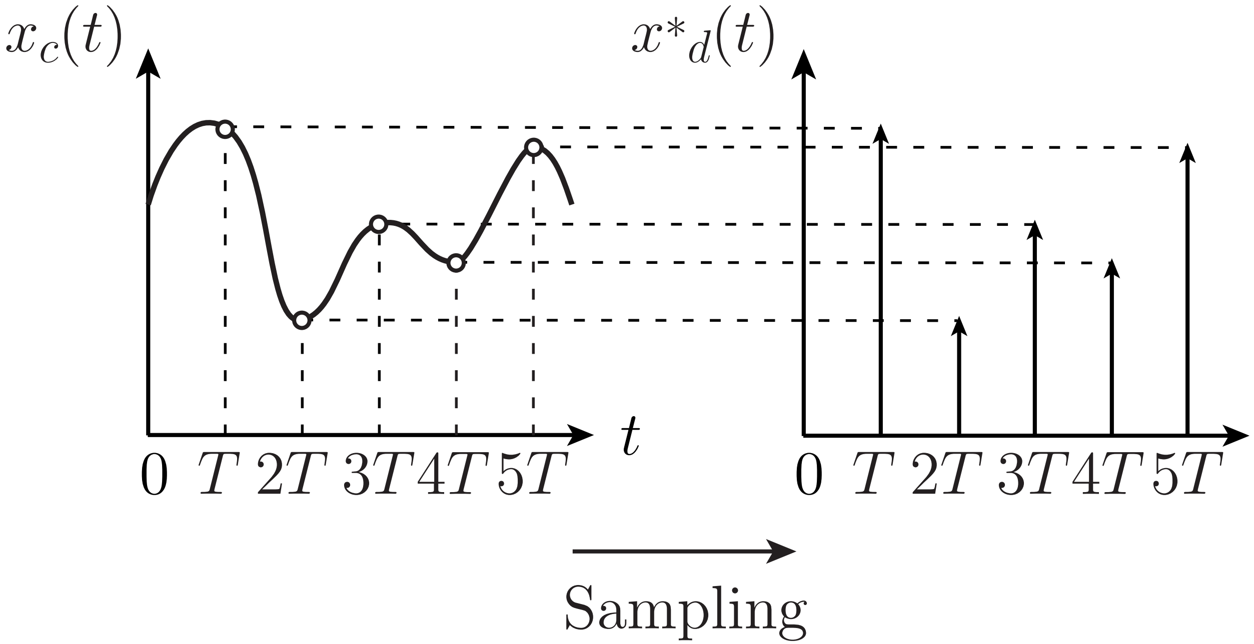
あるt_0について$|t|>t_0$のときは$\varphi(t)=0$となり，

何回でも微分可能な関数.

$$x_d^*(t) = \sum_{n=-\infty}^{\infty} x_c(t)\delta(t-nT)$$

$$\int_{-\infty}^{\infty} \delta(t)\varphi(t)dt = \varphi(0)$$

離散時間信号の連続時間仮表現



$$x_d^*(t) = \sum_{n=-\infty}^{\infty} x_c(t)\delta(t-nT) = x_c(t)\delta_T(t)$$

$$X_d(j\omega) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x_c(t)\delta(t-nT)e^{-j\omega t}dt$$

$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} x_c(t)\delta(t-nT)e^{-j\omega t}dt$$

$$= \sum_{n=-\infty}^{\infty} x_c(nT)e^{-j\omega nT}$$

$x_c(nT)=x_d(n)$であり，zを$e^{j\omega T}$で置き換えると

(両側)z変換

z変換の定義：$X_d(z) = \sum_{n=-\infty}^{\infty} x_d(n)z^{-n}$

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

z変換の性質(1)

$$\sum_{n=-\infty}^{\infty} h(n-1) z^{-n} = \sum_{l=-\infty}^{\infty} h(l) z^{-(l+1)} = \left(\sum_{l=-\infty}^{\infty} h(l) z^{-l} \right) z^{-1} = H(z) z^{-1}$$

z変換の性質(2)

$H(z)$の周波数特性　　$z=e^{j\omega T}$を代入

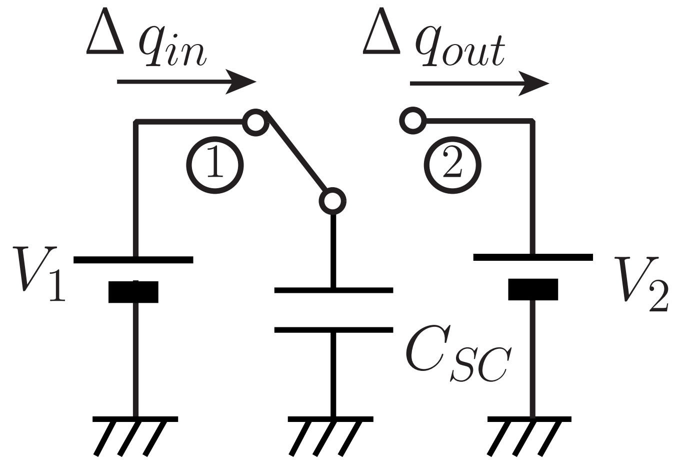
z変換の性質(3)　　$s_i=\sigma_i+j\omega_i$と置いたとき

$|z_i|=|e^{s_i T}|$に$s_i=\sigma_i+j\omega_i$を代入すると

$$|z_i|=|e^{(\sigma_i+j\omega_i)T}|=|e^{\sigma_i T}||e^{j\omega_i T}|=|e^{\sigma_i T}|<1$$

$\sigma_i<0$なら安定

スイッチトキャパシタ回路の動作原理



$q_1 = C_{SC} V_1$　　　　$q_2 = C_{SC} V_2$

$q_2 + (q_1 - q_2) = q_1$ → $\Delta q_{in} = q_1 - q_2 = C_{SC}(V_1 - V_2)$

$q_1 - (q_1 - q_2) = q_2$ → $\Delta q_{out} = q_1 - q_2 = C_{SC}(V_1 - V_2)$

$$\Delta q = \Delta q_{out} = \Delta q_{in}$$

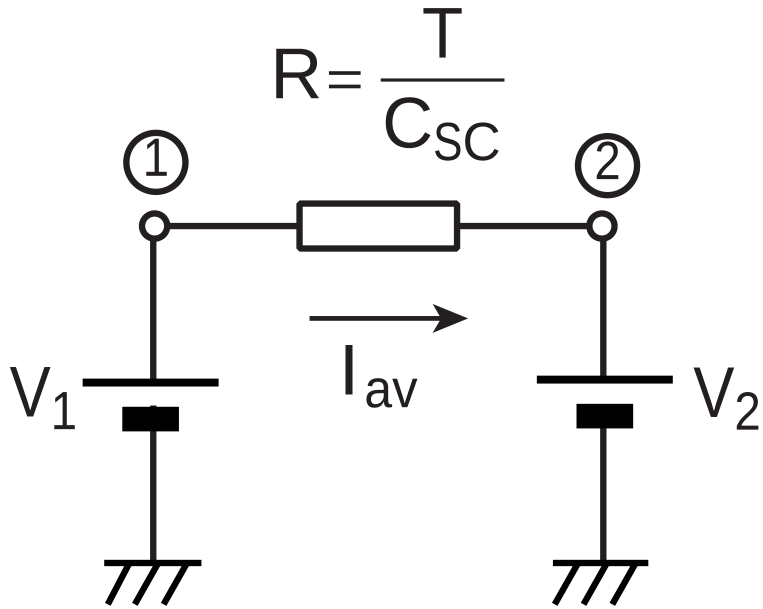
近似モデル

$\Delta q = \Delta q_{out} = \Delta q_{in}$　　**スイッチング周期：T**

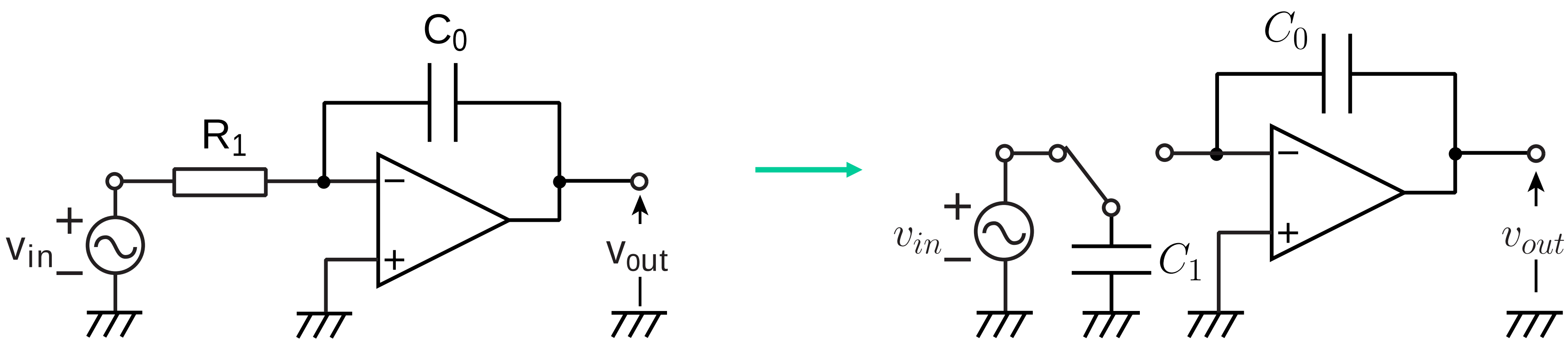
T秒間にΔqの電荷が移動

平均電流： $I_{av} = \dfrac{\Delta q}{T} = \dfrac{C_{SC}(V_1 - V_2)}{T}$

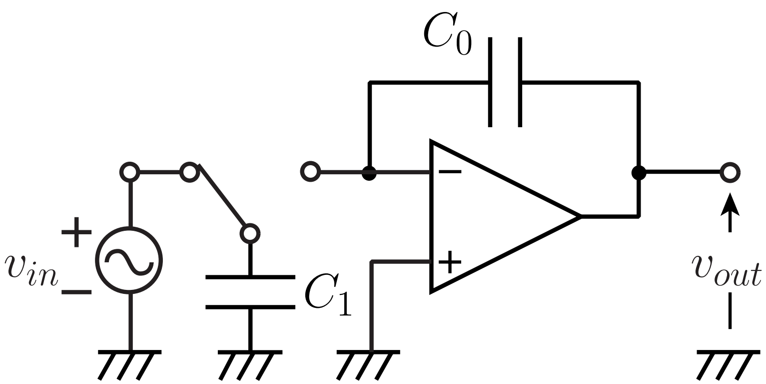
等価抵抗： $R = \dfrac{T}{C_{SC}}$



基本スイッチトキャパシタ積分回路



問　スイッチと容量を等価抵抗で置き換えることにより，スイッチトキャパシタ積分回路の伝達特性$\frac{v_{out}}{v_{in}}$を求めよ．ただし，スイッチング周期をTとする．



$$T(s)=\frac{v_{out}}{v_{in}}=\frac{-C_1}{sC_0T}$$

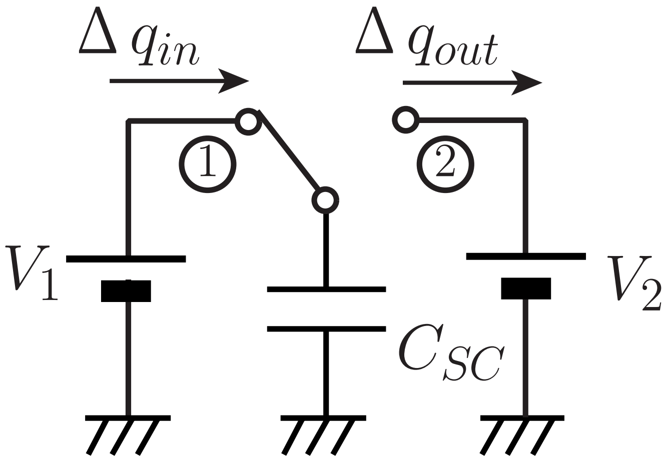
特性：　容量比　＆　スイッチング周期

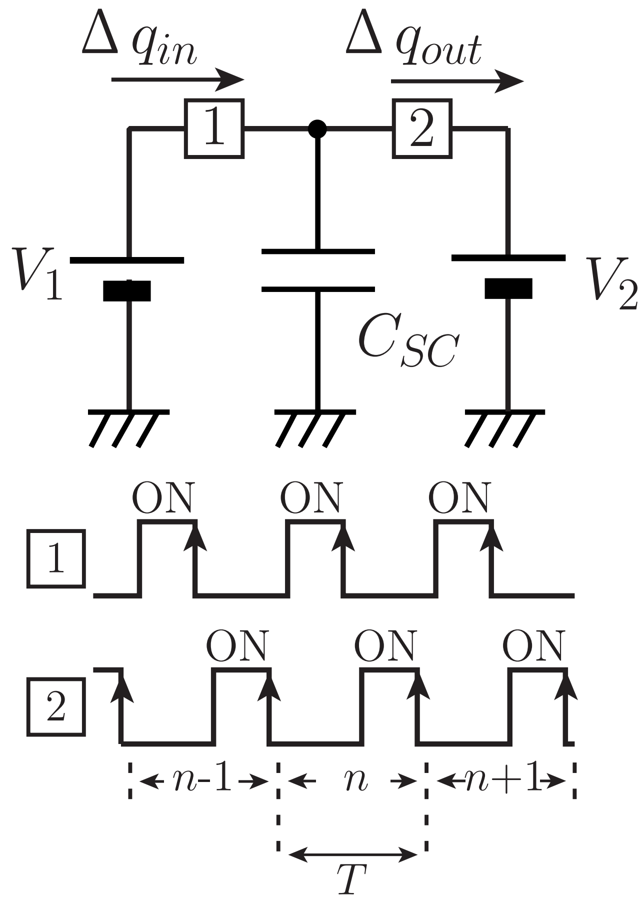
C_0=1nF，　C_1=1nF　　＝　　C_0=1pF，　C_1=1pF

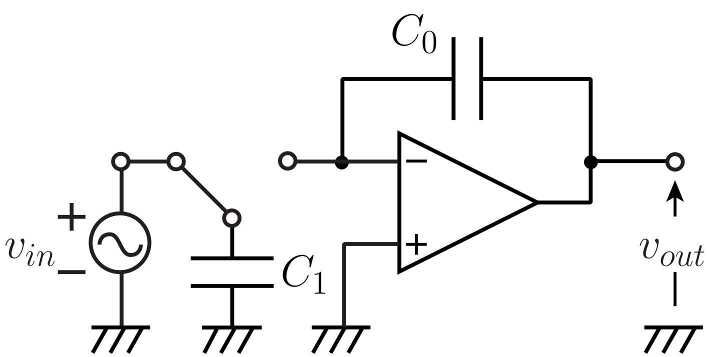
T：水晶発振回路により正確

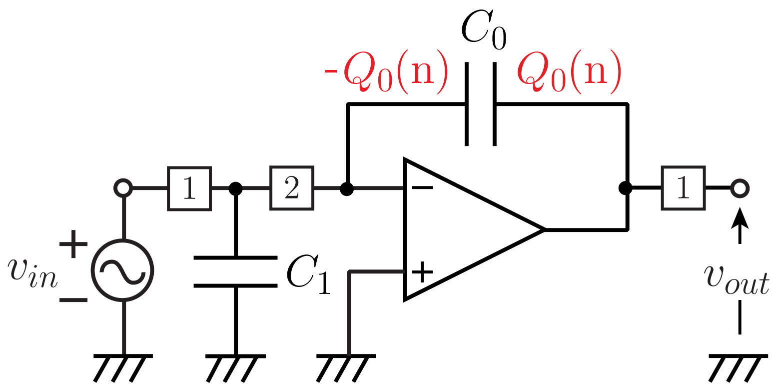
容量，演算増幅器，スイッチ：実現容易

スイッチトキャパシタ回路の解析

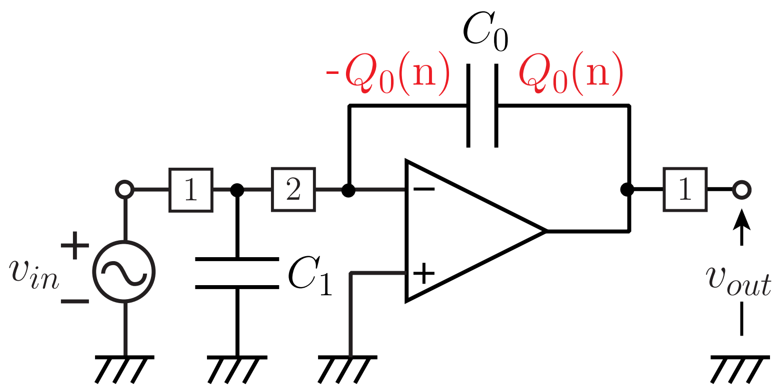


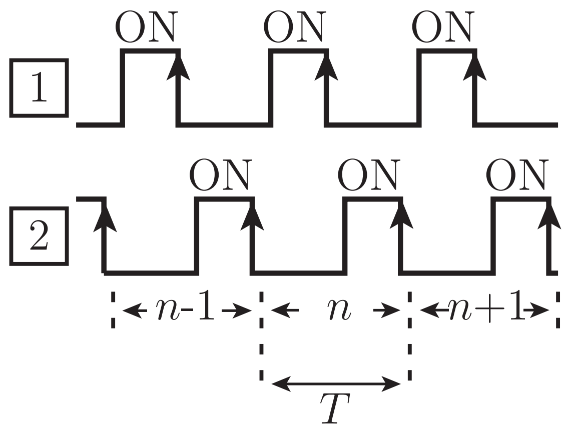






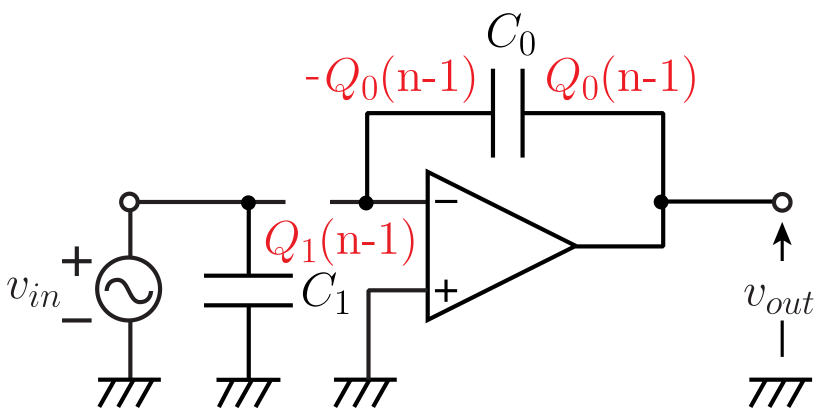
基本スイッチトキャパシタ積分回路の解析



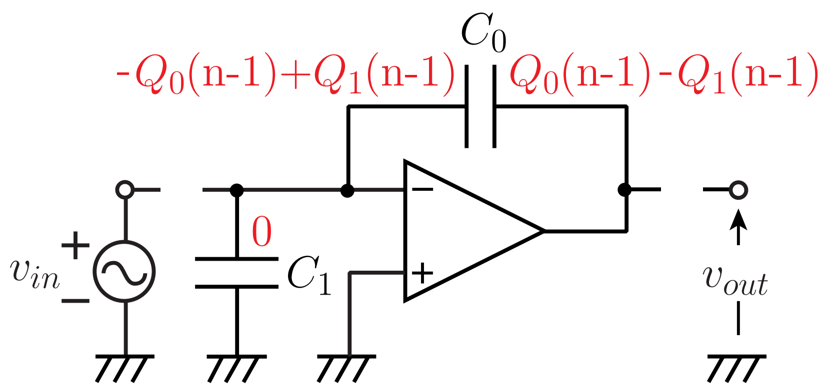


時刻n-1

スイッチ1が短絡
スイッチ2が開放

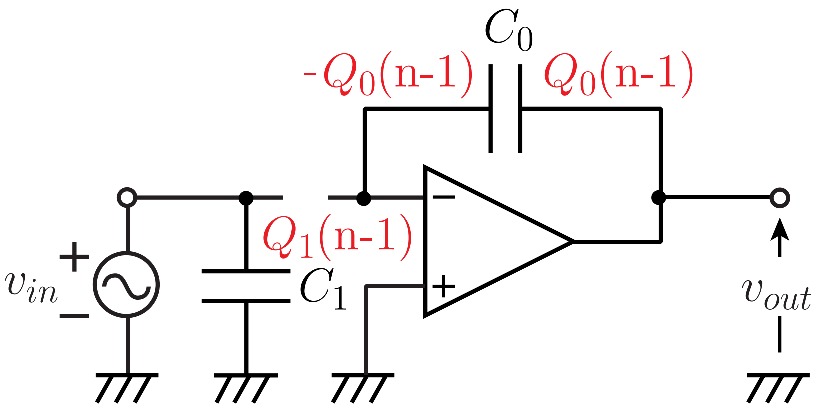


スイッチ1が開放
スイッチ2が短絡

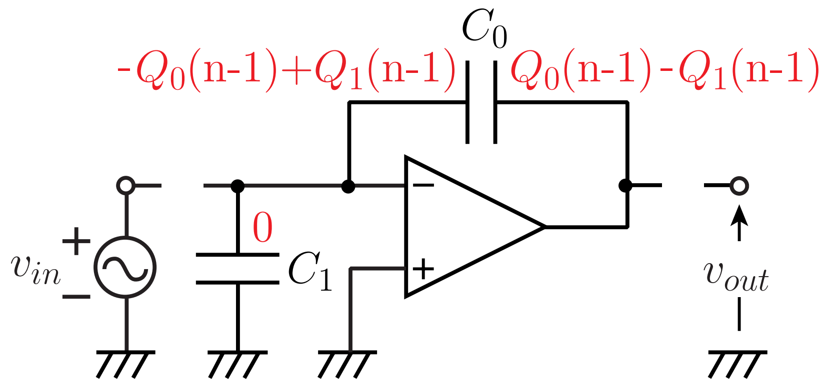


時刻n-1

スイッチ1が短絡
スイッチ2が開放



スイッチ1が開放
スイッチ2が短絡



$$-Q_0(n)=-Q_0(n-1)+Q_1(n-1)$$

$$Q_1(n-1)=C_1 v_{in}(n-1)$$

$$Q_0(n-1)=C_0 v_{out}(n-1)$$

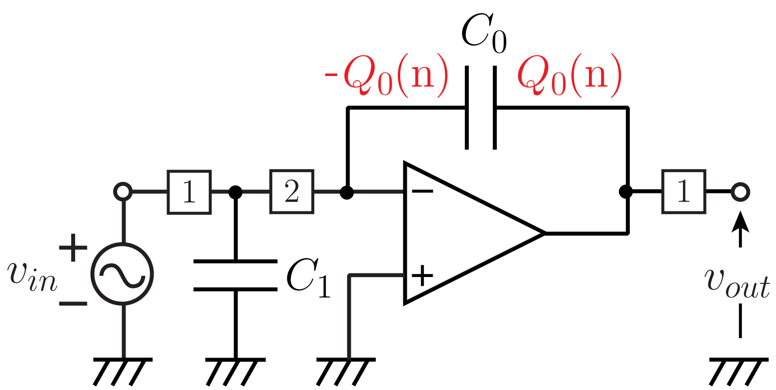
$$Q_0(n)=C_0 v_{out}(n)$$

$$-Q_0(n)=-Q_0(n-1)+Q_1(n-1) \rightarrow Q_0(n)=Q_0(n-1)-Q_1(n-1)$$

$$Q_0(n)=C_0 v_{out}(n) \quad Q_0(n-1)=C_0 v_{out}(n-1) \quad Q_1(n-1)=C_1 v_{in}(n-1)$$

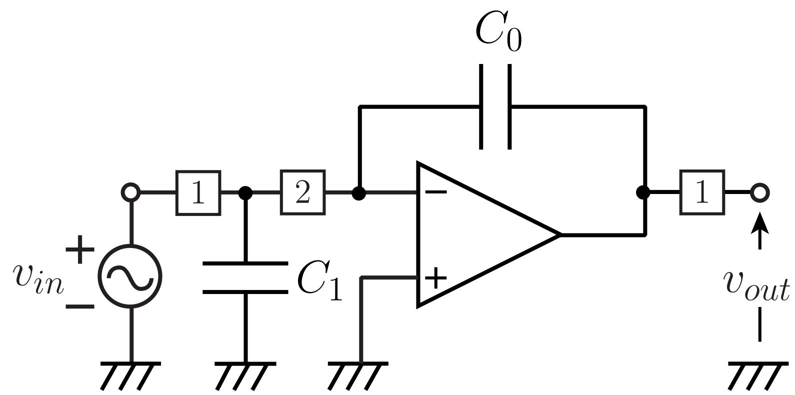
$$C_0 v_{out}(n)=C_0 v_{out}(n-1)-C_1 v_{in}(n-1)$$

$$C_0 V_{out}(z)=C_0 V_{out}(z)z^{-1}-C_1 V_{in}(z)z^{-1}$$



$$\frac{V_{out}(z)}{V_{in}(z)}=\frac{-C_1 z^{-1}}{C_0(1-z^{-1})}$$

$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1 z^{-1}}{C_0(1-z^{-1})}$$



$z = e^{j\omega T}$を代入

$$H(j\omega) = \frac{-C_1 e^{-j\omega T}}{C_0(1-e^{-j\omega T})} = \frac{-C_1 e^{-j\omega T/2}}{C_0(e^{j\omega T/2} - e^{-j\omega T/2})}$$

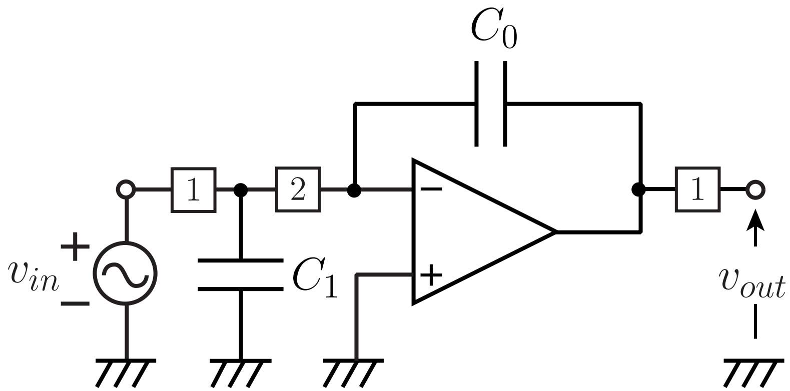
$$= \frac{-C_1 e^{-j\omega T/2}}{C_0\{\cos(\omega T/2) + j\sin(\omega T/2) - \cos(\omega T/2) + j\sin(\omega T/2)\}}$$

$$= \frac{-C_1 e^{-j\omega T/2}}{j2C_0 \sin(\omega T/2)} \approx \frac{-C_1}{j\omega C_0 T} \quad (\because x \ll 1 \text{のとき} \sin x \approx x)$$

スイッチトキャパシタ積分回路の構成

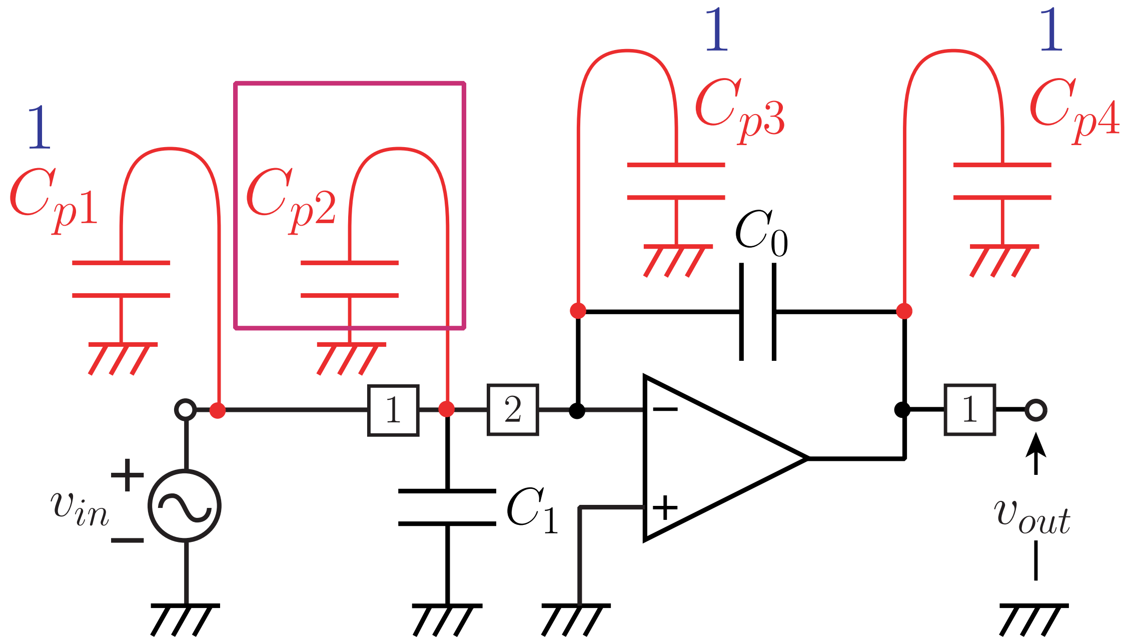
基本スイッチトキャパシタ積分回路の問題点

寄生容量



寄生容量が特性に影響を与えない3条件

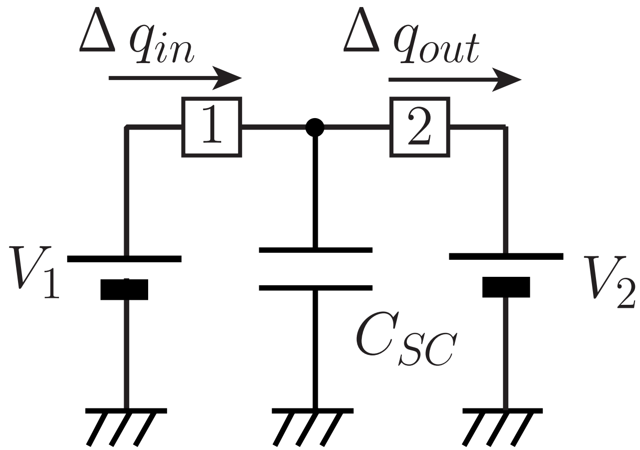
1. 常に電圧源または接地端子, 仮想接地端子のいずれかに接続されている寄生容量
2. 電圧源と接地端子に交互に接続される寄生容量
3. 仮想接地と接地端子に交互に接続される寄生容量

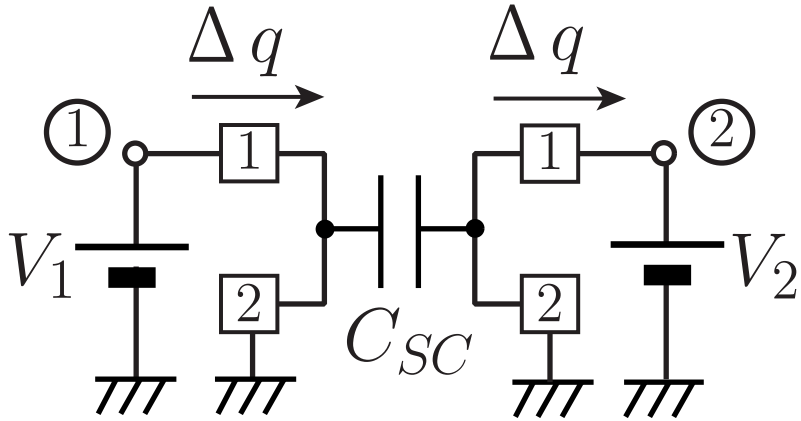


寄生容量に有感なスイッチトキャパシタ回路

1. 常に電圧源または接地端子，仮想接地端子のいずれかに接続されている寄生容量
2. 電圧源と接地端子に交互に接続される寄生容量
3. 仮想接地と接地端子に交互に接続される寄生容量

寄生容量への対策



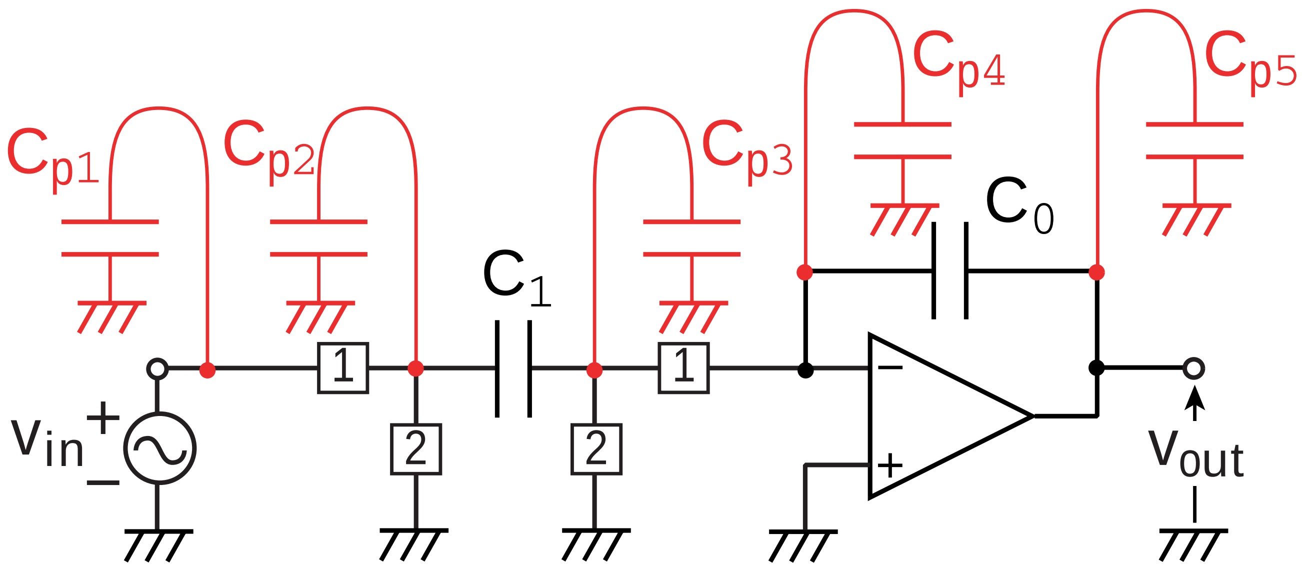


$$\Delta q_{out} = \Delta q_{in} = C_{SC}(V_1 - V_2)$$

$$\Delta q = C_{SC}(V_1 - V_2)$$

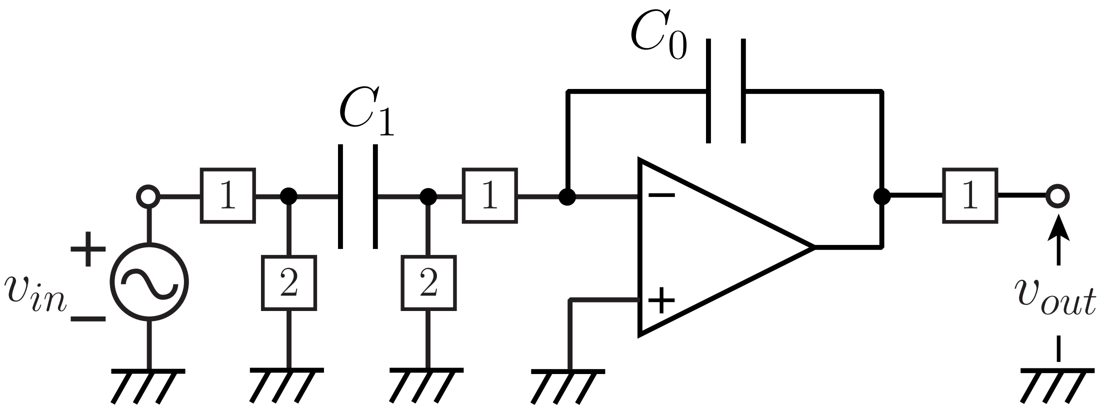
電荷の移動量：同じ

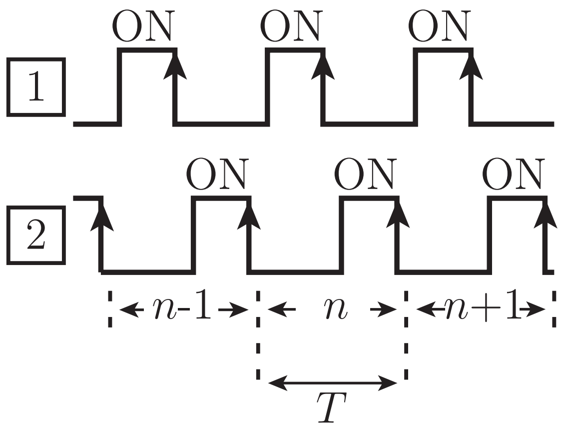
電荷の移動のタイミング：異なる



問　上の回路は寄生容量の影響を受けないスイッチトキャパシタ積分回路である．また，全ての節点と接地間の寄生容量も示されている．この積分回路が，これら寄生容量がスイッチトキャパシタ積分回路に影響を与えない理由を3条件から選べ．

寄生容量に不感なスイッチトキャパシタ積分回路 (1)



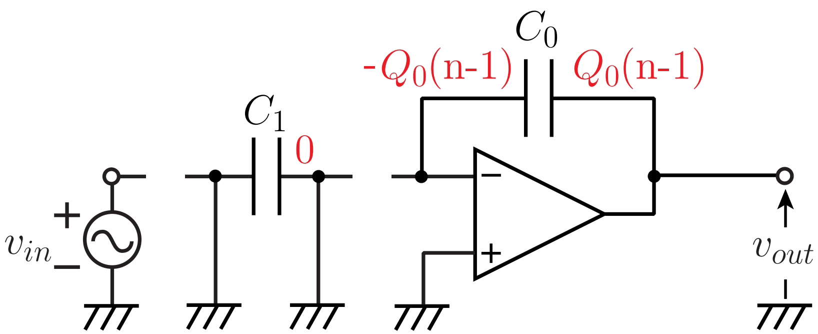


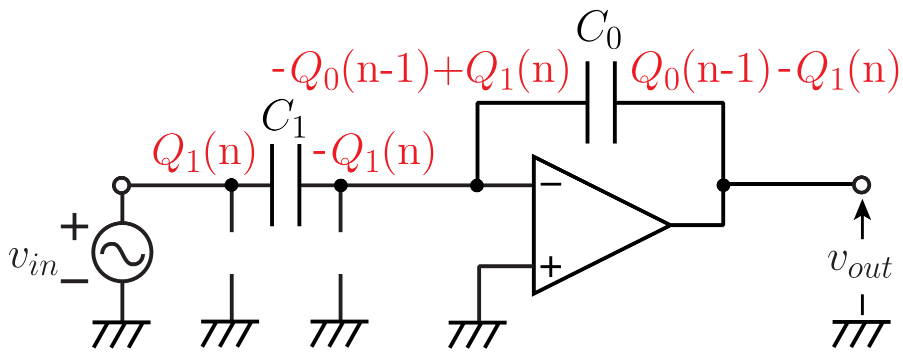
時刻n-1

スイッチ1が開放
スイッチ2が短絡

時刻n

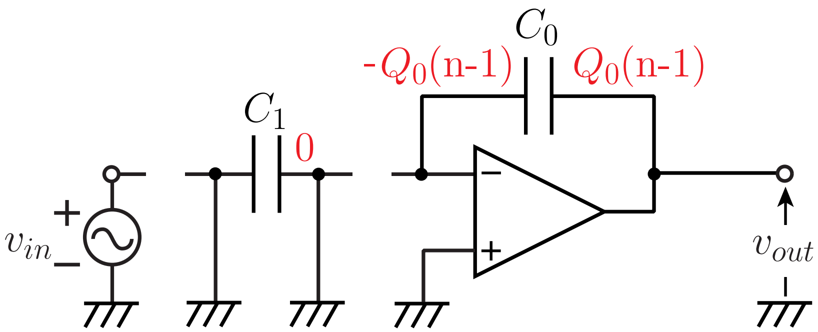
スイッチ1が短絡
スイッチ2が開放

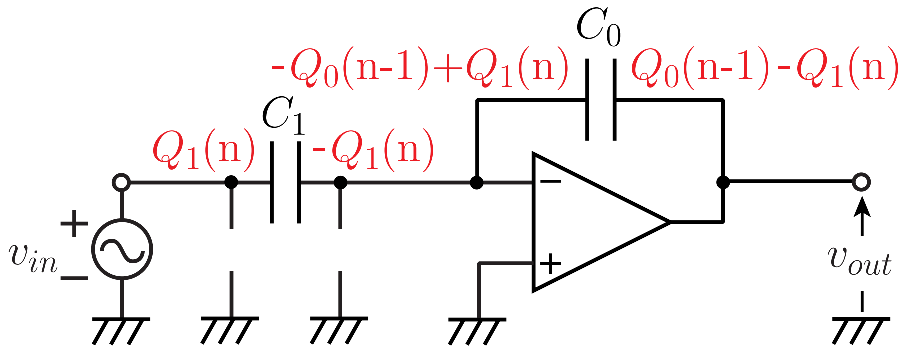




時刻n-1 スイッチ1が開放 スイッチ2が短絡

時刻n スイッチ1が短絡 スイッチ2が開放





$$-Q_0(n) = -Q_0(n-1) + Q_1(n)$$

$$Q_1(n) = C_1 v_{in}(n)$$

$$Q_0(n-1) = C_0 v_{out}(n-1)$$

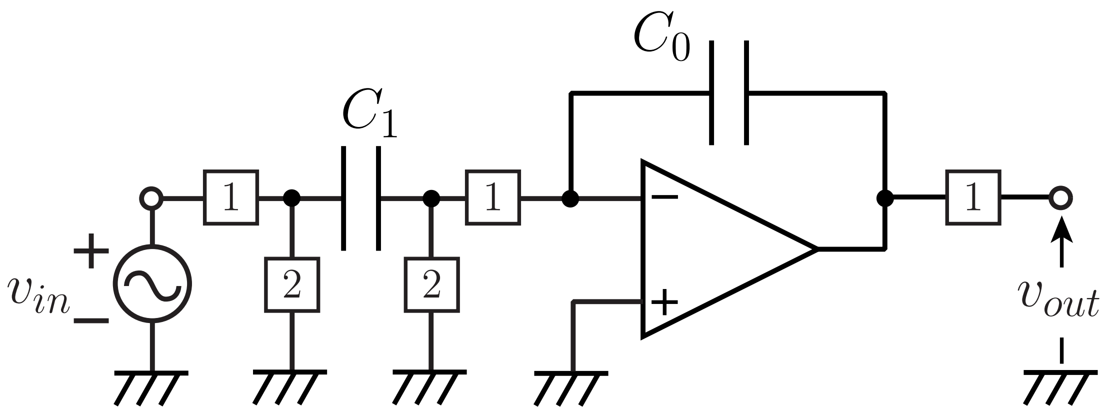
$$Q_0(n) = C_0 v_{out}(n)$$

$$-Q_0(n) = -Q_0(n-1) + Q_1(n) \rightarrow Q_0(n) = Q_0(n-1) - Q_1(n)$$

$$Q_0(n) = C_0 v_{out}(n) \quad Q_0(n-1) = C_0 v_{out}(n-1) \quad Q_1(n) = C_1 v_{in}(n)$$

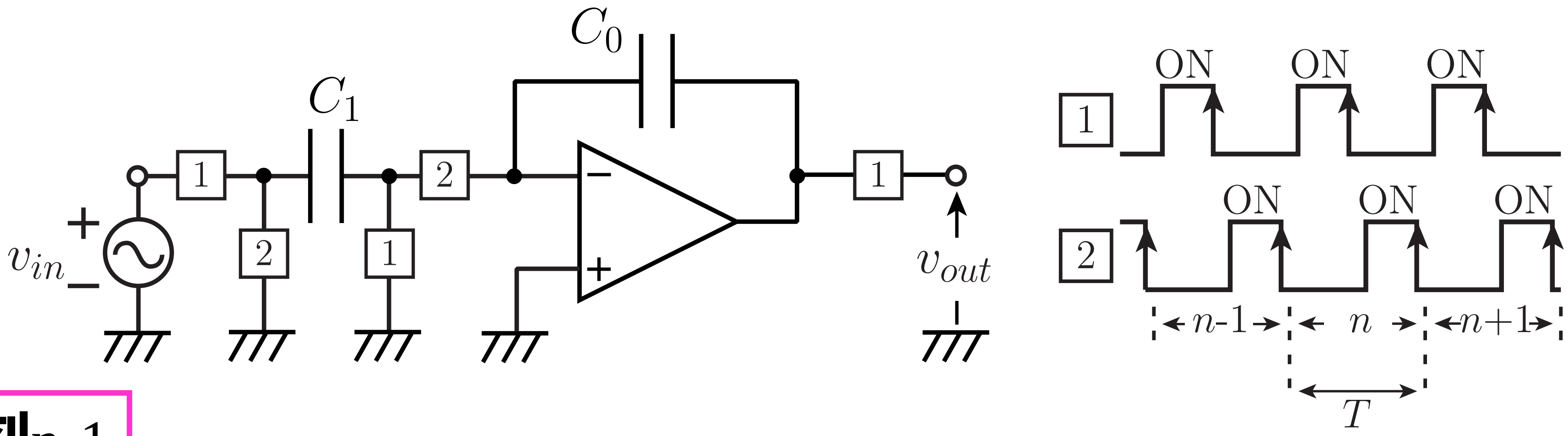
$$C_0 v_{out}(n) = C_0 v_{out}(n-1) - C_1 v_{in}(n)$$

$$C_0 V_{out}(z) = C_0 V_{out}(z) z^{-1} - C_1 V_{in}(z)$$



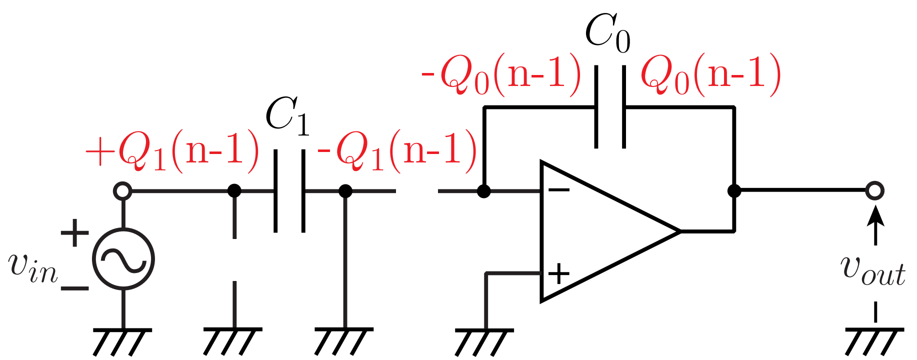
$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1}{C_0(1-z^{-1})}$$

寄生容量に不感なスイッチトキャパシタ積分回路 (2)

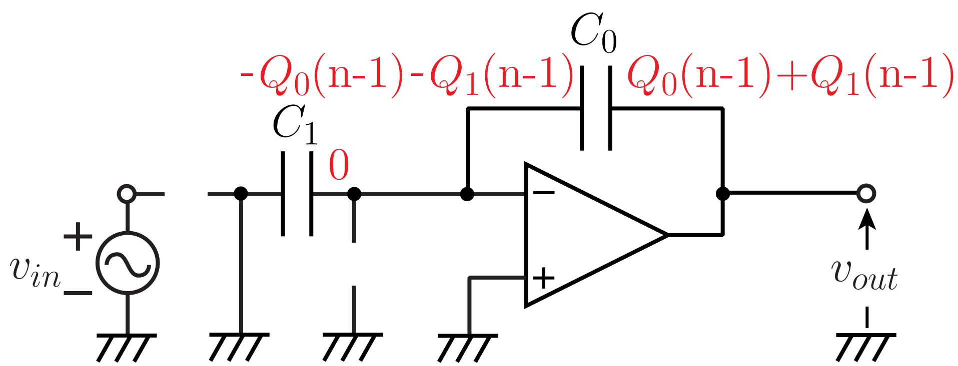


時刻n-1

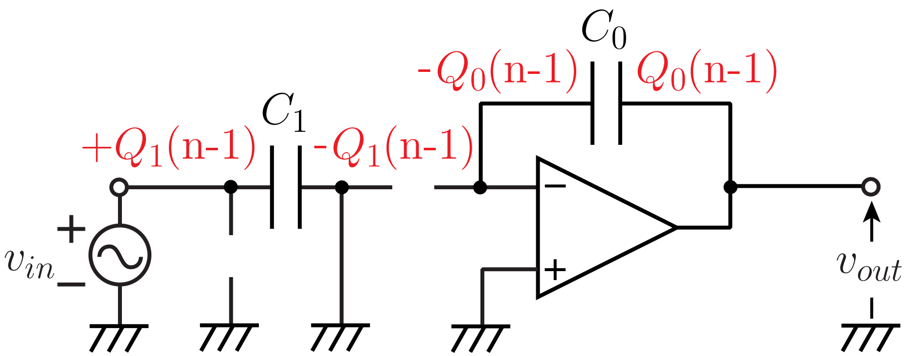
スイッチ1が短絡
スイッチ2が開放



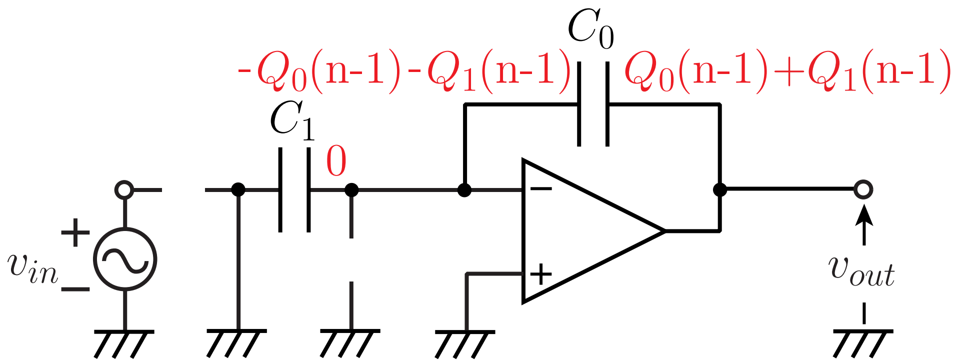
スイッチ1が開放
スイッチ2が短絡



スイッチ1が短絡
スイッチ2が開放



スイッチ1が開放
スイッチ2が短絡



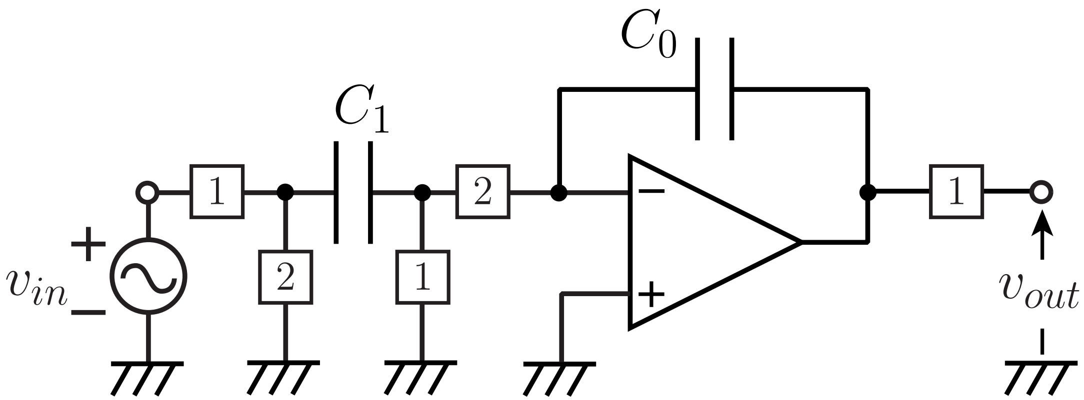
$$-Q_0(n) = -Q_0(n-1) - Q_1(n-1) \quad \rightarrow \quad Q_0(n) = Q_0(n-1) + Q_1(n-1)$$

$$Q_0(n) = C_0 v_{out}(n) \quad Q_0(n-1) = C_0 v_{out}(n-1) \quad Q_1(n-1) = C_1 v_{in}(n-1)$$

$$C_0 v_{out}(n) = C_0 v_{out}(n-1) + C_1 v_{in}(n-1)$$

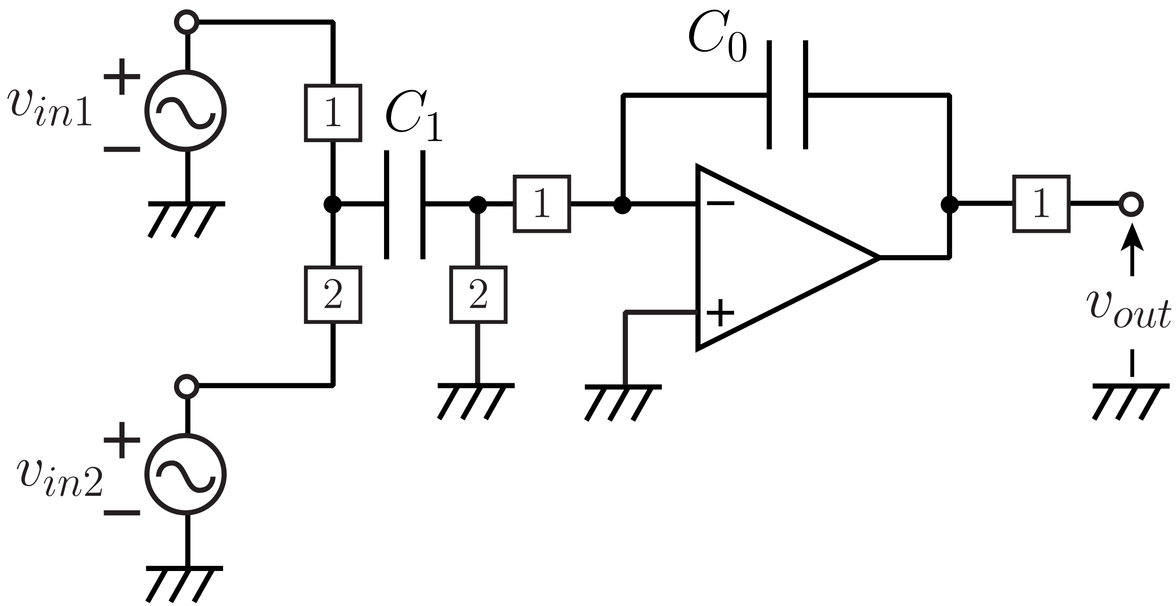
$$C_0 v_{out}(n) = C_0 v_{out}(n-1) + C_1 v_{in}(n-1)$$

$$C_0 V_{out}(z) = C_0 V_{out}(z) z^{-1} + C_1 V_{in}(z) z^{-1}$$



$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{C_1 z^{-1}}{C_0 (1 - z^{-1})}$$

寄生容量に不感なスイッチトキャパシタ積分回路 (3)



問　上の回路は寄生容量の影響を受けないスイッチトキャパシタ積分回路である．この回路の出力電圧v_{out}をv_{in1}とv_{in2}，C_1，C_2を用いて表せ.

スイッチトキャパシタフィルタの構成

双一次s-z変換

$$s \to \frac{2(1-z^{-1})}{T(1+z^{-1})}$$

$s = j\Omega,\quad z = e^{j\omega T}$ を代入

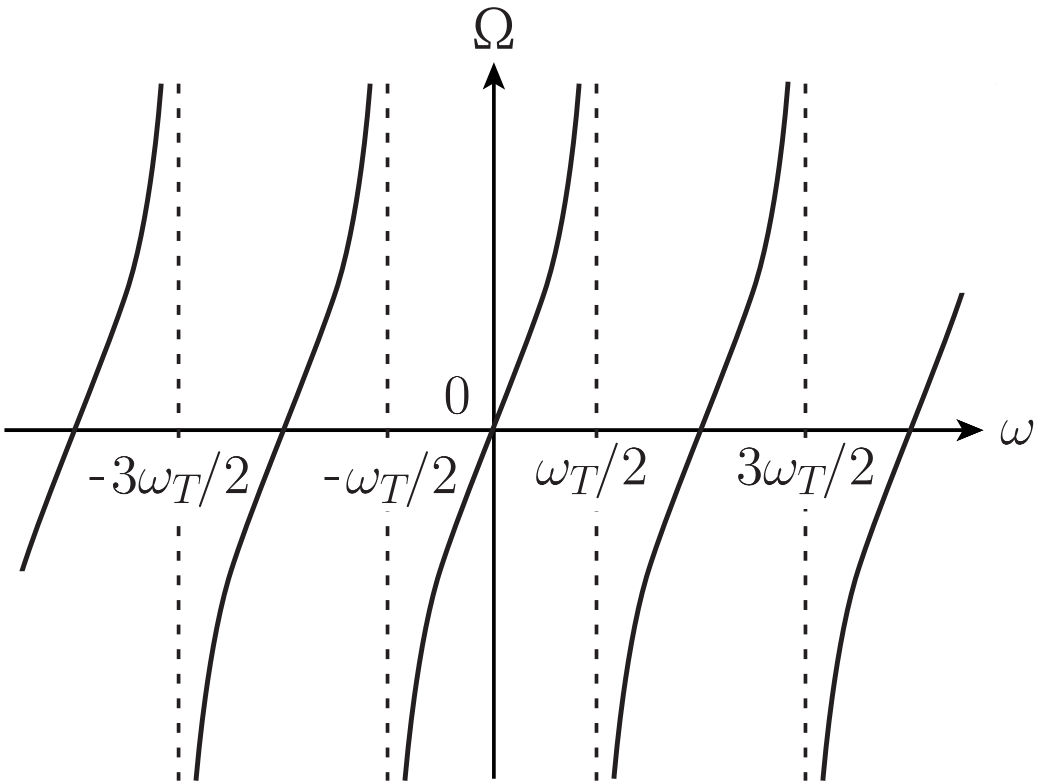
$$\frac{2(1-z^{-1})}{T(1+z^{-1})} = \frac{2(1-e^{-j\omega T})}{T(1+e^{-j\omega T})} = \frac{2(e^{j\omega T/2}-e^{-j\omega T/2})}{T(e^{j\omega T/2}+e^{-j\omega T/2})}$$

$$= \frac{2\{\cos(\omega T/2)+j\sin(\omega T/2)-\cos(\omega T/2)+j\sin(\omega T/2)\}}{T\{\cos(\omega T/2)+j\sin(\omega T/2)+\cos(\omega T/2)-j\sin(\omega T/2)\}}$$

$$= \frac{4j\sin(\omega T/2)}{2T\cos(\omega T/2)} = j\frac{2}{T}\tan\frac{\omega T}{2}$$

$$s \to \frac{2(1-z^{-1})}{T(1+z^{-1})}$$

$$\Omega \to \frac{2}{T}\tan\left(\frac{\omega T}{2}\right)$$



$$\omega_T = \frac{2\pi}{T}$$

$\omega T << 1$ のとき $\Omega = \frac{2}{T}\tan\left(\frac{\omega T}{2}\right) \approx \frac{2}{T} \times \frac{\omega T}{2} = \omega$

LDI変換 (Lossless Discrete Integrator変換)

$$s \to \frac{z^{1/2} - z^{-1/2}}{T}$$

$s = j\Omega$, $z = e^{j\omega T}$を代入

$$\frac{z^{1/2} + z^{-1/2}}{T} = \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{T}$$

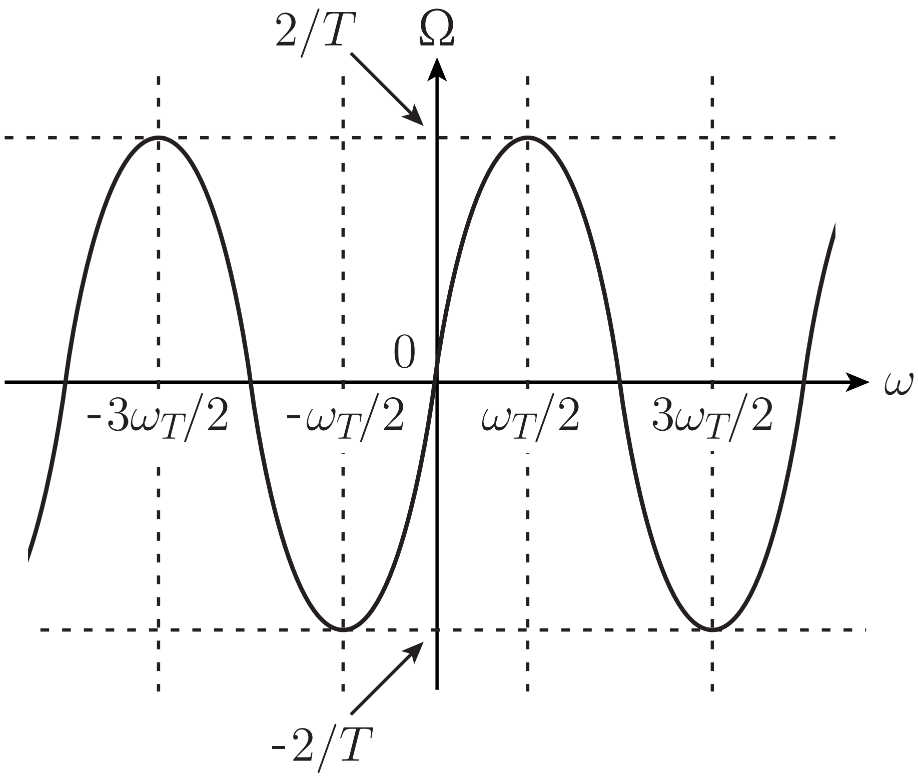
$$= \frac{\{\cos(\omega T/2) + j\sin(\omega T/2)\} - \{\cos(\omega T/2) - j\sin(\omega T/2)\}}{T}$$

$$= j\frac{2}{T}\sin(\omega T/2)$$

$$\Omega \to \frac{2}{T}\sin\left(\frac{\omega T}{2}\right)$$

$$\Omega \rightarrow \frac{2}{T}\sin\left(\frac{\omega T}{2}\right) \approx \omega$$

(∵$x<<1$のとき$\sin x \approx x$)



$$\omega_T = \frac{4\pi}{T}$$

Ωの一部がωへ写像

スイッチトキャパシタ2次区間回路

$$T_{second}(s)=\frac{N(s)}{s^2+\frac{\Omega_0}{Q}s+\Omega_0^{\ 2}}$$

＋ **双一次s-z変換**

$$T_{SC2}(z)=T_{second}\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)=\frac{N\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)}{\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)^2+\frac{\Omega_0}{Q}\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)+\Omega_0^{\ 2}}$$

$$=\frac{N\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)(1+z^{-1})^2}{\frac{4}{T^2}(1-z^{-1})^2+\frac{\Omega_0}{Q}(1-z^{-2})+\Omega_0^{\ 2}(1+z^{-1})^2}$$

$$T_{SC2}(z)=\frac{N\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)(1+z^{-1})^2}{\frac{4}{T^2}(1-z^{-1})^2+\frac{\Omega_0}{Q}(1-z^{-2})+\Omega_0^{\,2}(1+z^{-1})^2}$$

低域通過型 $N(s)=K\Omega_0^{\,2}$ $N_{SC2}(z)\propto(1+z^{-1})^2$

帯域通過型 $N(s)=K\frac{\Omega_0}{Q}s\Rightarrow K\frac{\Omega_0}{Q}\cdot\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}$ $N_{SC2}(z)\propto(1-z^{-2})$

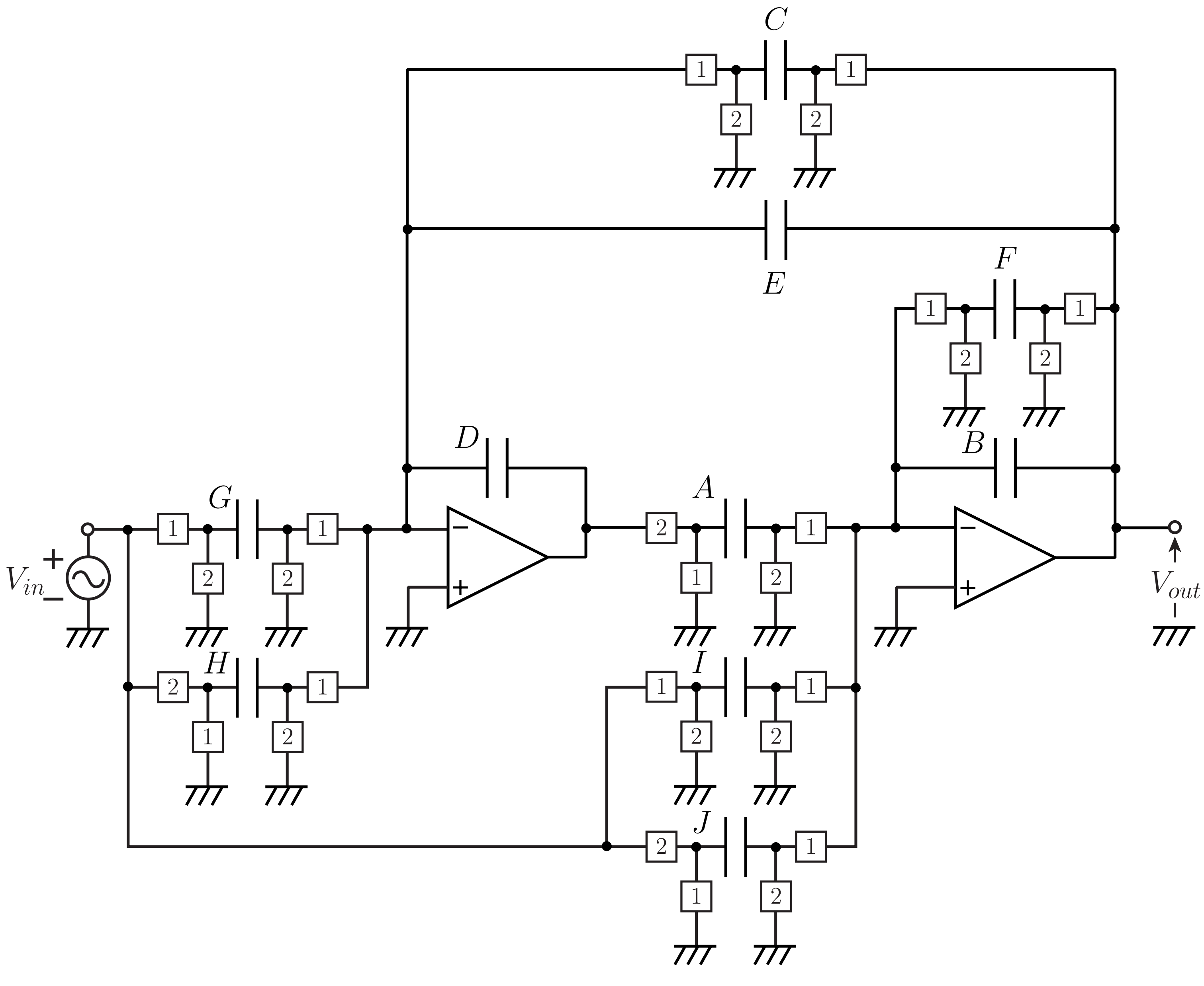
高域通過型 $N(s)=Ks^2\Rightarrow K\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)^2$ $N_{SC2}(z)\propto(1-z^{-1})^2$

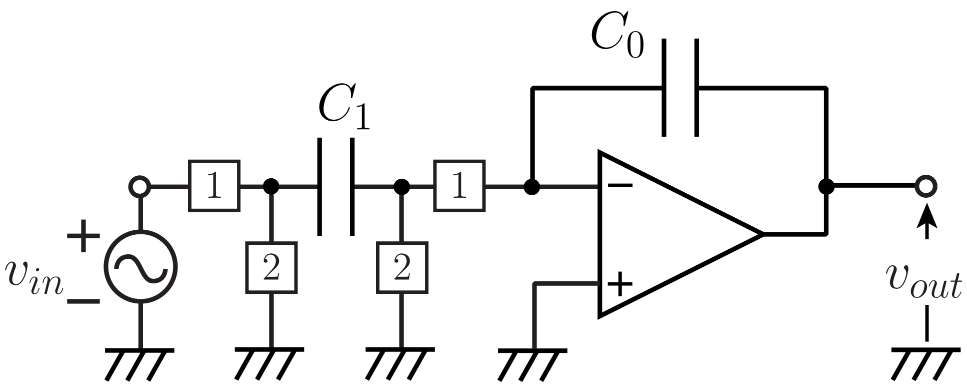
例：低域通過型伝達関数

$$T_{SC2}(z)=\frac{K\Omega_0^{\ 2}}{\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)^2+\frac{\Omega_0}{Q}\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)+\Omega_0^{\ 2}}$$

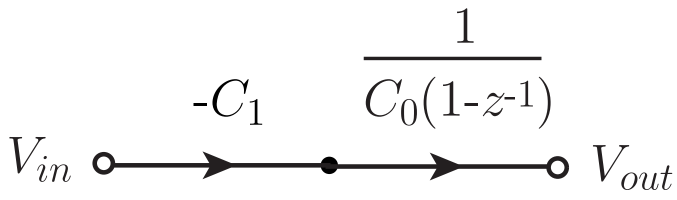
$$=\frac{\dfrac{K\Omega_0^{\ 2}}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^{\ 2}}(1+z^{-1})^2}{1-2\dfrac{\frac{4}{T^2}-\Omega_0^{\ 2}}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^{\ 2}}z^{-1}+\dfrac{\frac{4}{T^2}-\frac{2\Omega_0}{TQ}+\Omega_0^{\ 2}}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^{\ 2}}z^{-2}}$$

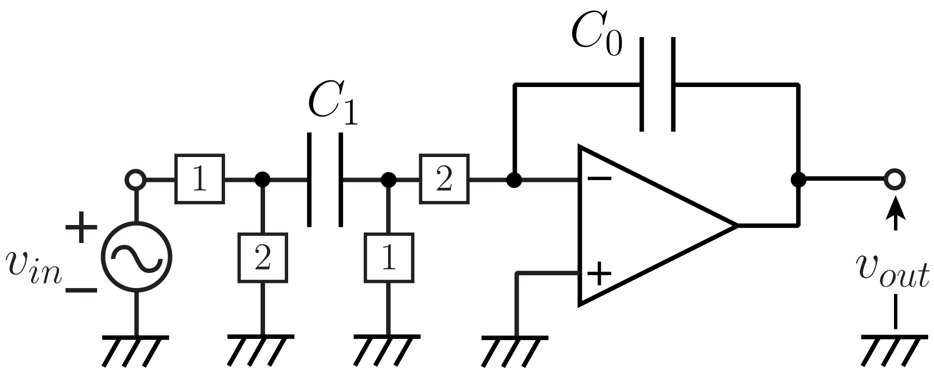
FleischerとLakerの回路



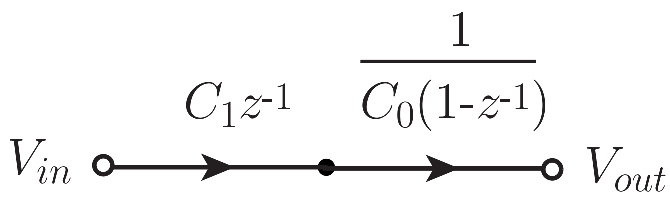


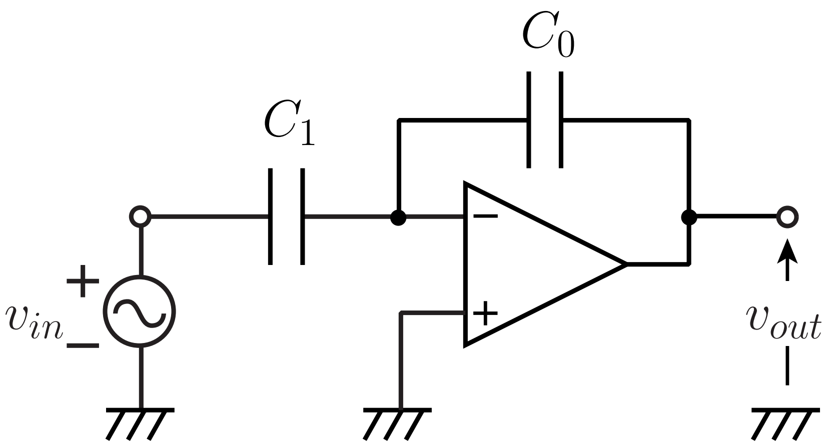
$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1}{C_0(1-z^{-1})} = -C_1 \times \frac{1}{C_0(1-z^{-1})}$$



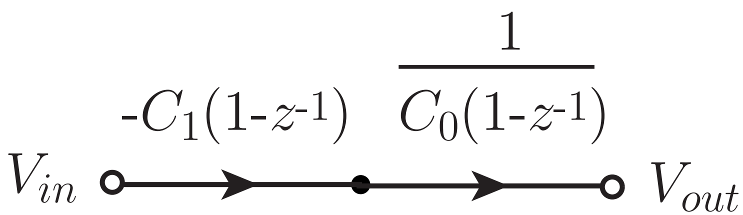


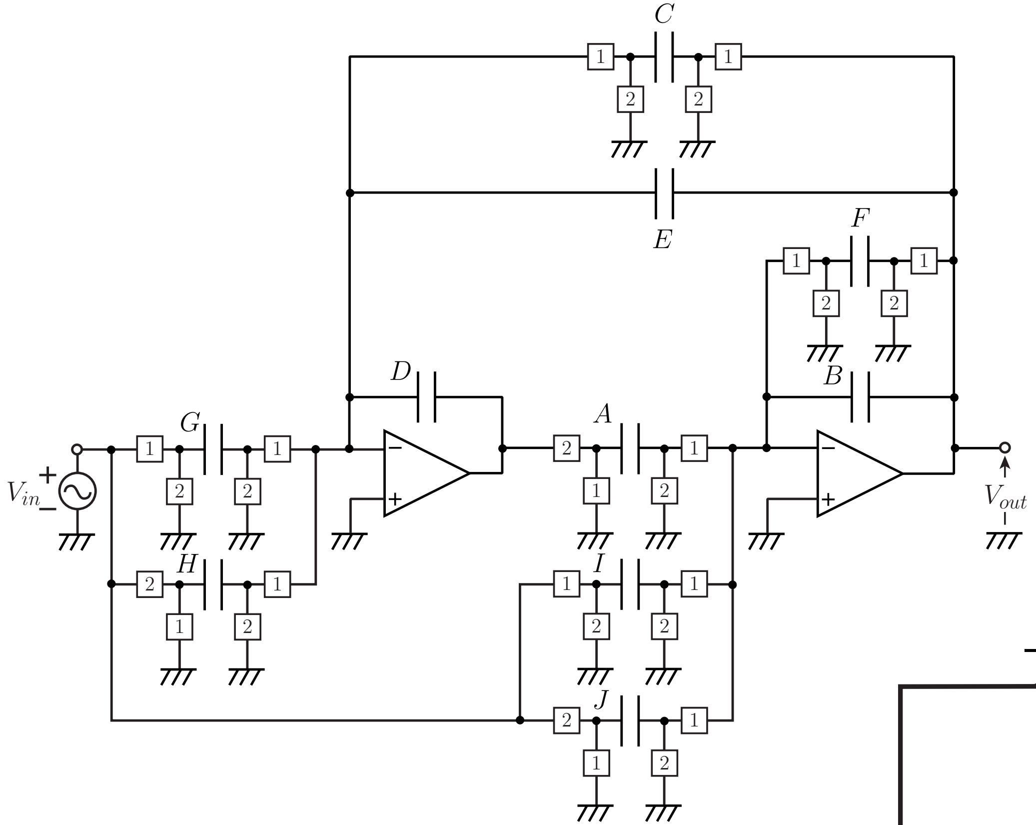
$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{C_1 z^{-1}}{C_0(1-z^{-1})} = C_1 z^{-1} \times \frac{1}{C_0(1-z^{-1})}$$



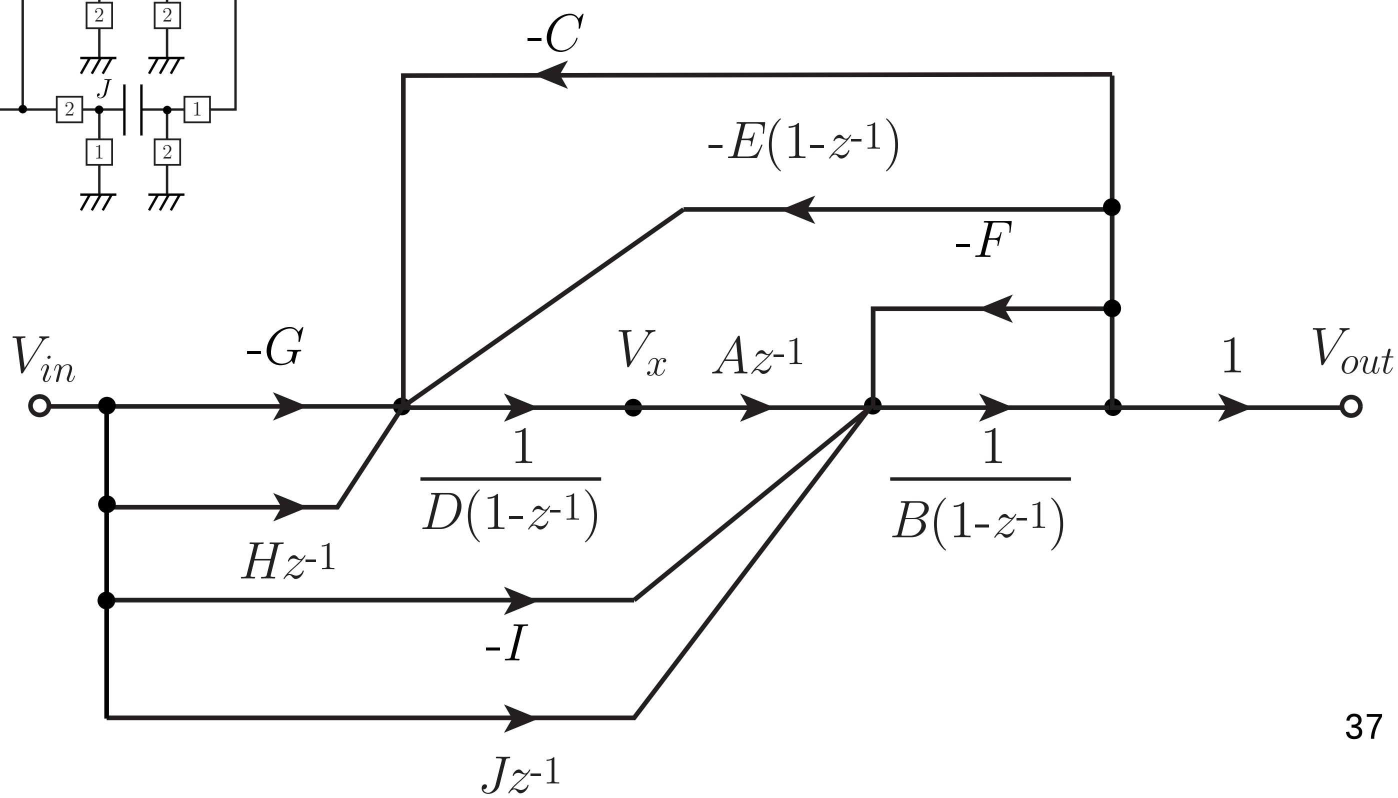


$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1}{C_0} = -C_1(1-z^{-1}) \times \frac{1}{C_0(1-z^{-1})}$$

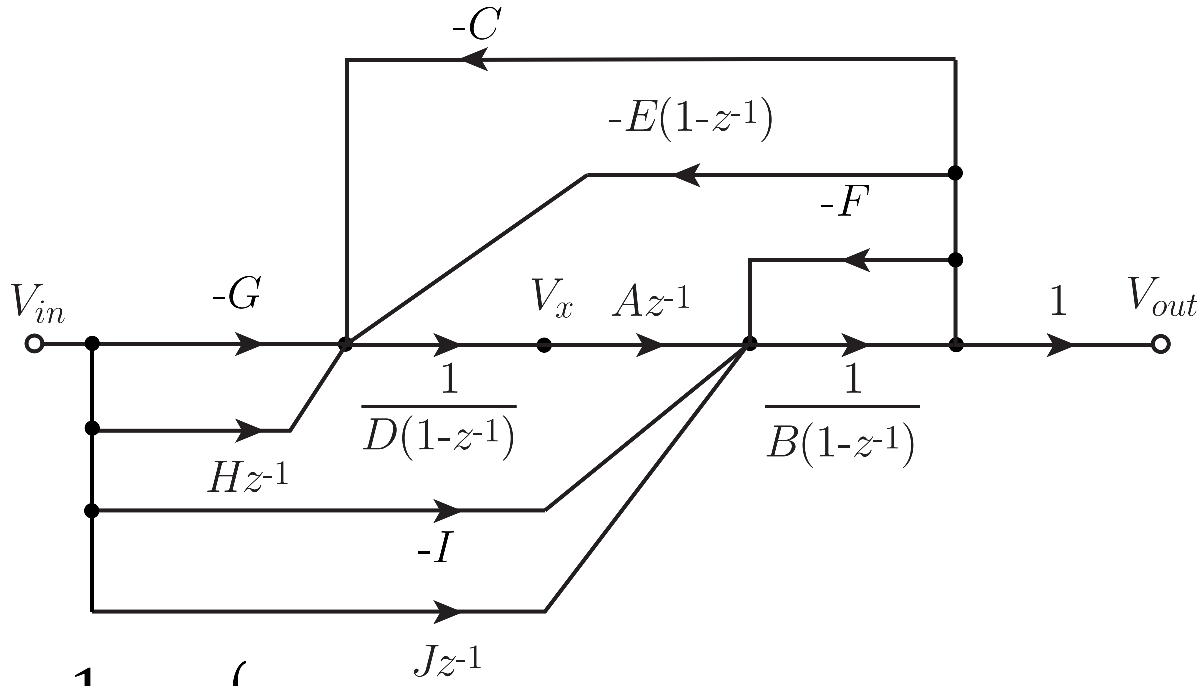




C
E
F
D
B
A
G
H
I
J
V_{in}
V_{out}



-C
-E(1-z-1)
-F
-G
V_{in}
V_x
Az-1
1
V_{out}
1/D(1-z-1)
1/B(1-z-1)
Hz-1
-I
Jz-1

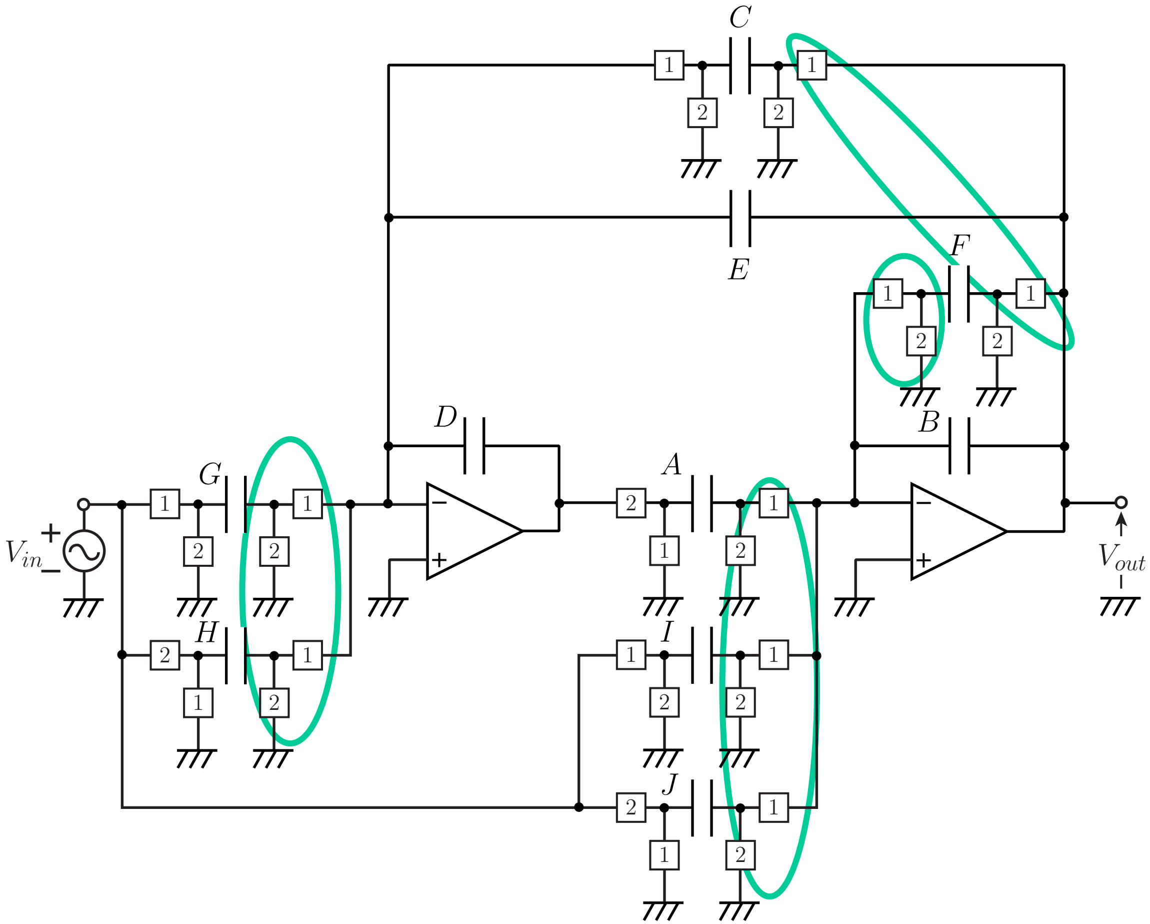


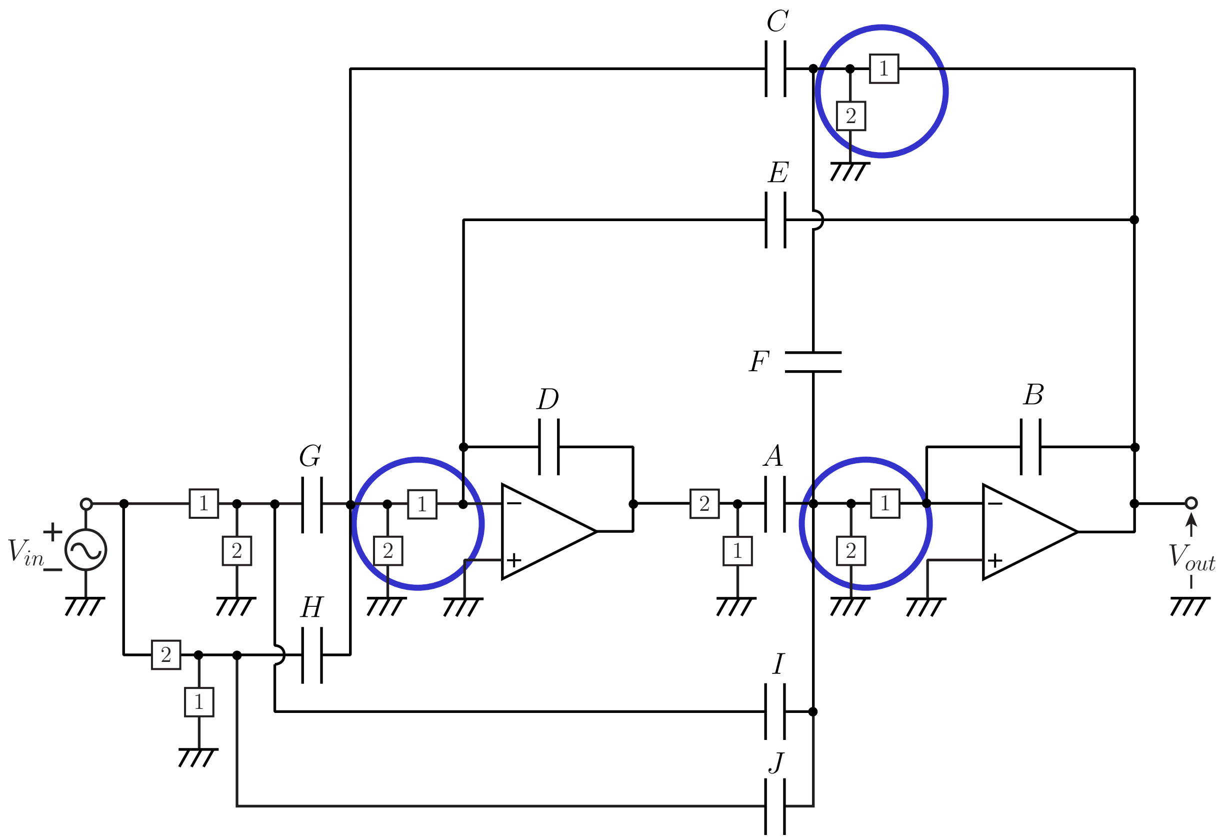
$$V_x = \frac{1}{D(1-z^{-1})}\left\{-GV_{in} + Hz^{-1}V_{in} - CV_{out} - E(1-z^{-1})V_{out}\right\}$$

$$V_{out} = \frac{1}{B(1-z^{-1})}\left\{Az^{-1}V_x - IV_{in} + Jz^{-1}V_{in} - FV_{out}\right\}$$

$$T_{FL}(z) = -\frac{DI + (AG - DI - DJ)z^{-1} + (DJ - AH)z^{-2}}{D(B+F) + (AC + AE - DF - 2BD)z^{-1} + (BD - AE)z^{-2}}$$

スイッチの共有





C
1
2
E
F
D
B
G
A
1
2
1
2
1
2
1
2
1
V_{in}
H
2
1
I
J
V_{out}

$$T_{FL}(z) = \frac{DI + (AG - DI - DJ)z^{-1} + (DJ - AH)z^{-2}}{D(B+F) + (AC + AE - DF - 2BD)z^{-1} + (BD - AE)z^{-2}}$$

特性：容量比で決定 → **規格化** → $A = B = D = 1$

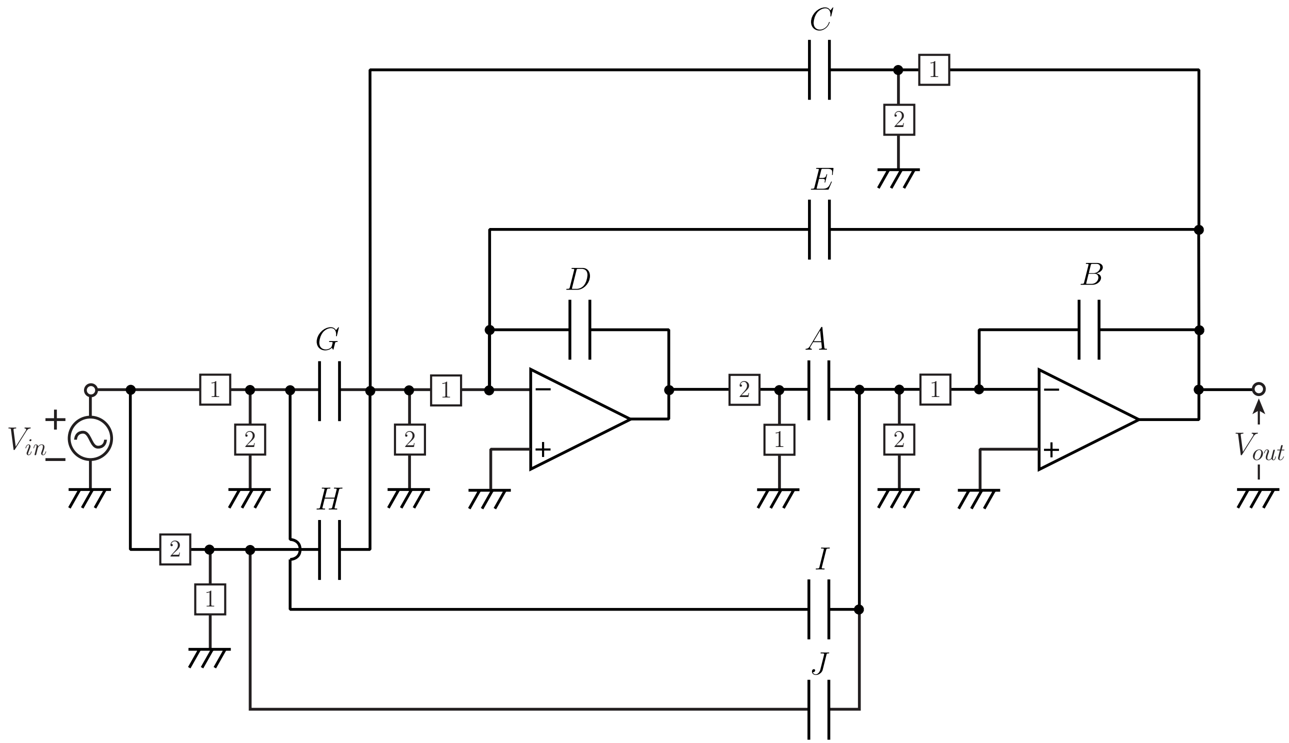
$$T_{FL}(z) = \frac{I + (G - I - J)z^{-1} + (J - H)z^{-2}}{1 + F + (C + E - F - 2)z^{-1} + (1 - E)z^{-2}}$$

安定性：分母多項式の根は単位円内

$1 + F > 1 - E$ $F = 0 \text{ or } E = 0$

例： $F = 0$

$$T_{FL}(z) = \frac{I + (G - I - J)z^{-1} + (J - H)z^{-2}}{1 + (C + E - 2)z^{-1} + (1 - E)z^{-2}}$$



FleischerとLakerのE回路

$$T_{SC2}(z)=\frac{\dfrac{K\Omega_0^{2}}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0^{2}}(1+z^{-1})^2}{1-2\dfrac{\dfrac{4}{T^2}-\Omega_0^{2}}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0^{2}}z^{-1}+\dfrac{\dfrac{4}{T^2}-\dfrac{2\Omega_0}{TQ}+\Omega_0^{2}}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0^{2}}z^{-2}}$$

$$T_{FL}(z)=\frac{I+(G-I-J)z^{-1}+(J-H)z^{-2}}{1+(C+E-2)z^{-1}+(1-E)z^{-2}}$$

$$E=\frac{\dfrac{4\Omega_0}{TQ}}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0^{2}}$$

$$C=\frac{4\Omega_0^{2}}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0^{2}}$$

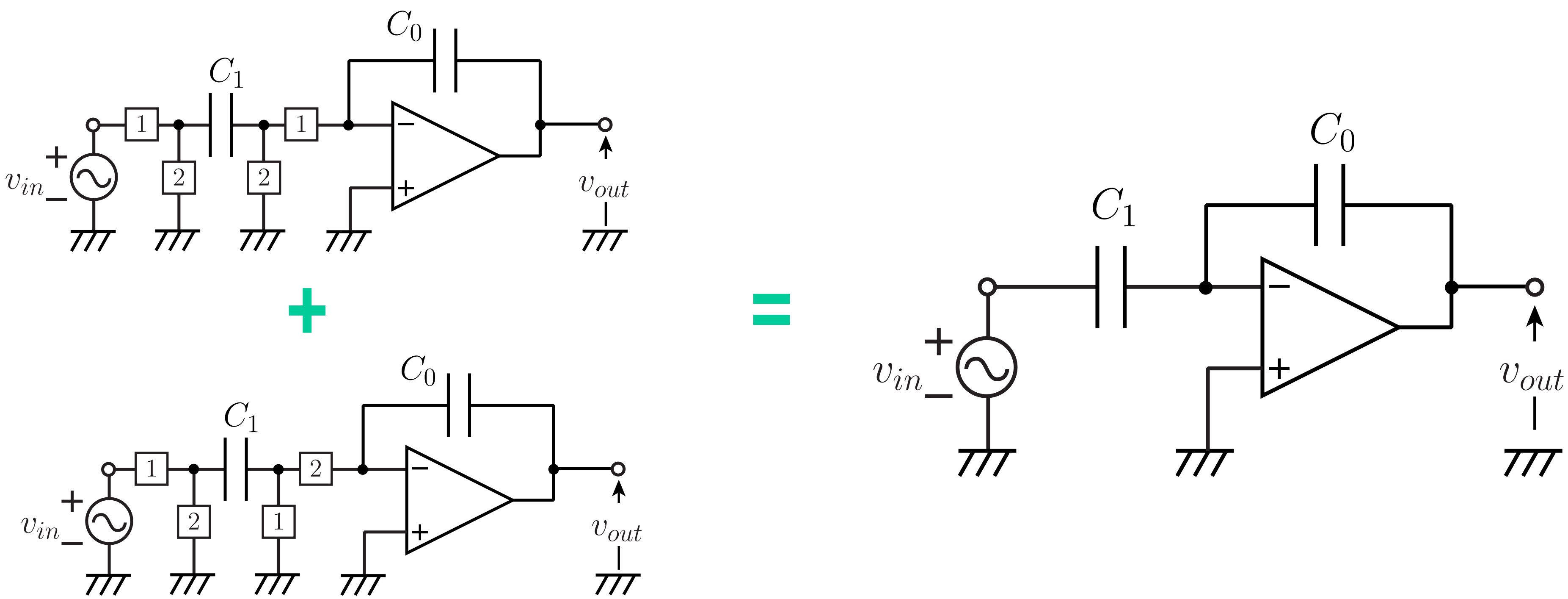
$$T_{FL}(z) = \frac{I + (G - I - J)z^{-1} + (J - H)z^{-2}}{1 + (C + E - 2)z^{-1} + (1 - E)z^{-2}}$$

$$T_{SC2}(z) = \frac{\dfrac{K\Omega_0{}^2}{\dfrac{4}{T^2} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2}(1 + 2z^{-1} + z^{-2})}{1 - 2\dfrac{\dfrac{4}{T^2} - \Omega_0{}^2}{\dfrac{4}{T^2} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2}z^{-1} + \dfrac{\dfrac{4}{T^2} - \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2}{\dfrac{4}{T^2} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2}z^{-2}}$$

$$I = \frac{K\Omega_0{}^2}{\dfrac{4}{T^2} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2} \qquad J = \frac{K\Omega_0{}^2}{\dfrac{4}{T} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2} \qquad H = 0 \qquad G = \frac{4K\Omega_0{}^2}{\dfrac{4}{T^2} + \dfrac{2\Omega_0}{TQ} + \Omega_0{}^2}$$

$$A = B = D = 1 \quad C = \frac{4\Omega_0^2}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + \Omega_0^2} \quad E = \frac{\frac{4\Omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + \Omega_0^2}$$

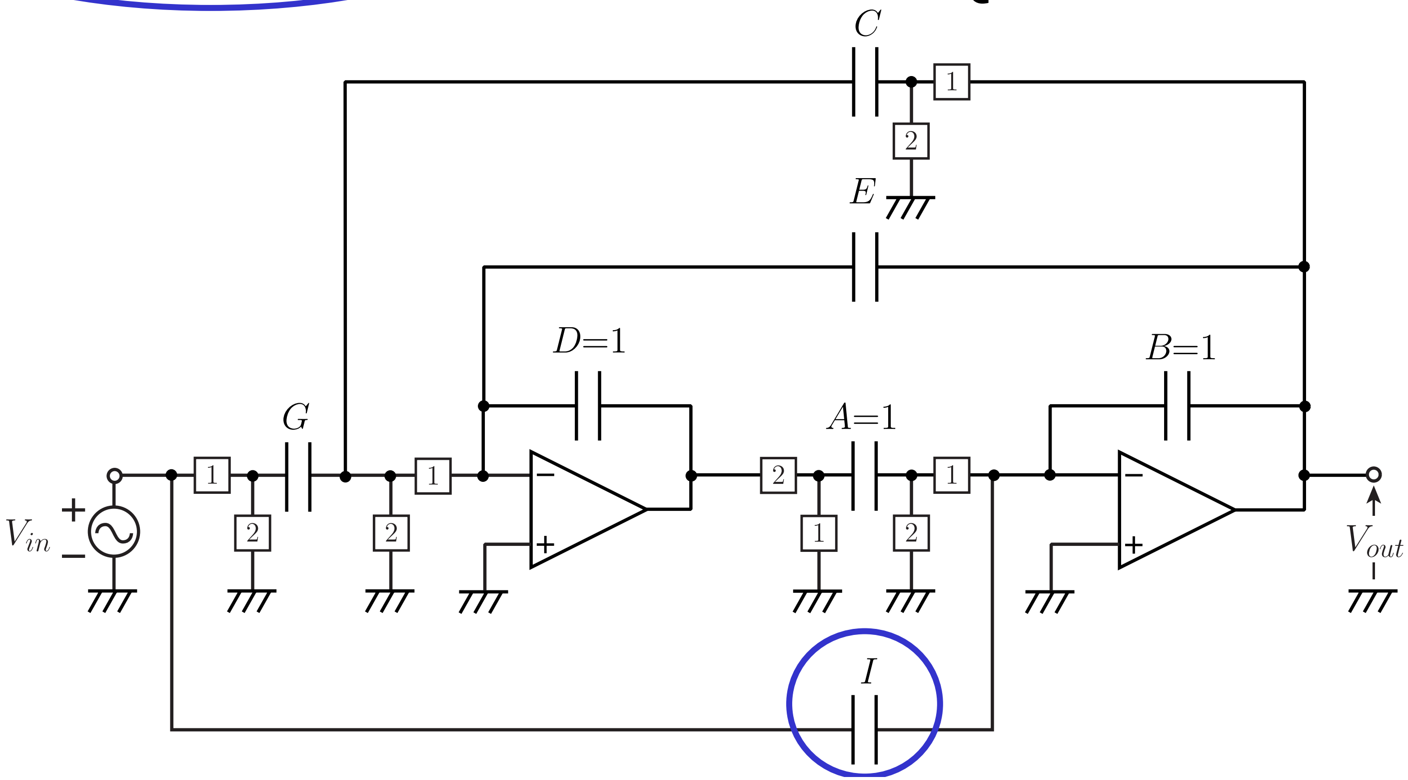
$$I = J = \frac{K\Omega_0^2}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + \Omega_0^2} \quad G = \frac{4K\Omega_0^2}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + \Omega_0^2} \quad H = 0$$



$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1}{C_0(1-z^{-1})} + \frac{C_1 z^{-1}}{C_0(1-z^{-1})} = \frac{-C_1(1-z^{-1})}{C_0(1-z^{-1})} = \frac{-C_1}{C_0}$$

$$A = B = D = 1 \quad C = \frac{4\Omega_0^{\,2}}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + \Omega_0^{\,2}} \quad E = \frac{\frac{4\Omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + \Omega_0^{\,2}}$$

$$I = J = \frac{K\Omega_0^{\,2}}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + \Omega_0^{\,2}} \quad G = \frac{4K\Omega_0^{\,2}}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + \Omega_0^{\,2}} \quad H = 0$$



帯域通過型関数:

$$T_{SC2}(z)=T_{second}\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)=\frac{N\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)}{\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)^2+\frac{\omega_0}{Q}\left(\frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right)+\omega_0{}^2}$$

$$=\frac{\dfrac{\dfrac{2K\Omega_0}{TQ}}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}(1-z^{-2})}{1-2\dfrac{\dfrac{4}{T^2}-\Omega_0{}^2}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}z^{-1}+\dfrac{\dfrac{4}{T^2}-\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}{\dfrac{4}{T^2}+\dfrac{2\Omega_0}{TQ}+\Omega_0{}^2}z^{-2}}$$

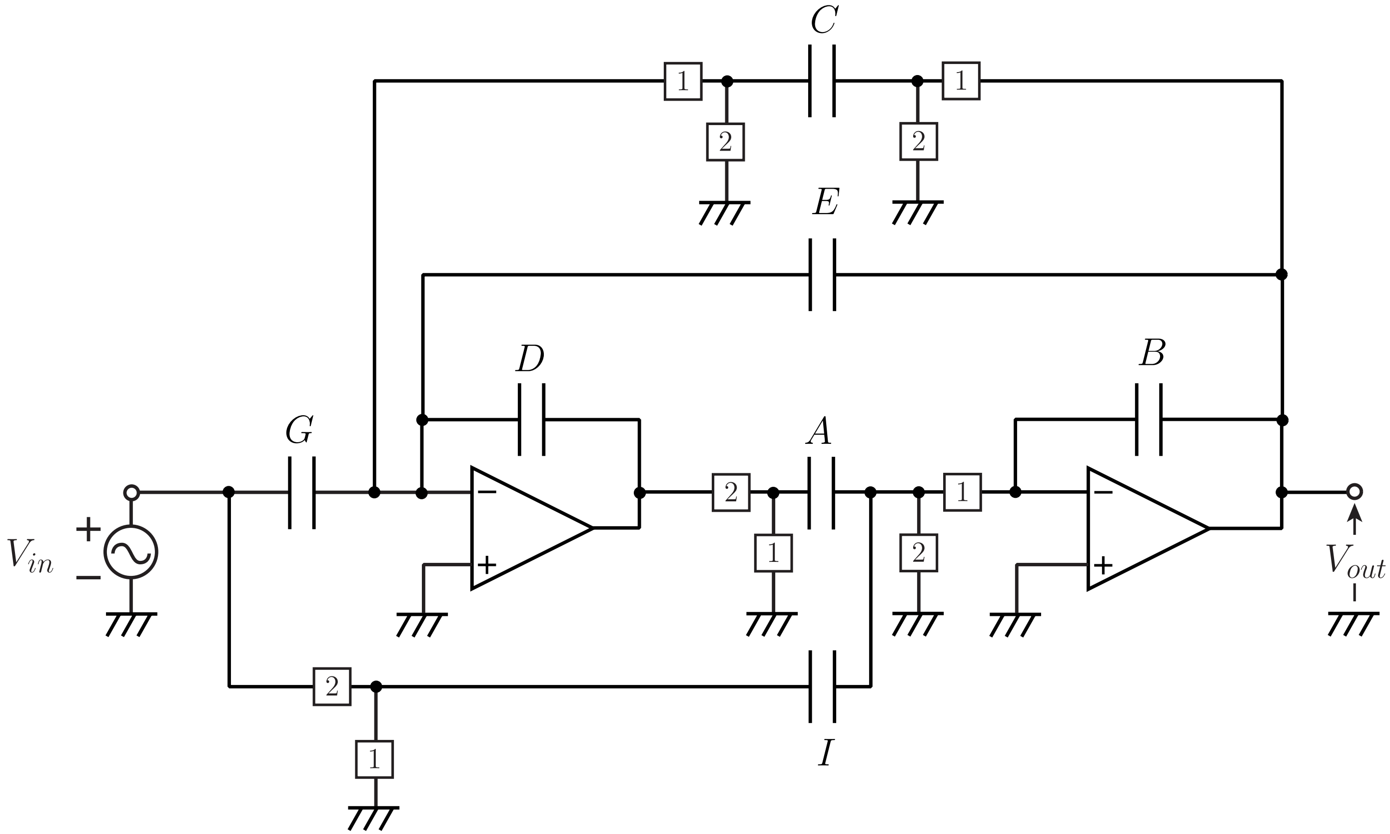
$$T_{SC2}(z)=\frac{\frac{\frac{2K\Omega_0}{TQ}}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^2}(1-z^{-2})}{1-2\frac{\frac{4}{T^2}-\Omega_0^2}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^2}z^{-1}+\frac{\frac{4}{T^2}-\frac{2\Omega_0}{TQ}+\Omega_0^2}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^2}z^{-2}}$$

$A=B=D=1$とすると

$$T_{FL}(z)=\frac{I+(G-I-J)z^{-1}+(J-H)z^{-2}}{1+(C+E-2)z^{-1}+(1-E)z^{-2}}$$

$$C=\frac{4\Omega_0^2}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^2}\quad E=\frac{\frac{4\Omega_0}{TQ}}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^2}\quad I=G=H=\frac{\frac{2K\Omega_0}{TQ}}{\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\Omega_0^2}\quad J=0$$

2次帯域通過フィルタ



例题 中心周波数1kHz，Q=5，T=1/20000s，K=1

$$\Omega \to \frac{2}{T}\tan(\frac{\omega T}{2})$$

$$\Omega_0 = 40000\tan(\frac{2\pi\times 1000}{2}\times\frac{1}{20000}) = 2\pi\times 1008$$

$$T(z^{-1}) = \frac{\frac{2K\Omega_0}{TQ}(1-z^{-2})}{\left(\frac{4}{T^2}+\frac{2\Omega_0}{TQ}+\omega_0{}^2\right)-2\left(\frac{4}{T^2}-\Omega_0{}^2\right)z^{-1}+\left(\frac{4}{T^2}-\frac{2\Omega_0}{TQ}+\Omega_0{}^2\right)z^{-2}}$$

$$= \frac{0.0300(1-z^{-2})}{1-1.85z^{-1}+0.940z^{-2}}$$

$$\Omega_0 = 2\pi \times 1008,\quad Q = 5,\quad T = 1/20000\text{s},\quad K = 1$$

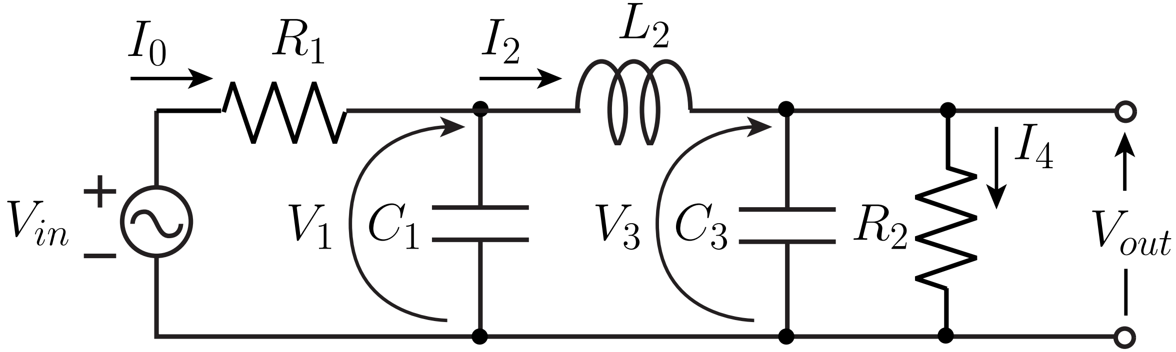
$$A = B = D = 1 \qquad \rightarrow 33.3\text{pF}$$

$$C = \frac{4\Omega_0^{\ 2}}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + \Omega_0^{\ 2}} = 0.0950 \qquad \rightarrow 3.17\text{pF}$$

$$E = \frac{\frac{4\Omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + \Omega_0^{\ 2}} = 0.0600 \qquad \rightarrow 2.00\text{pF}$$

$$I = G = H = \frac{K\frac{2\Omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\Omega_0}{TQ} + \Omega_0^{\ 2}} = 0.0300 \qquad \rightarrow 1.00\text{pF}$$

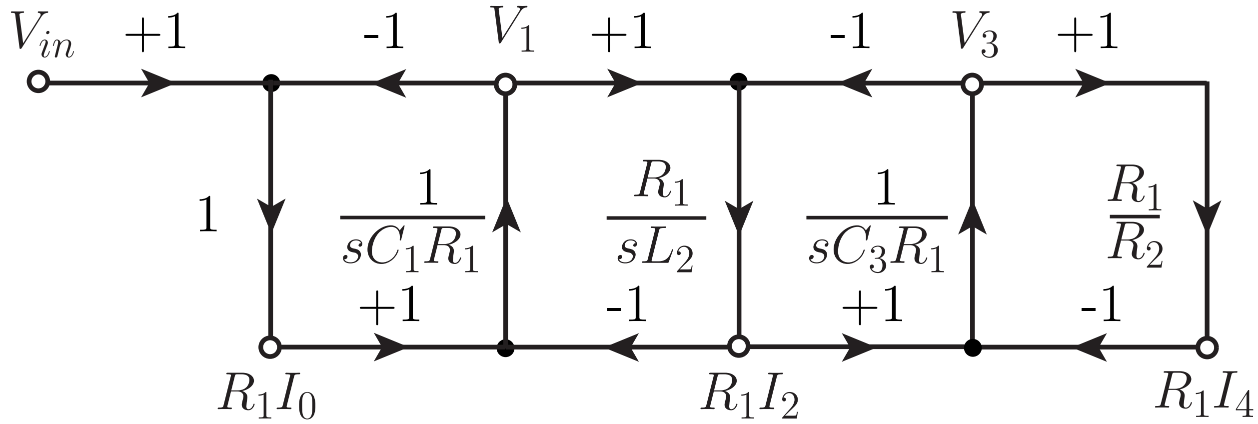
スイッチトキャパシタ回路による リープフロッグ・シミュレーション



$$R_1 I_0 = V_{in} - V_1$$

$$V_1 = \frac{1}{sC_1R_1}(R_1I_0 - R_1I_2)$$

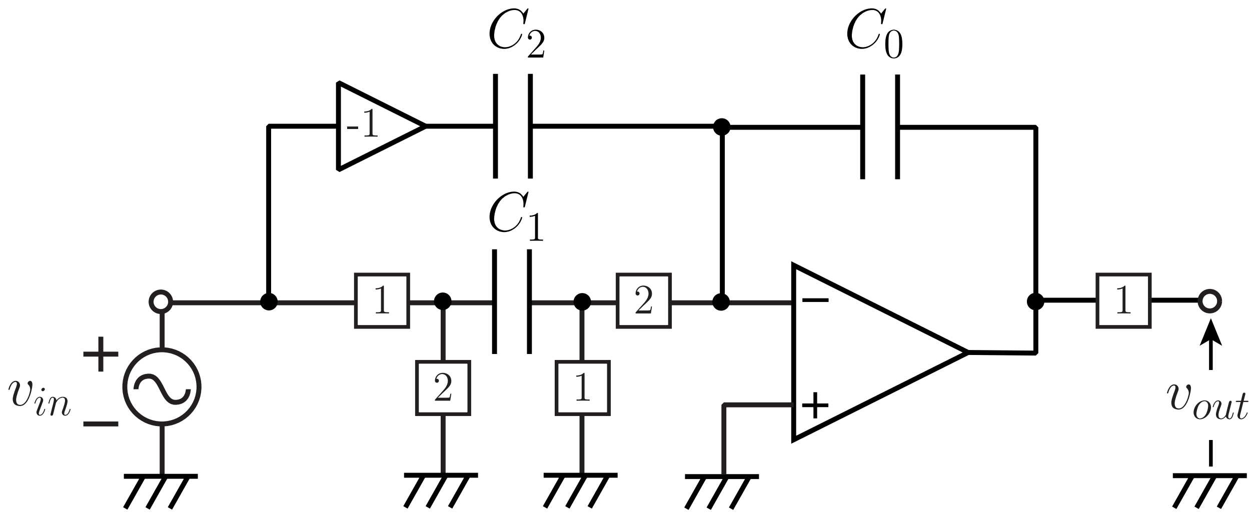
$$R_1I_2 = \frac{R_1}{sL_2}(V_1 - V_3) \quad V_3 = \frac{1}{sC_3R_1}(R_1I_2 - R_1I_4) \quad R_1I_4 = \frac{R_1}{R_2}V_3$$



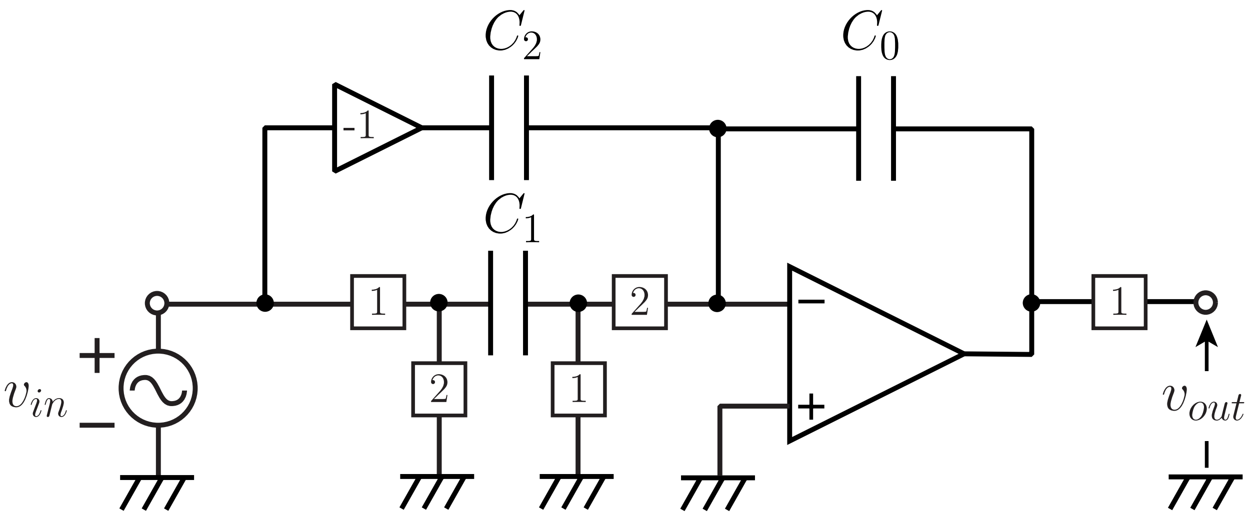
(a)

積分回路の選択

双一次積分回路

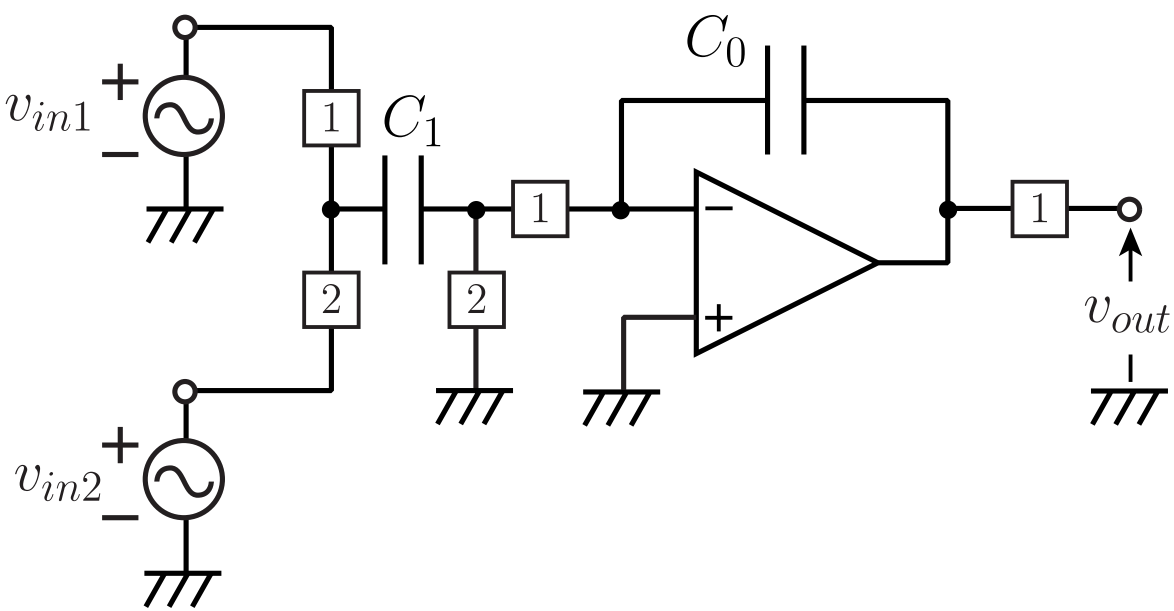


問　$C_1=2C_2$のとき，上の回路の伝達特性$\frac{v_{out}}{v_{in}}$をC_0とC_1を用いて表せ.

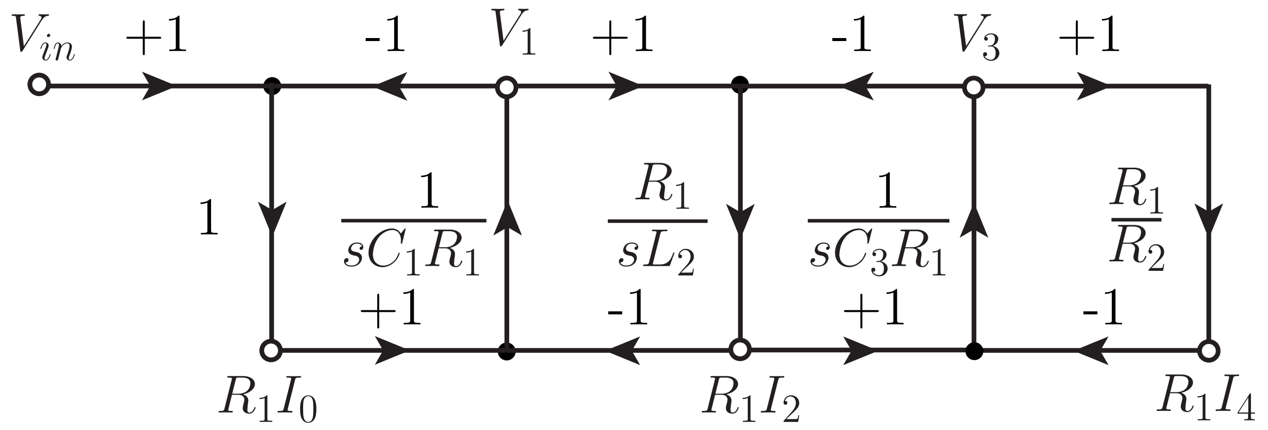


C_1=$2C_2$のとき

双一次積分回路
複　雑

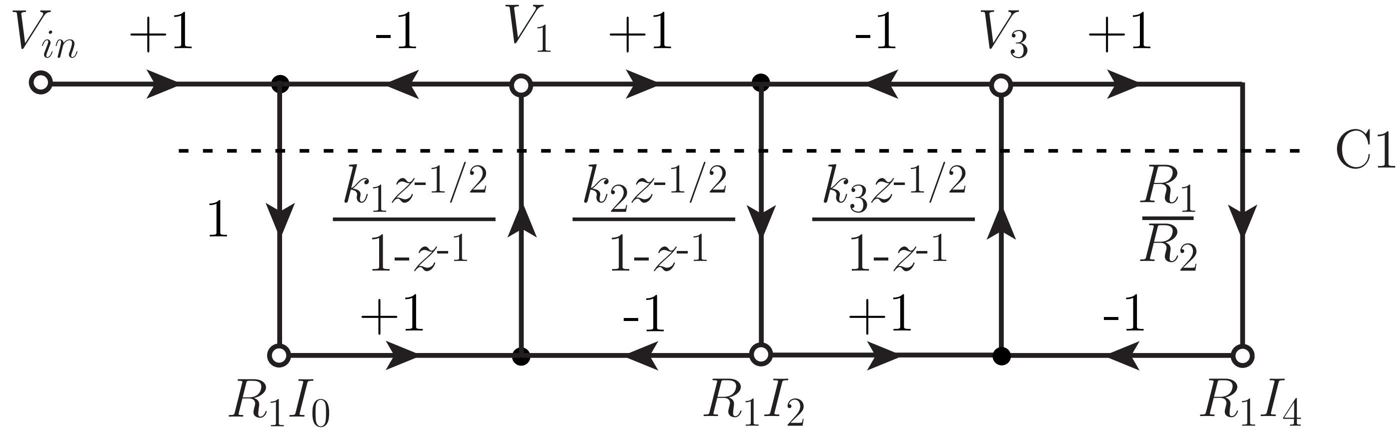


LDI積分回路
簡　単



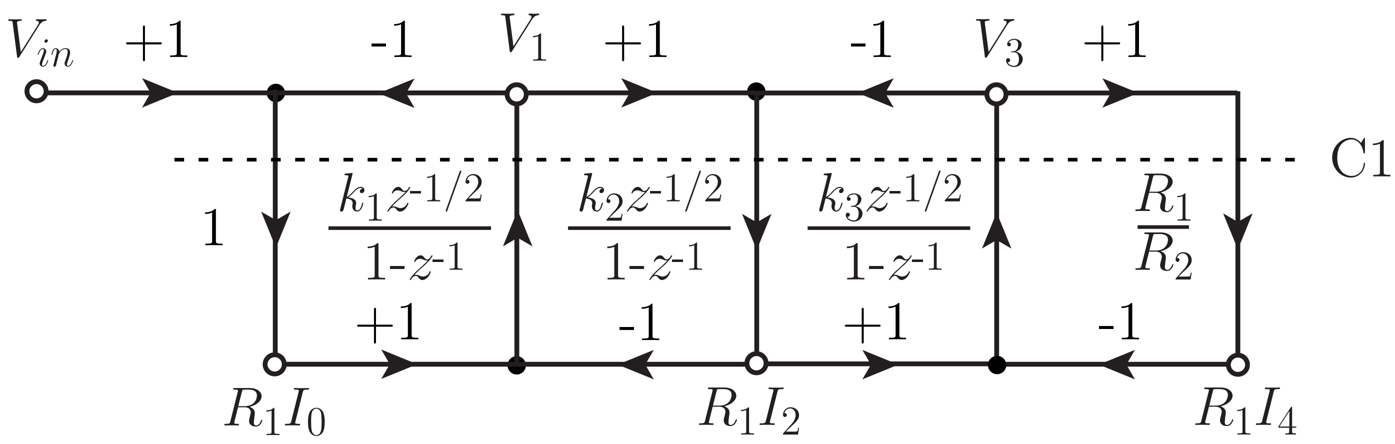
V_{in}
+1
-1
V_1
+1
-1
V_3
+1
1
$\frac{1}{sC_1R_1}$
$\frac{R_1}{sL_2}$
$\frac{1}{sC_3R_1}$
$\frac{R_1}{R_2}$
+1
-1
+1
-1
R_1I_0
R_1I_2
R_1I_4

(a)

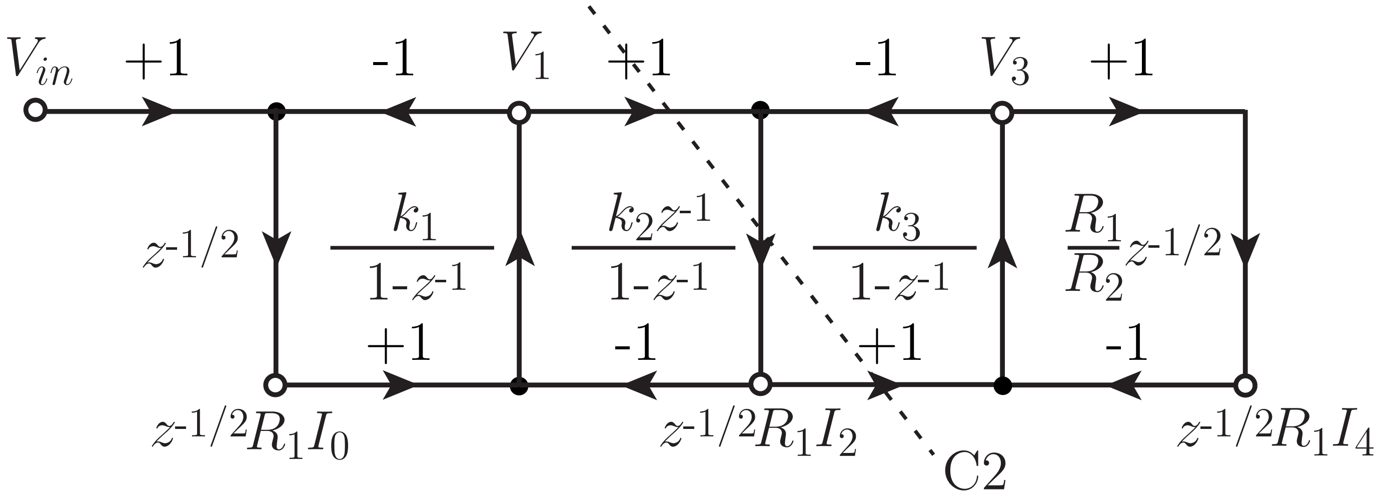


V_{in}
+1
-1
V_1
+1
-1
V_3
+1
C1
1
$\frac{k_1z^{-1/2}}{1-z^{-1}}$
$\frac{k_2z^{-1/2}}{1-z^{-1}}$
$\frac{k_3z^{-1/2}}{1-z^{-1}}$
$\frac{R_1}{R_2}$
+1
-1
+1
-1
R_1I_0
R_1I_2
R_1I_4

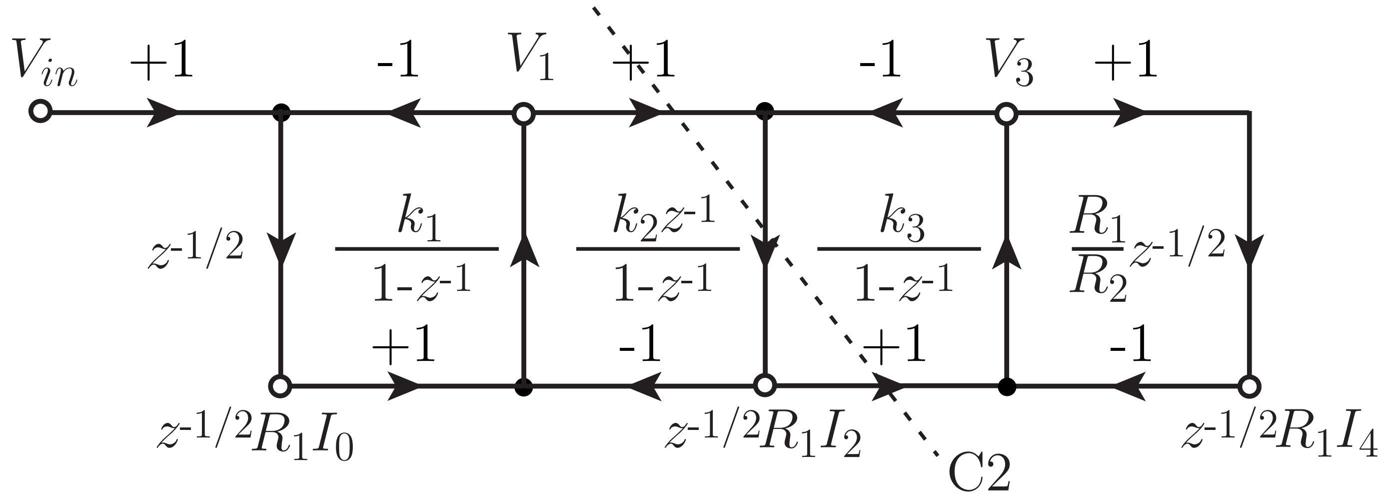
(b)


V_{in}
+1
-1
V_1
+1
-1
V_3
+1
C1
1
$\frac{k_1 z^{-1/2}}{1-z^{-1}}$
$\frac{k_2 z^{-1/2}}{1-z^{-1}}$
$\frac{k_3 z^{-1/2}}{1-z^{-1}}$
$\frac{R_1}{R_2}$
+1
-1
+1
-1
$R_1 I_0$
$R_1 I_2$
$R_1 I_4$

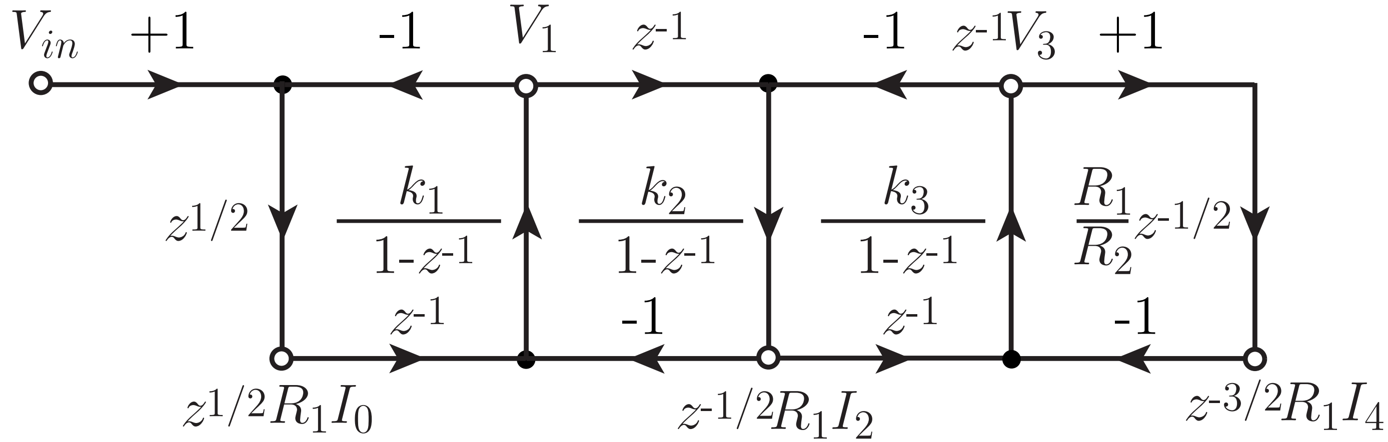
(b)


V_{in}
+1
-1
V_1
+1
-1
V_3
+1
$z^{-1/2}$
$\frac{k_1}{1-z^{-1}}$
$\frac{k_2 z^{-1}}{1-z^{-1}}$
$\frac{k_3}{1-z^{-1}}$
$\frac{R_1}{R_2} z^{-1/2}$
+1
-1
+1
-1
$z^{-1/2} R_1 I_0$
$z^{-1/2} R_1 I_2$
$z^{-1/2} R_1 I_4$
C2

(c)

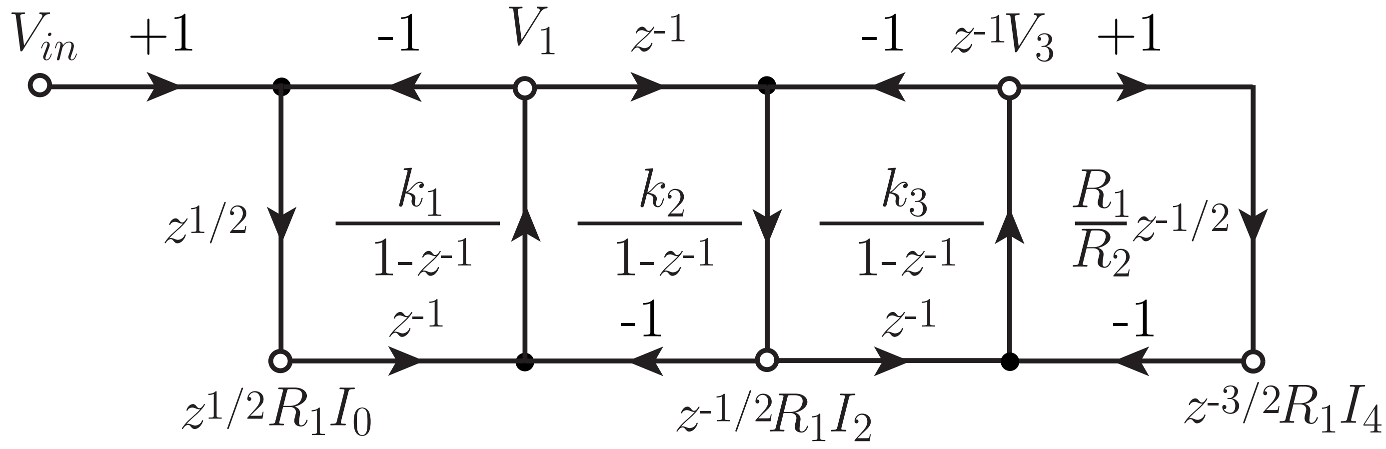


(c)



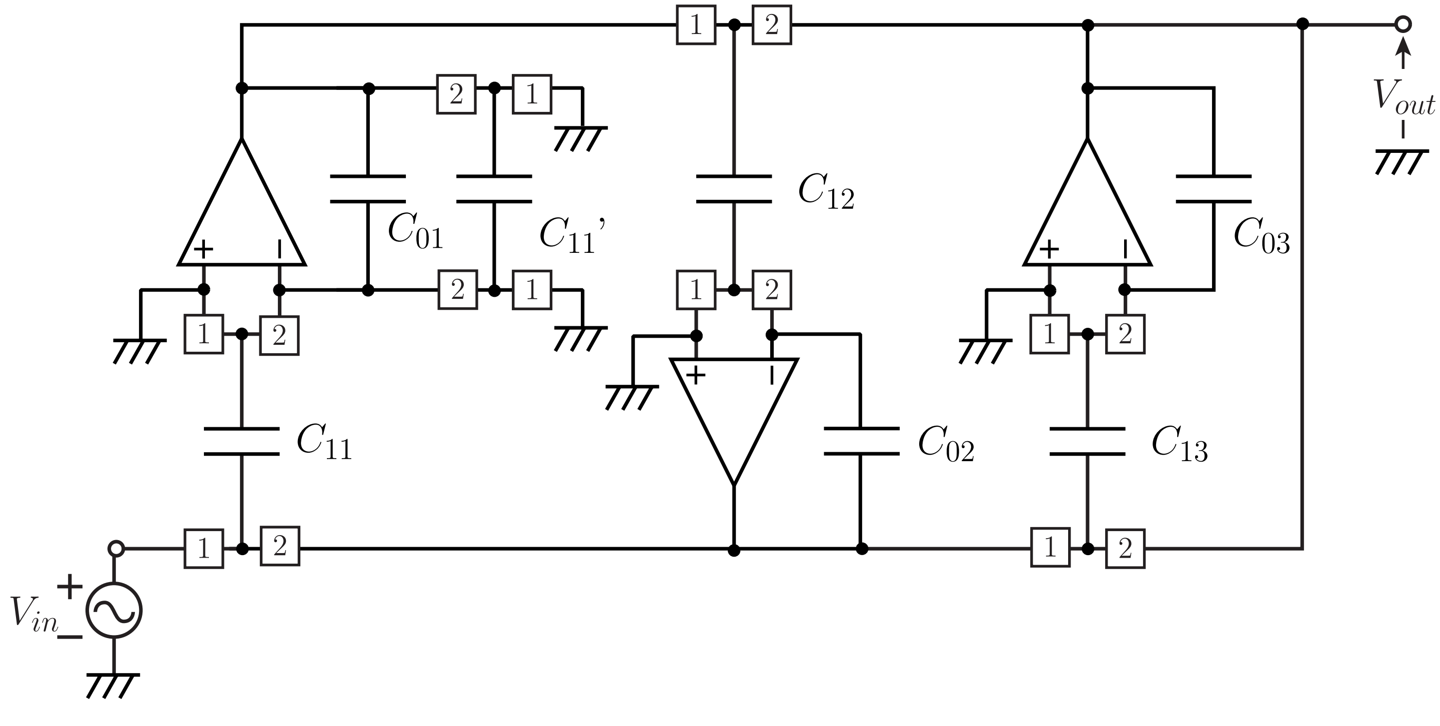
(d)

$$k_1 = \frac{T}{C_1 R_1} \qquad k_2 = \frac{R_1 T}{L_2} \qquad k_3 = \frac{T}{C_3 R_1}$$



(d)

$$k_1=\frac{T}{C_1R_1} \qquad k_2=\frac{R_1T}{L_2} \qquad k_3=\frac{T}{C_3R_1}$$



LDI変換の問題点

伝達関数T (s)の極： $s_i=\sigma_i+j\,\omega_i$

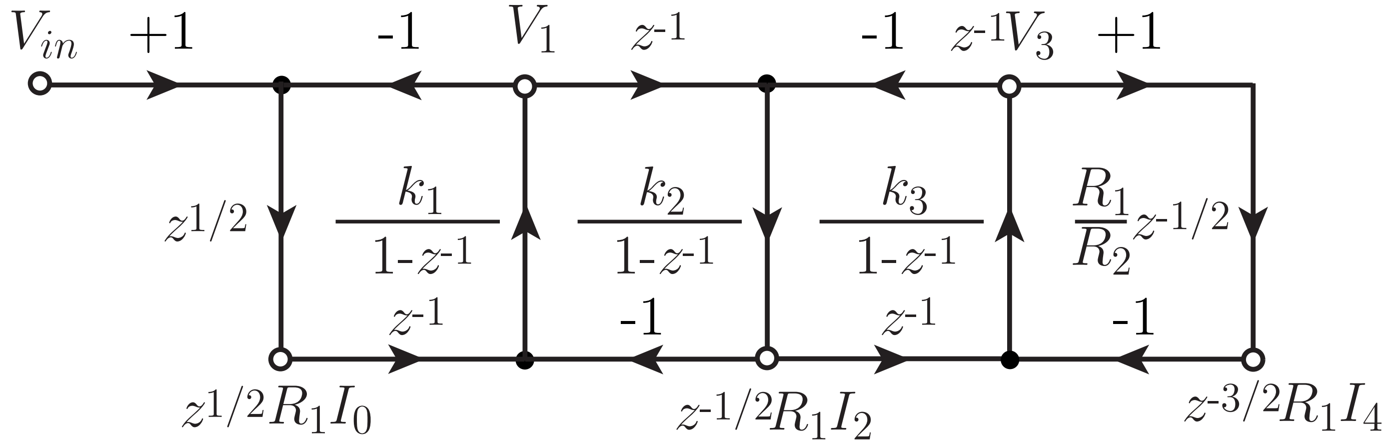
$$s_i=\frac{z^{1/2}-z^{-1/2}}{T} \longrightarrow s_i^{\,2}=\frac{z-2+z^{-1}}{T^2}$$

解： $z_i \longrightarrow z_i^{\,-1}$

$|z_i|<1 \longrightarrow$ **安定**

$\downarrow$

$|z_i^{\,-1}|>1 \longrightarrow$ **不安定**



(d)

$$k_1=\frac{T}{C_1R_1} \qquad k_2=\frac{R_1T}{L_2} \qquad k_3=\frac{T}{C_3R_1}$$

誤差要因

不安定性を除去

例題　　**遮断周波数1kHzの3次低域通過フィルタ**

$$\Omega \to \frac{2}{T}\sin(\frac{\omega T}{2})$$

$T = 1/20000\mathrm{s}$

$$\Omega_0 = 40000\sin(\frac{2\pi\times 1000}{2}\times\frac{1}{20000}) = 2\pi\times 996$$

$R_1 = R_2 = 1\mathrm{k}\Omega$　　$C_1 = C_3 = 0.159\mu\mathrm{F}$　　$L_2 = 0.318\mathrm{H}$

↓

$R_1 = R_2 = 1\mathrm{k}\Omega$　　$C_1 = C_3 = 0.160\mu\mathrm{F}$　　$L_2 = 0.319\mathrm{H}$

$$k_1 = \frac{T}{C_1 R_1} = \frac{C_{11}}{C_{01}} = \frac{C'_{11}}{C_{01}} = 0.319$$

$$k_2 = \frac{TR_1}{L_2} = \frac{C_{12}}{C_{02}} = 0.160$$

$$k_3 = \frac{T}{C_3 R_1} = \frac{C_{13}}{C_{03}} = 0.319$$

$C_{01}=C_{02}=C_{03}=1$とすれば

$C_{11}=C'_{11}=C_{13}=0.319$, $C_{12}=0.160$

$C_{01}=C_{02}=C_{03}=6.25\text{pF}$, $C_{11}=C'_{11}=C_{13}=1.99\text{pF}$, $C_{12}=1.00\text{pF}$

容量の総和 :25.7pF

$$k_1=\frac{T}{C_1R_1}=\frac{C_{11}}{C_{01}}=\frac{C'_{11}}{C_{01}}=0.319$$

$$k_2=\frac{TR_1}{L_2}=\frac{C_{12}}{C_{02}}=0.160$$

$$k_3=\frac{T}{C_3R_1}=\frac{C_{13}}{C_{03}}=0.319$$

$C_{01}=C_{03}=1$, $C_{02}=2$とすれば

$$C_{11}=C'_{11}=C_{12}=C_{13}=0.319$$

$C_{01}=C_{03}=3.13$pF, $C_{02}=6.25$pF, $C_{11}=C'_{11}=C_{12}=C_{13}=1.00$pF

容量の総和 :13.4pF