

# Filtering: Signal Conditioning and Processing

## Agenda

- Review of Filter & Signal Processing
- Linear & Non-linear Signal Processing
- Filter Design & Synthesis
- Gaussian Filter
- Nyquist Filter
- Partial Response Filter

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## Review of Filter & Signal Processing

- 1) Filter = **Hardware** and/or **Algorithm**
- 2) **Stochastic** vs. **Deterministic**

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- Deterministic:

How to realize a filter circuit which has a desired frequency characteristics

- Linear Signal Processing
  - Noise & Interference Suppression
  - Inter-Symbol Interference Problem

(Negative) Remove → Nyquist Filter (1920's)  
Nyquist Criteria  
(Positive) Utilize → Partial Response Filter (1960's)  
Spectrum Shaping

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– Non-Linear Signal Processing

- **Envelope Detection** (Diode + LPF) : No phase Information
- **PLL** (Phase Comparator + LPF + VCO) : Frequency Synthesizer
- **Pre-emphasis** in FM System

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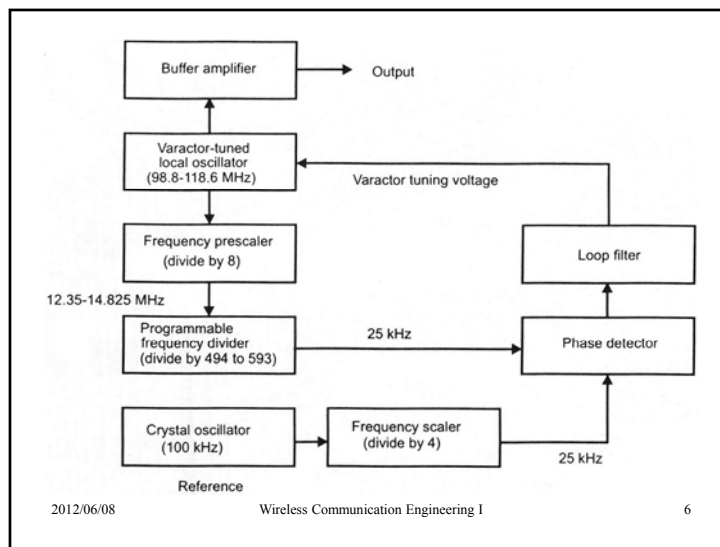
## PLL (Phase Lock Loop) Principle

- Reference Frequency by Stable Crystal Oscillator
- Pre-scaler
- VCO ( Voltage Controlled Oscillator )

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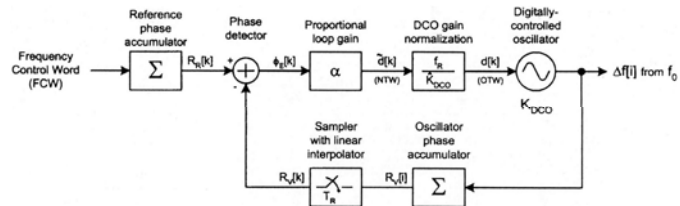
## Principle of ADPLL ( All- Digital PLL)

- Digital Loop Filter
- Digital Controlled Oscillator
- TDC ( Time-to-Digital Converter )

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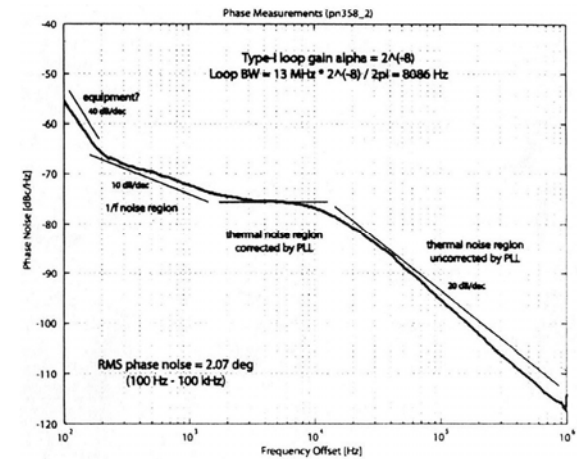
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## History of Filter Design

Design Theory: Butterworth (1930's)  
Chebyshev (1950's),  
Elliptic (1960's)

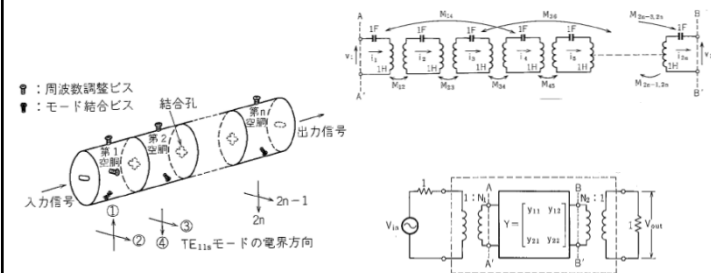
Hardware: LCR, Active, Digital, Ceramic,  
SAW, SC, Waveguide

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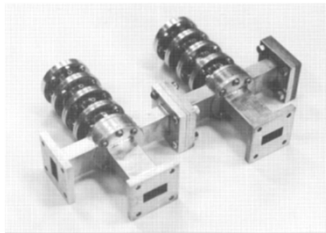
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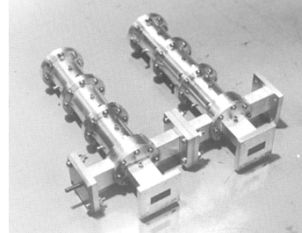
## Dual-mode Filter In-Line (Longitudinal) Type



## Multiplexer/Demultiplexer



(a) Multiplexer (TE<sub>111</sub>)



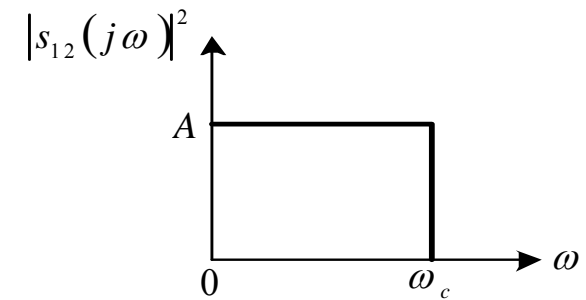
(b) Demultiplexer (TE<sub>113</sub>)

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## Design Procedure

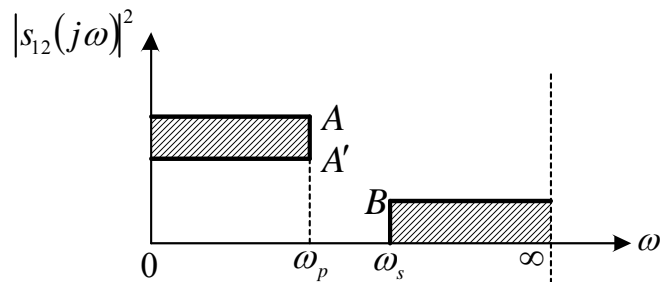


Ideal low-pass prototype response

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Low-pass prototype specification

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	Pass-band	Stop-band
Butterworth	Flat	Flat
Chebyshev	Equal-Ripple	Flat
Inv. Chebyshev	Flat	Equal-Ripple
Elliptic	Equal-Ripple	Equal-Ripple

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- Maximally Flat (Butterworth)

$$|S_{12}(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 \omega^{2n}}$$

- Equal Ripple (Chebyshev)

$$|S_{12}(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 T_n^2(\omega)}$$

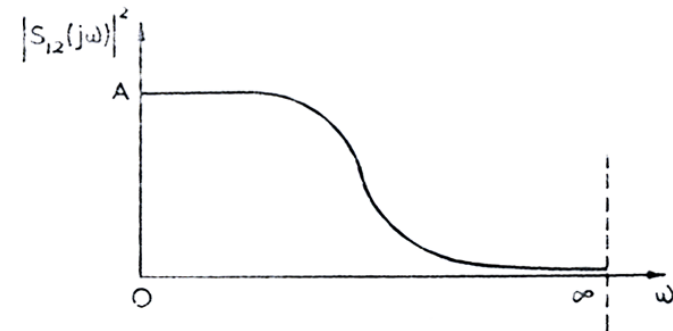
$T_n(\omega)$ :  $n$  - th order Chebyshev Polynomial

$\varepsilon$ : Ripple Level

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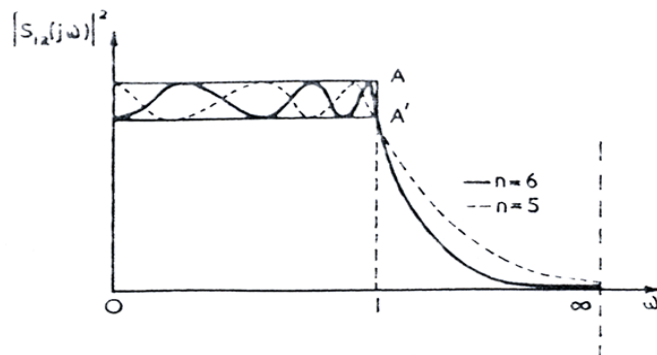


Maximally flat response

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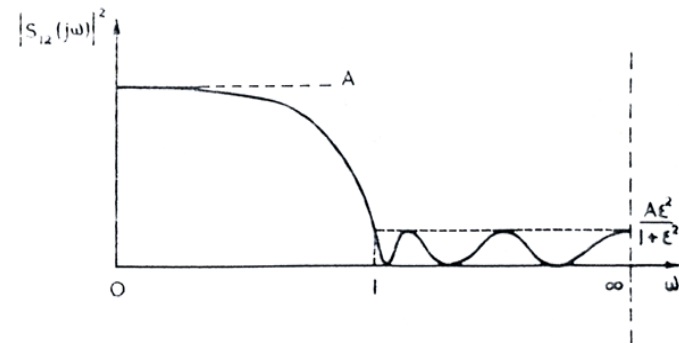


Chebyshev response

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Inverse Chebyshev response

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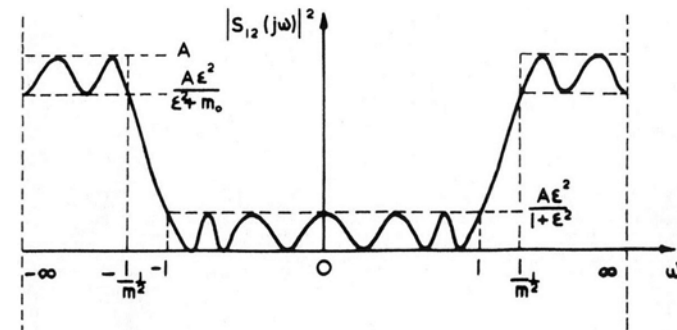
## Elliptic Filter

- Sharp Transition
- Equal-Ripple Characteristics both in PB and SB
- Elliptic function is used for the design of Filter Transfer Function

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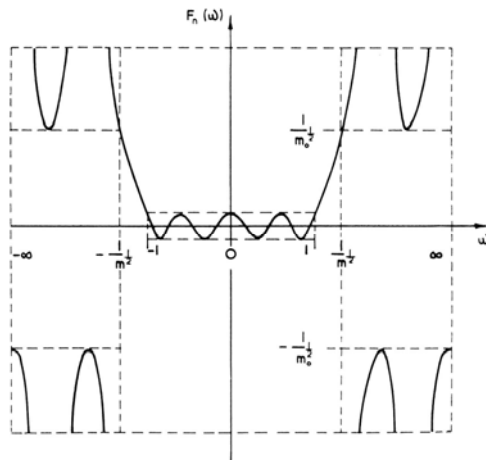
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## Design Example

- $LA=50\text{dB}$ ,  $LR=20\text{dB}$ ,  $\omega_s/\omega_p=2$ :  
 Elliptic Filter  $n=5$   
 Chebyshev Filter  $n=7$   
 Maximally Flat Filter  $n=12$

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## Filter Synthesis

$$|S_{21}(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 \omega^{2n}}$$

$$|S_{11}(j\omega)|^2 = 1 - |S_{21}(j\omega)|^2 = \frac{\varepsilon^2 \omega^{2n}}{1 + \varepsilon^2 \omega^{2n}} = S_{11}(s) S_{11}(-s)$$

[ Factorization Technique]

$$s = j\omega$$

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$$Z_{in}(s) = \frac{1 - S_{11}(s)}{1 + S_{11}(s)}$$

Continued Fraction Technique

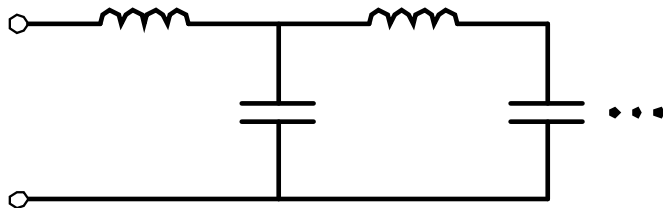
$$\text{e.g. } \frac{3s^4 + 5s^2 + 1}{2s^3 + s} = \frac{3}{2}s + \frac{1}{\frac{4}{9}s + \frac{1}{\frac{49}{6}s + \frac{1}{\frac{3}{7}s}}}}$$

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## LC-Ladder Circuit with $n$ -elements

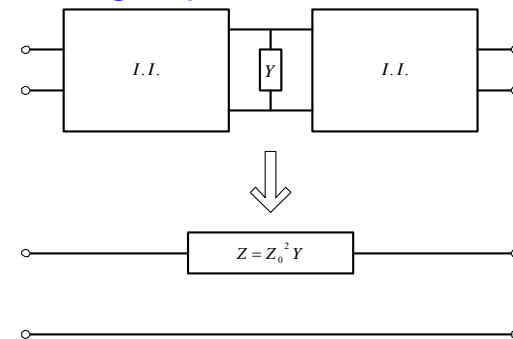


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## Parallel to Serial Transform by using Impedance Inverters



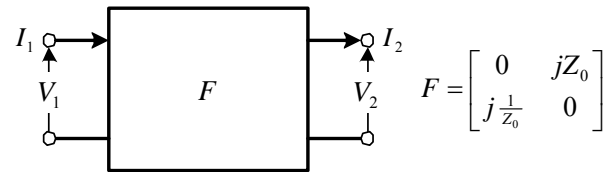
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## Impedance Inverters

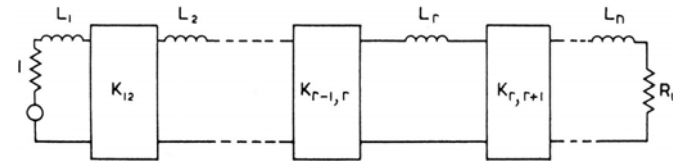
- $\lambda/4$  Transmission Lines (Passive)
- Operational Amplifiers (Active)



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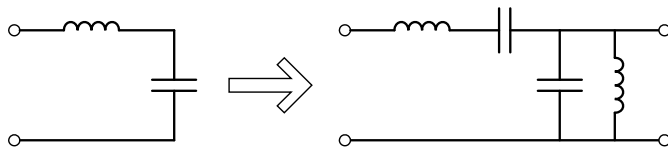
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## Frequency Transformation

$$S \xrightarrow{\text{LPF}} \frac{S}{\omega_0} + \frac{\omega_0}{S} \xrightarrow{\text{BPF}}$$

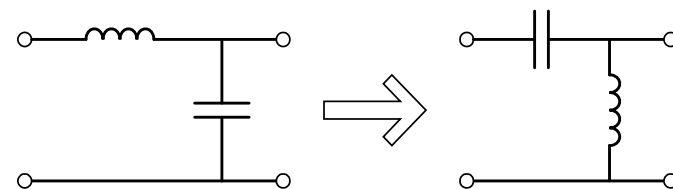


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$$S \xrightarrow{\text{LPF}} \frac{1}{S} \xrightarrow{\text{HPF}}$$



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- Stochastics:

How to select signal and noise

*Estimation and Prediction Theory*

- Gauss (1795):

Least Square Mean Concept →

Astronomy (Prediction of Satellite Orbit),

→ Gauss Distribution

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- Wiener and Kolmogorov (1940's):

Linear Prediction for **Stationary Stochastic Process** using 2-nd order statistics (Correlation Matrix)

**Generalized Harmonic Analysis**  
(Stochastic Theory + Fourier Analysis)

**Wiener-Hopf Integral Equation**  
(Semi-infinite Singular Boundary Value Problem)  
Communication + Control ⇒ **Cybernetics**

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## Wiener Filter based on Correlation Function

$$x(t) = s(t) + n(t)$$

$$y(t) = \int_0^\infty x(t-\tau)h(\tau)d\tau$$

$$\text{Min } E \left[ |y(t) - s(t)|^2 \right]$$

→ **Wiener-Hopf Equation** for  $h(\tau)$

$$\int_0^\infty [R_{ss}(\tau - \tau') + R_{nn}(\tau - \tau')] h(\tau') d\tau' = R_{ss}(\tau)$$

$R_{ss}$  = Signal – Auto – Correlation

$R_{nn}$  = Noise – Auto – Correlation

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- Kalman (1960's):

**Non-stationary** Process Prediction by using Kalman algorithm

State Space Approach, Linear System Theory, Control Theory, **Controllability**, **Observability**, Optimum Regulator, Optimum Filter, Stability, etc.

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– Godard (1974):

Learning Theory, Adaptive Equalizer for Wired Transmission  
Unknown state variables = Transmission Characteristics

– RLS (Recursive LSM) (1990):

→ Inter-symbol Interference Canceller, Multi-user Detection for Wireless Communication

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## Frequency Characteristics and Impulse Response

- Transfer Function of Linear Filter:

[Linearity + Time-Invariance]

→ Impulse response function  $h(t)$  is enough for system description.

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Output signal  $y(t)$  is given by a convolution of Input signal  $x(t)$  and Impulse response function  $h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$$

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## Linear System

- Linear Time-Invariant : Impulse Function
- Linear Periodic-variant : Multi-rate System
- Application : Band aggregation, Rate Transform

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→ Exponential time function

$\exp(at)$  = eigen-function

→ Fourier Analysis

$$Y(f) = X(f)H(f)$$

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→ Transfer Function  $H(f)$

$$H(f) = \int_{-\infty}^{\infty} h(t) \exp(-j2\pi ft) dt$$

$|H(f)|$ : Amplitude Characteristics

$\angle H(f)$ : Phase Characteristics

$-\partial \angle H(f) / \partial f$ : Delay - time Characteristics

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- Ideal Filter and Physical Realizability:  
Causality

- Ideal Low Pass Filter: Flat Amplitude, Sharp Cutoff, Linear Phase

$$H(f) = A \cdot \text{rect}\left(\frac{f}{2W}\right) \exp(-j2\pi f\tau)$$

where

$$\text{rect}(x) = \begin{cases} 1 & \text{for } |x| \leq \frac{1}{2} \\ 0 & \text{for } |x| > \frac{1}{2} \end{cases}$$

$W$ : Bandwidth    $\tau$ : delay time

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- Impulse Response : **sinc** function, equal-distance zero-crossing

$$h(t) = 2AW \frac{\sin[2\pi W(t - \tau)]}{2\pi W(t - \tau)}$$

⇒ Non-causal !

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– **Uncertainty Principle:**  $\Delta f \cdot \Delta t \geq 1/4\pi$

Impulse function ( $\Delta t \rightarrow 0$ ) has flat spectrum  
 ( $\Delta f \rightarrow \infty$ ) Sinusoidal function ( $\Delta f \rightarrow 0$ ) is widely spread ( $\Delta t \rightarrow \infty$ )  
 (cf. In Quantum Physics,  $\Delta E \cdot \Delta t \geq h/4\pi$ ,  $E$ : Energy,  $h$ : Planck constant)  
 Gaussian function is optimum with respect to the product of time spread and frequency spread;  $\Delta t \cdot \Delta f$ .

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– Finiteness of system:  
 → Transfer function is a **Rational function** of  $f$

– Causality  $\Leftrightarrow h(t) = 0$  for  $t < 0$   
 $\Leftrightarrow$  Wiener-Paley Condition

$$\int_{-\infty}^{\infty} \frac{|H(f)|^2}{1+f^2} df < \infty$$

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→ Real part,  $R(f)$

← **Hilbert Transform** → Imaginary Part,  $X(f)$

$$R(f) = -\frac{2}{\pi} \int_0^{\infty} \frac{u}{u^2 - f^2} X(u) du + R(\infty)$$

$$X(f) = \frac{2}{\pi} \int_0^{\infty} \frac{f}{u^2 - f^2} R(u) du$$

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– For Minimum Phase System:

Amplitude Characteristics  $|H(f)|$  determines Phase Characteristics  $\angle H(f)$

But when delayed waves are larger than the first arriving wave in the multi-path environment, it becomes **Non-minimum Phase**.

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## Gaussian Filter

- Transfer Function:  $H(f) = \exp(-(f/f_0)^2)$
- Impulse Response:  $h(t) = f_0 \sqrt{\pi} \exp(-(\pi f_0 t)^2)$
- Step Response:  $s(t) = 1 - \frac{1}{2} \operatorname{erfc}(\pi f_0 t)$   
where  $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-u^2) du$ :  
complementary error function
- Mono pulse ( $T$ ) response:  
 $g(t) = s(t) - s(t-T)$   
 $= \frac{1}{2} [\operatorname{erfc}(\pi f_0 t(\frac{t}{T} - 1)) - \operatorname{erfc}(\pi f_0 t(\frac{t}{T}))]$

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- For Random pulse sequence  $\{a_n\}$ ,

$$g_r(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT)$$

- **Eye pattern** is determined by  $f_0 T$   
 $f_0 T \rightarrow$  large, Good eye pattern
- Bessel Filter of 5-th order  $\approx$  Gaussian Filter  
(Maximally Flat in delay characteristics)

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## Nyquist Filter

- No interference condition at sampling time
  - Roll-off Filter
- $$R(f) = \begin{cases} 1 & \text{for } 0 \leq |fT| \leq \frac{1-\alpha}{2} \\ \frac{1}{2} \left[ 1 - \sin\left(\frac{\pi}{2\alpha} (2fT - 1)\right) \right] & \text{for } \frac{1-\alpha}{2} \leq |fT| \leq \frac{1+\alpha}{2} \\ 0 & \text{for } \frac{1+\alpha}{2} \leq |fT| \end{cases}$$

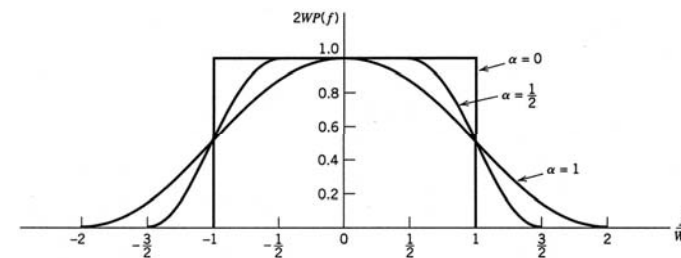
- Roll-off Response

$$r(t) = \frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\alpha t/T)}{\alpha t/T} \quad \alpha : \text{roll-off factor} (0 \leq \alpha \leq 1)$$

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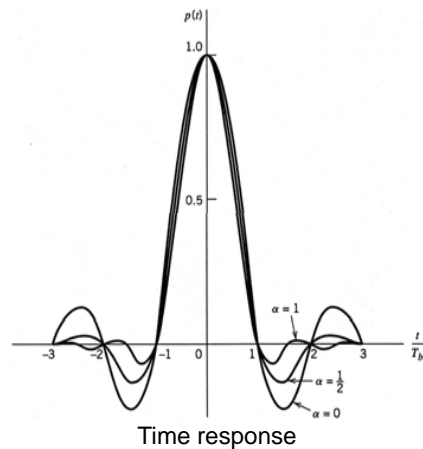


Frequency response

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## Partial Response Filter

### Controlled Interference

- Class of Partial Response Filter  
Partial Response Filter: Binary sequence  $\rightarrow$  Multi-valued sequence  $\rightarrow$  Spectrum Shaping  
Partial Response Filter:  
FIR Filter with Integer coefficient
- Similar concept: **Thomlinson-Harashima Precoding in Dirty-paper Coding**

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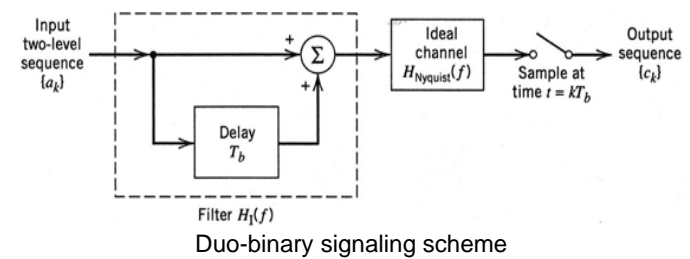
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Class	$c_0, c_1, c_2, c_3, c_4$	$H(f),  f  \leq \frac{1}{2T}$	Impulse response
1	1, 1	$2T \cos \pi f T$	$\frac{4}{\pi} \frac{\cos(\pi f T)}{1 - 4(f/T)^2}$
	1, -1	$-2T \sin \pi f T$	$\frac{8f/T}{\pi} \frac{\cos(\pi f T)}{4(f/T)^2 - 1}$
2	1, 2, 1	$4T \cos^2 \pi f T$	$\frac{2}{\pi f T} \frac{\sin(\pi f T)}{1 - (f/T)^2}$
3	2, 1, -1	$T(1 + \cos 2\pi f T + 3j \sin 2\pi f T)$	$\frac{3f/T - 1}{\pi f T} \frac{\sin(\pi f T)}{(f/T)^2 - 1}$
4	1, 0, -1	$j2T \sin \pi f T$	$\frac{2}{\pi} \frac{\sin(\pi f T)}{(f/T)^2 - 1}$
5	1, 0, 2, 0, 1	$-4T \sin^2 2\pi f T$	$\frac{8}{\pi f T} \frac{\sin(\pi f T)}{(f/T)^2 - 4}$

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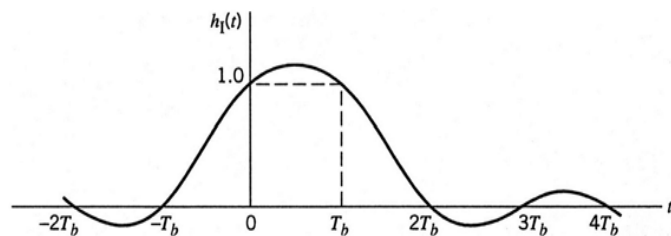


Duo-binary signaling scheme

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Impulse response of the duo-binary conversion filter

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- Error Propagation and Pre-coder

Full response system : No error propagation

Partial response system: Error propagation

Pre-coder is necessary for prevention of error propagation

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### Pre-coder in TX

Source information  $\{a_n\} \rightarrow$

Pre-coded information  $\{s_n\}$

Digital calculation (Logical calculation)

$$s_n = \sum_{i=0}^k c_i \cdot s_{n-i} \pmod{2}$$

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Pre-coded information  $\{s_n\} \rightarrow$

Partial response information  $\{g_n\}$

Analog calculation (Physical calculation)

$$g_n = \sum_{i=0}^k c_i \cdot s_{n-i}$$

Decoding in RX

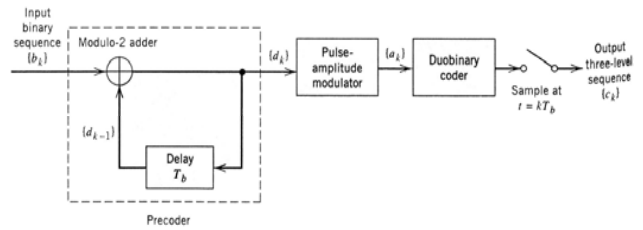
$$a_n = g_n \pmod{2}$$

Apparently, error propagation is eliminated.

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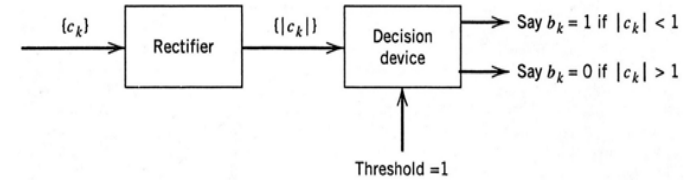
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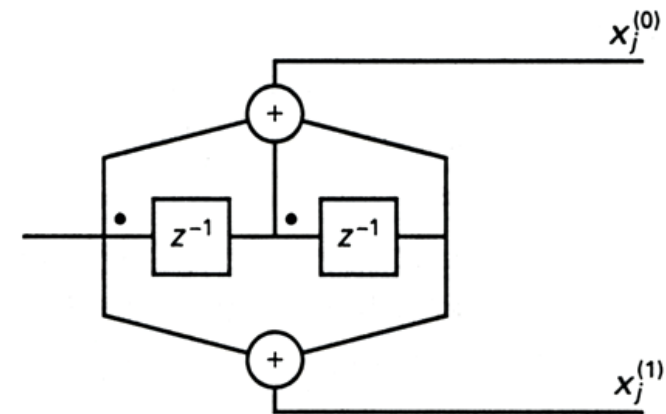
PR-VA (Partial Response & Viterbi Algorithm) is a most powerful recording method in **magnetic recording**.

Convolutional Code also utilizes a partial response in the codeword

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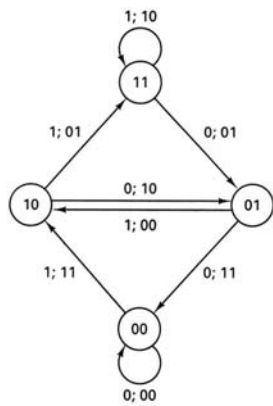
$R = \frac{1}{2}$ , nonsystematic,  $m = 2$  encoder

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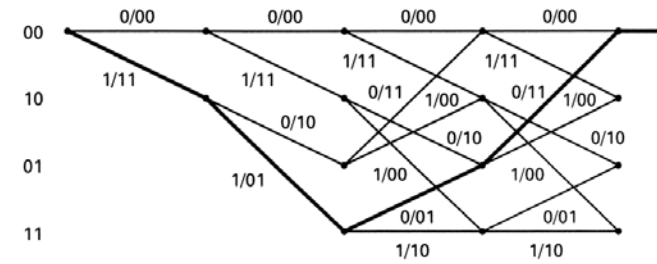


State transition diagram for  $R = \frac{1}{2}$ ,  $m = 2$  code

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Trellis for four levels,  $R = \frac{1}{2}$ ,  $v = 2$  code.

Heavy line denotes route for message  $\mathbf{u} = (11000\dots)$ .

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