

## Fading Theory

- In many circumstances, it is too complicated to describe all reflection, diffraction, and scattering processes that determine the different Multi-path Components. Rather, it is often preferable to describe the probability that a channel parameter attains a certain value.

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## Stochastic Signal Processing

- Realized value of random number is known → e.g. MRC
- Pdf of random number is known → e.g. Wiener Filtering
- Moments of random number is known → e.g. Decoupling Circuit Design

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## Deterministic vs. Stochastic

- Deterministic case : “ $x=y$ ” means  $2=2$ .
- Stochastic case : “ $x=y$ ” means “ $p(x)=p(y)$ ”.
- For example,  $x = 1-x$  holds when  $x$  is a uniform distributed random variable in the interval  $[0,1]$   
 $z$ : zero-mean Complex Gaussian Noise  
 $\therefore “z = -z^* = -z^*”$   
 $Z$ : zero-mean Complex Gaussian Independent Vector  
 $\therefore “Z = UZ”$  where  $U$ : Unitary matrix

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## Stochastic Process in Wireless Communication

- Noise (white spectrum)
- Signal (modulated bandwidth)
- Channel (Doppler frequency)
- Noise > Signal > Channel

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## Contents

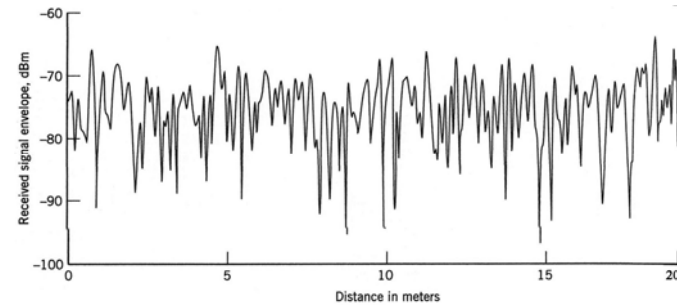
- Path Loss Formula
- Log-normal distribution
- Rayleigh/Rice distribution
- Envelope/Phase distribution
- Power Spectrum & Doppler effect
- Fading Coefficient
- MAP Estimation of Fading Channel in PHS

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- Received signal fluctuates dramatically
- Fading (Long-range, Medium-range, Short-range)



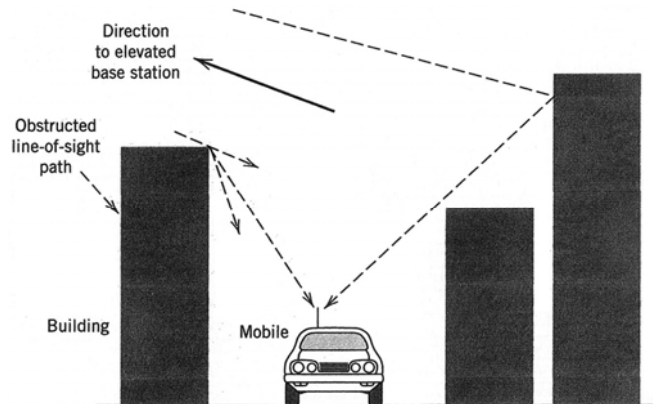
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## Mobile Communication Channel

In addition to Direct wave, there are many Reflection, Refraction and Diffraction waves.



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## Hierarchical stochastic structure

- **Path loss** : The large-scale mean itself depends on the “distance” between transmitter and receiver.
- **Log-normal** : Mean power, averaged over about 10 wavelengths, itself shows fluctuations due to “shadowing” by large objects.
- **Rayleigh and Nakagami-Rice** : On a very-short-distance scale, power fluctuates around a local mean value due to “interference” between different MPCs.

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## Path loss and Power Control

- For 3G Wireless Communication System, i.e. W-CDMA (Wideband Code Division Multiple Access) **Power Control** is used in order to alleviate “**Near-Far Problem**”.  
Dynamic Range for Power Control is required more than **74dB**.

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- **Middle** distance variation (**Log-normal** distribution: Shadowing) Median over several ten or hundred wavelengths obeys a log-normal distribution.

$$E_r = T_1 \times T_2 \times T_3 \times \cdots \times E_s$$

$E_r$  : Signal Strength at the receiver

$E_s$  : Signal Strength at the transmitter

$T_i$  : Transmission coefficient at the  $i$ -th obstacle

$$\therefore \log E_r = \log T_1 + \log T_2 + \cdots + \log E_s$$

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## Path Loss Formula

- **Land mobile electromagnetic wave propagation**  
Propagation characteristics are important in designing a cell size, a transmitter and a receiver.
  - **Long** distance variation (Okumura curve): The CCIR adopted the basic formula for the median path loss, based on Okumura's measurements.

$$L = 69.55 + 26.16 \log(f) - 13.82 \log(H_b) + [44.9 - 6.55 \log(H_b)] \log(d) + a_x(H_m)$$

$f$  : frequency in MHz

$H_b$  : Base station antenna height in meter

$d$  : Range in Km

$H_m$  : Mobile station antenna height in meter

$a_x(H_m)$  : Correction factor

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## Central Limit Theorem

- The sum of statistically independent and identically distributed random variables with finite mean and variance approaches a **Gaussian distribution** as the number of variables increases.
- **Gaussian distribution** is characterized only by mean and variance ( 2 parameters ).
- An instantaneous complex amplitude of OFDM signal can be also approximated by Gaussian variable.

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## Shadowing effect

- Typical shadowing range is around 4-10dB
- 3GPP Channel model:
  - Suburban Macro 8dB
  - Urban Macro 8dB
  - Urban Micro 10dB(NLOS) 4dB(LOS)

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## BER Performance in Rayleigh Fading Channel

- BER ( Bit Error Rate ) is proportional to an **exponential function of SNR in non-fading channel** ( AWGN channel).
- BER is proportional to an **inverse of SNR in fading channel**.
- Because SNR in fading channel is a random variable of which **PDF ( probability density function ) is an exponential function**.

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## Rayleigh Fading

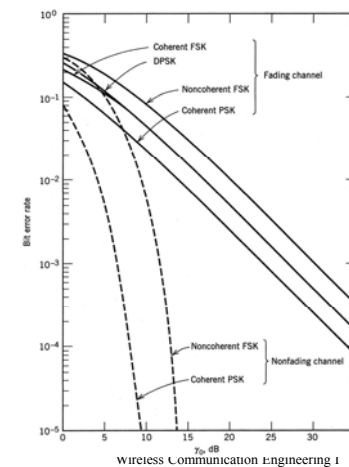
- **Short** distance variation (Rayleigh Fading) There are so many reflection and diffraction waves to generate a complicate standing wave pattern. The mobile station moves through there.

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- Fading significantly deteriorates **QoS** (i.e. bit error rate).



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## BER in Rayleigh channel

- Instantaneous BER:  $Pe(\gamma) \cong \exp(-\gamma)/2$
- Averaged BER:  $\overline{Pe(\gamma)} = \int Pe(\gamma) \times P(\gamma) d\gamma = 1/\{2(\Gamma + 1)\}$
- Pdf of SNR:  $P(\gamma) = \exp(-\gamma/\Gamma)/\Gamma$   
where  $\Gamma$  : average SNR

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## – Envelope and phase distribution

Received signal  $e(t)$  is composed of  $N$  elementary waves.

$$\begin{aligned} e(t) &= \sum_{n=1}^N e_n(t) \\ &= \text{Re} \left[ \sum_{n=1}^N z_n(t) \exp(j2\pi f_c t) \right] \\ z(t) &= \sum_{n=1}^N z_n(t) \\ &= x(t) + jy(t) \end{aligned}$$

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## Interference between Multi-path Components

### • Rayleigh Fading Model

The  $n$ -th elementary arriving wave  $e_n(t)$  at an angle of  $\phi_n$

$$e_n(t) = \text{Re}[z_n(t) \exp(j2\pi[f_c + f_D \cos(\phi_n)]t)]$$

$\text{Re}[\ ]$  : Real part complex number

$z_n(t)$  : Complex envelope =  $R_n e^{j\theta_n}$

$f_c$  : Carrier frequency

$f_D$  : Maximum Doppler frequency shift ( $= v/\lambda$ )

$v$  : Velocity of mobile station

$\lambda$  : Wavelength ( $= c/f_c$ )

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$$\begin{aligned} x(t) &: \text{In - phase component} = R(t) \cos(\theta(t)) \\ y(t) &: \text{Quadrature component} = R(t) \sin(\theta(t)) \end{aligned}$$

In the limit ( $N \rightarrow \infty$ ),  $x(t)$  and  $y(t)$  become an independent Gaussian random variable with zero mean.

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Thus, a joint pdf (probability density function) of  $x$  and  $y$

$$p(x, y) = \exp\left(-\frac{x^2 + y^2}{2b_0}\right) / 2\pi b_0$$

where  $2b_0$  : average received power =  $E[x^2 + y^2] = E[R^2]$

A joint pdf of  $R$  and  $\theta$  is

$$p(R, \theta) = \frac{R}{2\pi b_0} \exp\left(-\frac{R^2}{2b_0}\right) = p(R) p(\theta)$$

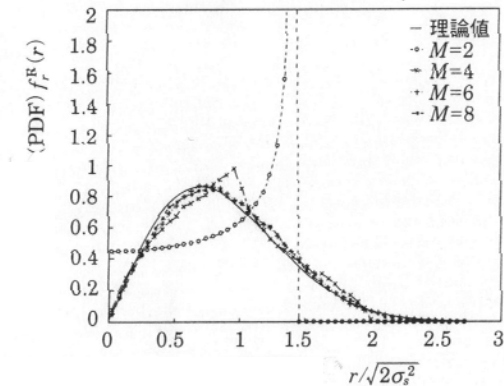
where  $R$  : envelope

$\theta$  : phase

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## Rayleigh Distribution

A pdf of envelope  $R$  is a **Rayleigh** distribution

$$p(R) = \frac{R}{b_0} \exp\left(-\frac{R^2}{2b_0}\right)$$

A pdf of phase  $\theta$  is a **uniform** distribution

$$p(\theta) = 1/2\pi$$

CNR (Carrier - to - noise ratio),  $\gamma = R^2/p_n$  is **exponential** distribution with noise power of  $p_n$

$$p(\gamma) = \frac{1}{\Gamma} \exp\left(-\frac{\gamma}{\Gamma}\right)$$

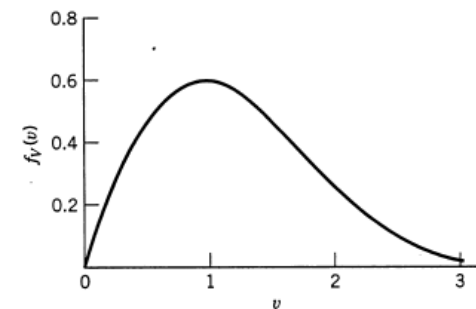
where  $\Gamma$  : Average CNR

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## Normalized Rayleigh Distribution

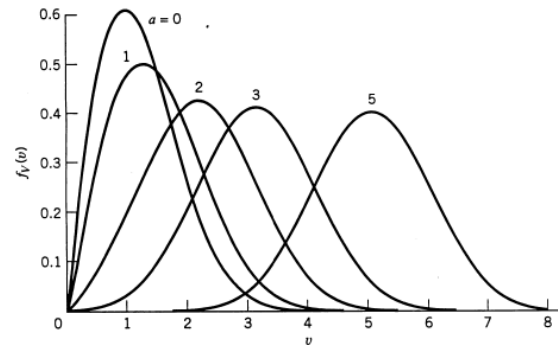


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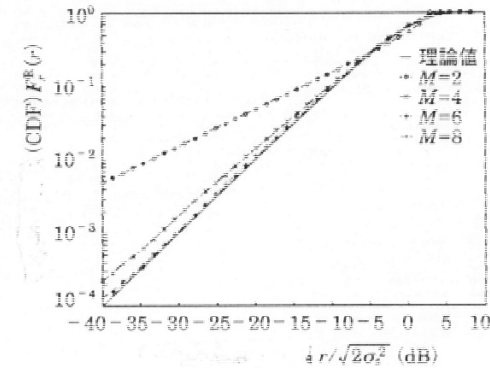
## Normalized Nakagami/Rice Distribution



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## Rician Distribution

$$f_v(v) = v \exp\left(-\frac{v^2 + a^2}{2}\right) I_0(av)$$

$I_0(\cdot)$ : 0-th Modified Bessel Function  
 $a$ : Rice factor = LOS / NLOS

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## • Power spectrum & Doppler effect

Elementary wave of arrival angle  $\phi$  has a different frequency from  $f_c$  due to the Doppler effect.

$$f = f_c + f_D \cos \phi$$

Arriving angle is uniformly distributed so that received power  $S(f)df$  in the range  $[f, f + df]$  is

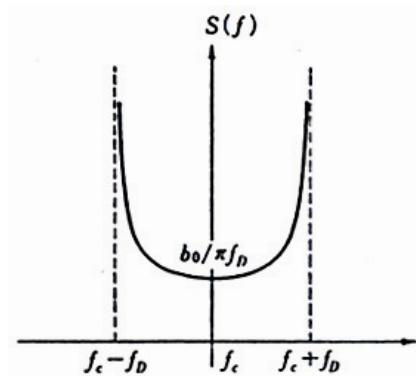
$$\begin{aligned} S(f)df &= 2 \times \frac{b_0}{2\pi} df \\ &= \frac{b_0}{\pi f_D \sqrt{1 - [(f - f_c)/f_D]^2}} df \end{aligned}$$

(cf.  $f_c = 1.5\text{GHz}$ ,  $v = 50\text{km/h}$ ,  $f_D = 135\text{Hz}$ )

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Power Spectrum

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## Joint pdf

$$\begin{aligned}
 p(x, y, \dot{x}, \dot{y}) &= \frac{1}{2\pi b_0} e^{-\frac{x^2 + y^2}{2b_0^2}} \frac{1}{2\pi b_2} e^{-\frac{\dot{x}^2 + \dot{y}^2}{2b_2^2}} \\
 p(R, \theta, \dot{R}, \dot{\theta}) &= \frac{R^2}{2\pi b_0} e^{-\frac{R^2}{2b_0^2}} \frac{1}{2\pi b_2} e^{-\frac{\dot{R}^2 + \dot{\theta}^2}{2b_2^2}} \\
 p(R, \dot{R}, \dot{\theta}) &= \frac{R^2}{b_0} e^{-\frac{R^2}{2b_0^2}} \frac{1}{\sqrt{2\pi b_2}} e^{-\frac{\dot{R}^2 + \dot{\theta}^2}{2b_2^2}} \\
 p(R, \dot{R}) &= \frac{R}{b_0} e^{-\frac{R^2}{2b_0^2}} \frac{1}{\sqrt{2\pi b_2}} e^{-\frac{\dot{R}^2}{2b_2^2}} = p(R) \cdot p(\dot{R}) \\
 p(R, \dot{\theta}) &= \frac{R^2}{b_0} e^{-\frac{R^2}{2b_0^2}} \frac{1}{\sqrt{2\pi b_2}} e^{-\frac{\dot{\theta}^2}{2b_2^2}} \\
 p(\theta) &= \frac{1}{2} \sqrt{\frac{b_0}{b_2}} \frac{1}{\left(1 + \frac{b_0}{b_2} \dot{\theta}^2\right)^{3/2}}
 \end{aligned}$$

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## Time derivative of random variables

$$dx(t)/dt = dR(t)/dt \times \cos(\theta(t)) - R(t) \times \sin(\theta(t)) \times d\theta(t)/dt$$

$$dy(t)/dt = dR(t)/dt \times \sin(\theta(t)) + R(t) \times \cos(\theta(t)) \times d\theta(t)/dt$$

$$pdf(x, y, dx/dt, dy/dt) \rightarrow pdf(R, \theta, dR/dt, d\theta/dt)$$

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## • Level crossing number & Fade duration

They are important parameters for mobile communication quality.

### – Level crossing number

$\dot{R}$  : time derivative of envelope  $R$

A joint pdf of  $R$  and  $\dot{R}$ ,  $p(R, \dot{R})$  is

$$p(R, \dot{R}) = \frac{R}{b_0} \exp\left[-\frac{R^2}{2b_0^2}\right] \frac{1}{\sqrt{2\pi b_2}} \exp\left[-\frac{\dot{R}^2}{2b_2^2}\right]$$

Level crossing number of envelope per unit time  $N(R_s)$  at the level  $R_s$

$$N(R_s) = \int_0^\infty \dot{R} \cdot P(R_s, \dot{R}) d\dot{R}$$

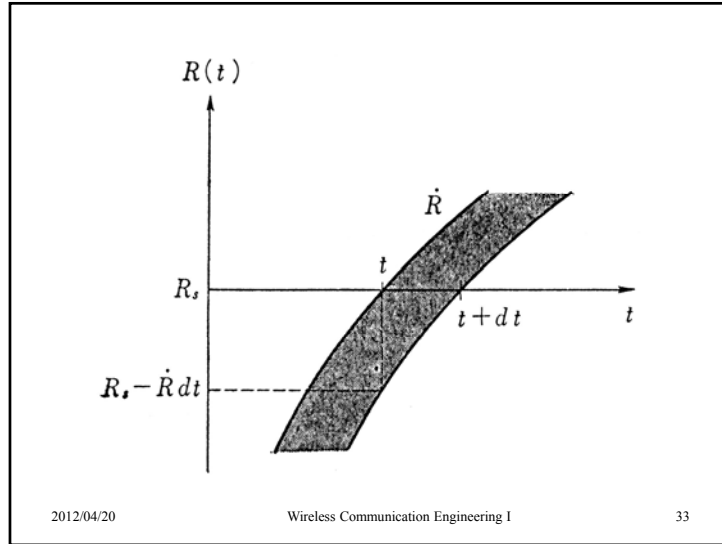
$$\text{where } b_2 = E[\dot{R}^2]$$

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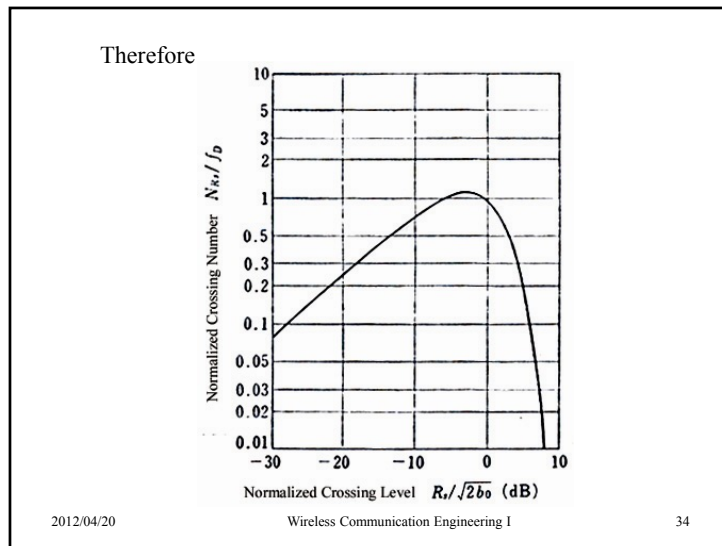
$$N(R_s) = \sqrt{\frac{b_2}{\pi b_0}} \frac{R_s}{\sqrt{2b_0}} \exp\left[-\frac{R_s^2}{2b_0}\right]$$

$$= \sqrt{2\pi} f_D \frac{R_s}{\sqrt{2b_0}} \exp\left[-\frac{R_s^2}{2b_0}\right]$$

$$N(\sqrt{b_0})_{\max} = f_D \sqrt{\pi/e}$$

$$2\pi f_0 = \sqrt{2b_2/b_0}$$

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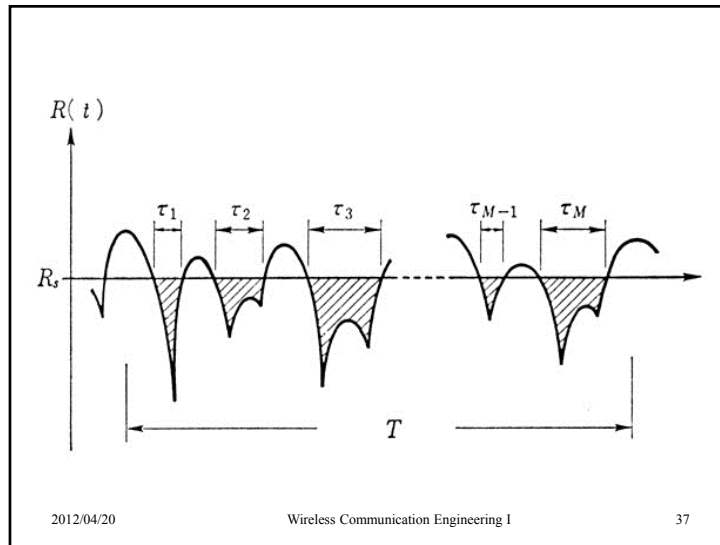
– Average fade duration time at the level  $R_s$ ,  $\bar{\tau}$

$$\bar{\tau} = \frac{\Pr[R(t) \leq R_s]}{N(R_s)}$$

$$= \frac{\sqrt{2b_0}}{\sqrt{2\pi} f_D R_s} \left[ \exp\left(\frac{R_s^2}{2b_0}\right) - 1 \right]$$

(cf. When  $R_s / \sqrt{2b_0} = 0.1$  (20dB down),  $f_c = 1.5\text{GHz}$ ,  $v = 50\text{km/h}$ ,  $\bar{\tau} = 2\text{ms}$ )

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- Random FM noise

$\theta(t)$  fluctuates randomly  $\rightarrow$  FM noise

A pdf of  $\dot{\theta}$ ,  $p(\dot{\theta})$  is

$$p(\dot{\theta}) = \frac{1}{2} \sqrt{\frac{b_0}{b_2}} \left[ 1 + \frac{b_0}{b_2} \dot{\theta}^2 \right]^{-3/2}$$

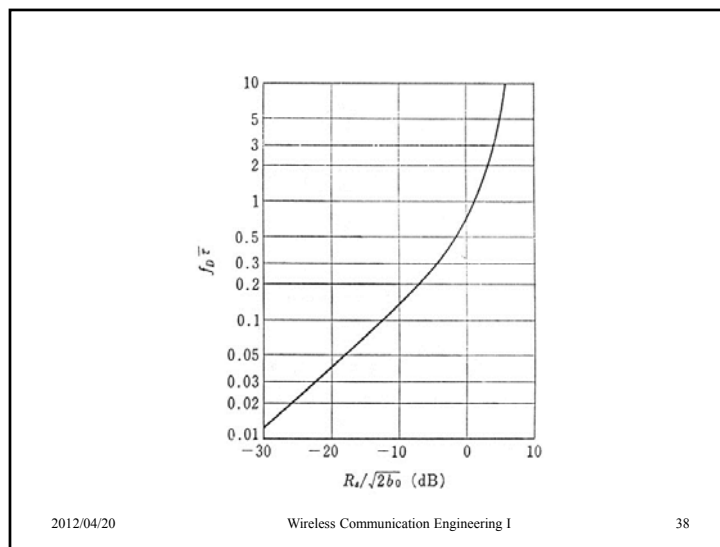
Random FM noise is independent on average received power.

This determines a lower bound of bit error rate.

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- **Fading correlation**

The correlation characteristics are necessary for the design of diversity system.

– Time correlation

$$\rho(\tau) = \frac{E[z^*(t)z(t+\tau)]}{E[z(t)^*z(t)]} = J_0(2\pi f_D \tau)$$

$J_0(\ )$ : 0 - th order Bessel function of the first kind

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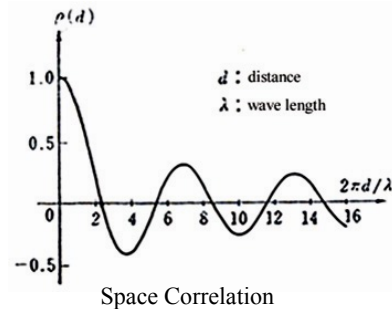
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### – Space correlation

Space distance  $d = v\tau$

$$\rho(d) = J_0(2\pi d/\lambda)$$

Around half wavelength spacing ( $d \sim \lambda/2$ )  $\rightarrow$  no correlation



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### – Frequency correlation

This is important parameter for Wide-band transmission.

$$\rho(\Omega) = \frac{1}{1 + j2\pi\Omega(\delta\ell/c)} \exp(j2\pi\ell_0/c)$$

$\ell_0$  : minimum path length

$\delta\ell$  : deviation in path length

(cf. For  $\delta\ell = 200\text{m}$ , coherent bandwidth is 400kHz)

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## MIMO Transmission and Antenna correlation

- Antenna correlation decreases MIMO channel capacity if average SNR at RX antenna is equal to each other.

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## A MAP Estimation of Rayleigh Fading Channel

– A Filter Theory of Complex Gaussian Process –  
and Its Application to PHS SDMA

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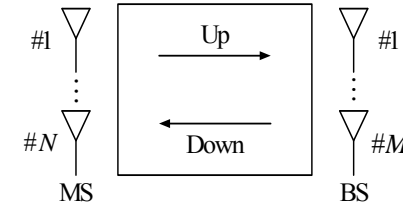
- Background & Motivation
- Complex Gaussian Stochastic Process
- Noisy Rayleigh Fading Channel
- MAP Estimation of Channel Transfer Coefficient
- Numerical Results
- Conclusion
- Future Work

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- Mobile Communication Channel with MIMO Systems  
– Time Variant Linear Reciprocal System



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## Background & Motivation

- Recursive Simulation Method for Rayleigh Fading Channel.
  - How to write a computer program ?
- Fading Channel Coefficients should be estimated in SDMA PHS Systems

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For  $(N + M)$ -port Circuit, a  $(N + M) \times (N + M)$  scattering matrix  $S$  is defined;

$$S(f, t) = \begin{bmatrix} \overset{\leftarrow N \rightarrow}{S_{MM}} & \overset{\leftarrow M \rightarrow}{S_{BM}} \\ S_{MB} & S_{BB} \end{bmatrix} \begin{matrix} \uparrow N \\ \downarrow M \end{matrix}$$

where

$S_{BM} : M \times N$  Transfer Matrix of Up-Link from MS to BS

$S_{MB} : N \times M$  Transfer Matrix of Down-Link from BS to MS

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By the reciprocity,

$$S = S^t$$

$$\therefore S_{MB}(f, t) = S_{BM}(f, t)^t$$

Thus, the **Down-Link** Transfer Characteristics can be determined by the **Up-Link** one.

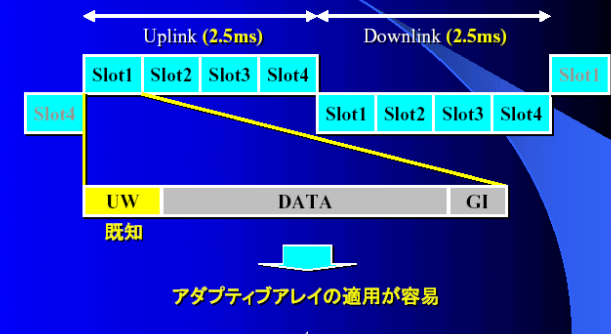
The above equality, however, holds only for the same **frequency and time**.

## 1.0 PHSとアダプティブアレイの親和性

SANYO

【PHSのフレーム構成】

・上り4スロット、下り4スロットのTDMA/TDD



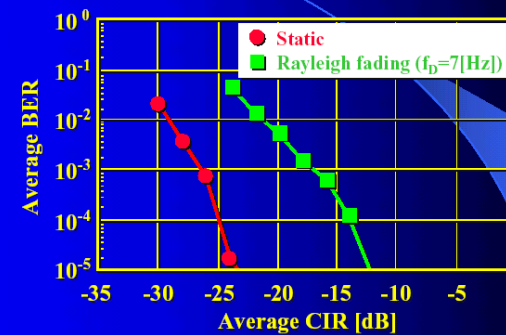
## PHS system

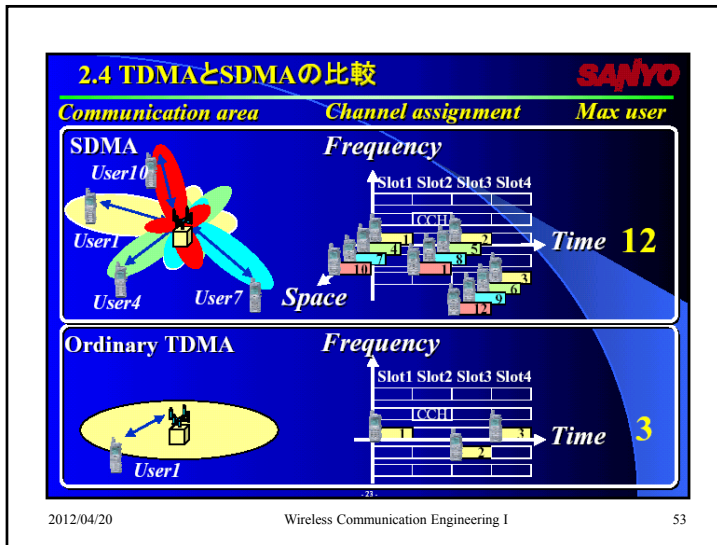
- TDD ( Time Domain Duplex )
- TDMA ( Time Domain Multiple Access )
- 4 Time Slot Segmentation
- Introduction of SDMA increases a channel capacity by 3 times or more.
- At the PHS base station, 4 antennas are installed.
- At most 4 data streams can be transmitted simultaneously by pre-coding at BS for down link.
- The idea is used in “i-Burst” system (IEEE802.20)

## 1.6.2 アダプティブアレイ基地局の特性

SANYO

CIR 対 BER特性





## Complex Gaussian Stochastic Process

- 1) Rayleigh (or Rice) Fading Coefficient :  $x(t)$
- 2) Random White Gaussian Noise :  $y(t)$
- ↓
- 3) Rayleigh Fading Coefficient contaminated with Noise:

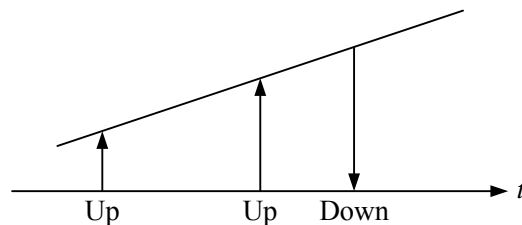
$$Z(t) = x(t) + y(t)$$

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- Conventionally
  - Linear Extrapolation for Channel coefficient is used.



- Noise Filtering is not taken into account.

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Stationary Gaussian Process can be characterized only by Autocorrelation Function

$$\begin{aligned} R_{ZZ}(\tau) &= \overline{Z(t)Z(t+\tau)} \\ &= R_{XX}(\tau) + R_{YY}(\tau) \end{aligned}$$

where

$$R_{XX}(\tau) = A J_0(2\pi f_D \tau)$$

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$A = \overline{|X(t)|^2}$  : Average Fading Level

$J_0$  : 0th Order Bessel Function of First Kind

$f_D$  : Maximum Doppler Frequency ( $= f_c \frac{v}{c}$ )

$f_c$  : Carrier Frequency

$v$  : velocity of MS

$c$  : velocity of Light

$$R_{YY}(\tau) = \begin{cases} N & (\tau = 0) \\ 0 & (\tau \neq 0) \end{cases}$$

$N = \overline{|Y(t)|^2}$  : Average Noise Level

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- MAP (LS) Estimation and Optimal Noise Reduction – Wiener-Hopf Equation

Optimal Linear Combination Estimator Vector :  $\mathbf{b}$

$$\begin{bmatrix} 1 + \frac{N}{A} & J_0(2\pi f_D(t_1 - t_0)) & \cdots & J_0(2\pi f_D(t_{n-1} - t_0)) \\ & \ddots & & \\ & & 1 + \frac{N}{A} \end{bmatrix} \mathbf{b} = \begin{bmatrix} J_0(2\pi f_D(t_n - t_0)) \\ \vdots \\ J_0(2\pi f_D(t_n - t_{n-1})) \end{bmatrix}$$

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For MAP Estimation, **Cross-correlation** Function is also needed

$$\begin{aligned} R_{ZX}(\tau) &= \overline{Z(t)X(t+\tau)} = \overline{(X(t) + Y(t))X(t+\tau)} \\ &= \overline{X(t)X(t+\tau)} = R_{XX}(\tau) \end{aligned}$$

$\therefore X(t)$  and  $Y(t)$  are independent.

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MAP Estimator for  $x(t_n)$

$$x(t_n)_{\text{MAP}} = \mathbf{b}^T \mathbf{z}$$

where

$\mathbf{z} = (z(t_0), \dots, z(t_{n-1}))$  : Observed Noisy Data

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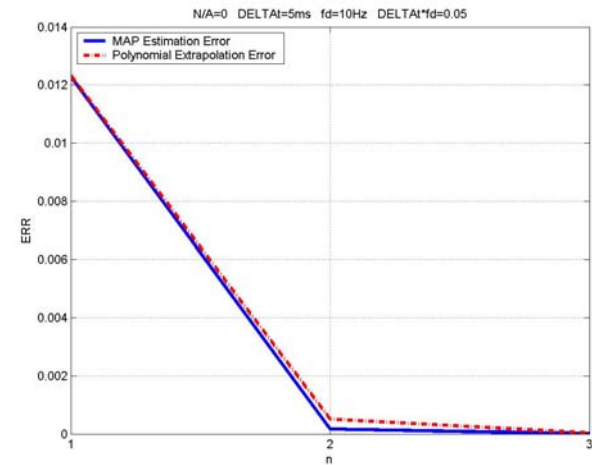
## Numerical Results

- (1) NoiseLevel :  $N/A = 0, 0.1, 1$   
 (2) DopplerFrequency:  $f_D = 10, 40[\text{Hz}]$   
 (3) No. of Data :  $n = 1, 2, 3$

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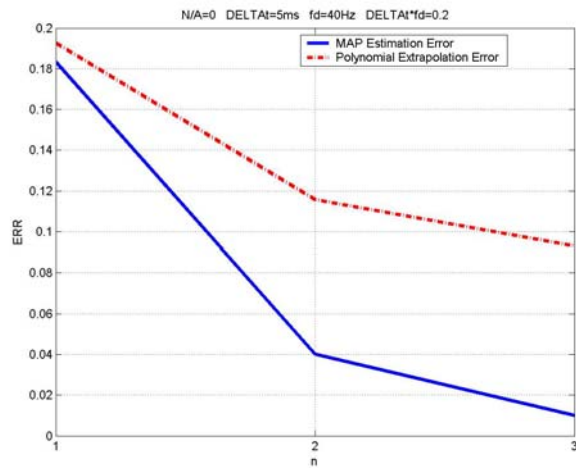
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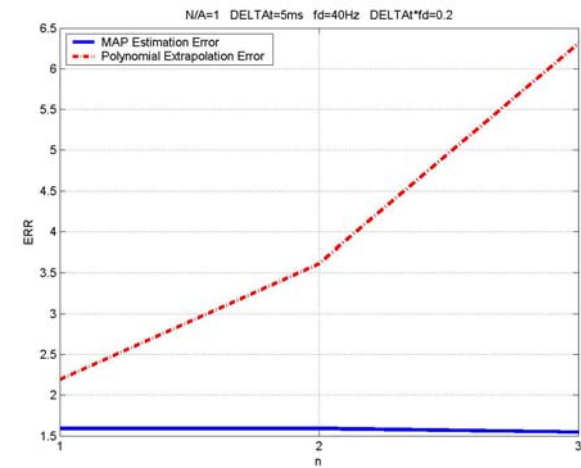
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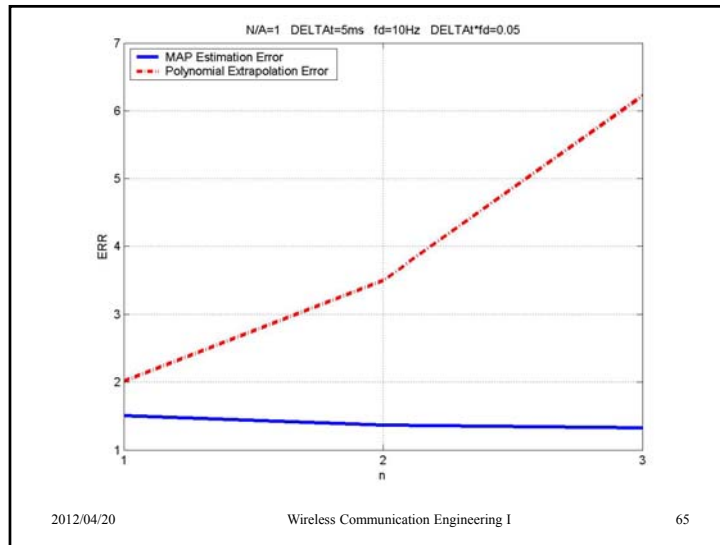


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## **Conclusion**

- Estimation of Fading Coefficient is useful for TDMA/ TDD.
- Conventional Estimation is not satisfactory.
- Estimation Error can be greatly reduced by MAP Estimation.