7.3 Transportation Simplex Method

How to pivot

Step 1 Determine the variables that should enter the basis.

Step 2 Find the loop involving the entering variable and some of the basic variables.

Step 3 Counting only cells in the loop. Label *even cells* and *odd cells*.

Step 4 Find the odd cell whose variable assume the smallest value. To perform the pivot, decrease the value of each odd cell and increase the value of each even cell.

0	1	2	3	4	5
(1,4) –	(3,4) –	(3,3) –	(2,3) –	(2,1) –	(1,1)

35			*	35
10	20	**20**		50
		10	30	40
45	20	30	30	

35-20			0+20	35
10+20	20	20-20		50
		10+20	30-20	40
45	20	30	30	

How to determine whether a bfs is optimal
How to determine which nonbasic variable should enter the basis

1. First supply constraint dropped ($u_1 = 0$)
2. $-u_i$: shadow price of the i th supply constraint

   $-v_j$: shadow price of the j th demand constraint
3. $c_{ij} - u_i - v_j \geq 0 \;\; or \;\; u_i + v_j - c_{ij} \leq 0$ for all nonbasic variables

   Then current bfs is optimal
4. Otherwise nonbasic variables with most positive value of $u_i + v_j - c_{ij}$ should enter the basis

35	8		6		10		9	35
10	9	**20**	12	**20**	13		7	50
	14		9	**10**	16	**30**	5	40
45		20		30		30		

$u_i + v_j - c_{ij} = 0$

for all BV

$u_1 = 0$

$u_1 + v_1 = 8$

$u_2 + v_1 = 9$

$u_2 + v_2 = 12$

$u_2 + v_3 = 13$

$u_3 + v_3 = 16$

$u_3 + v_4 = 5$

$\bar{c}_{ij} = u_i + v_j - c_{ij}$

$\bar{c}_{12} = u_1 + v_2 - c_{12}$

$= 0 + 11 - 6 = 5$

$\bar{c}_{13} = 2, \bar{c}_{14} = -8$

$\bar{c}_{24} = -5, \bar{c}_{31} = -2$

$\bar{c}_{32} = 6$

How to determine whether a bfs is optimal

1

35	8		6		10		9	35
10	9	**20**	12	**20**	13		7	50
	14		9	**10**	16	30	5	40
45		20		30		30		

2

	v_j								
	8		11		12		7		
0	**35**	8		6		10		9	35
u_i 1	**10**	9	**10**	12	**30**	13		7	50
-2		14	**10**	9		16	**30**	5	40
	45		20		30		30		

3

	v_j								
	8		6		12		2		
0	**25**	8	**10**	6		10		9	35
u_i 1	**20**	9		12	**30**	13		7	50
3		14	**10**	9		16	**30**	5	40
	45		20		30		30		

4

	v_j								
	6		6		10		2		
0		8	**10**	6	**25**	10		9	35
u_i 3	**45**	9		12	**5**	13		7	50
3		14	**10**	9		16	**30**	5	40
	45		20		30		30		

$$
\begin{aligned}
z &= 8x_{11} + 6x_{12} + 10x_{13} + 9x_{14} + 9x_{21} + 12x_{22} + 13x_{23} + 7x_{24} \\
&\quad + 14x_{31} + 9x_{32} + 16x_{33} + 5x_{34} \\
&= 6(10) + 10(25) + 9(45) + 13(5) + 9(10) + 5(30) \\
&= 1020
\end{aligned}
$$

Solution of Exercise

1

	7	3	6	4					
0	**12**	7	**6**	3		2		10	18
0		9	**14**	3	**8**	6		8	22
2		8		7	**8**	8	**18**	6	26
	12	20	16	18					

$\bar{c}_{13}=4, \bar{c}_{14}=-6, \bar{c}_{21}=-2, \bar{c}_{24}=-4, \bar{c}_{31}=1, \bar{c}_{32}=-2$

2

	7	-1	2	0					
0	**12**	7		3	**6**	2		10	18
4		9	**20**	3	**2**	6		8	22
6		8		7	**8**	8	**18**	6	26
	12	20	16	18					

$\bar{c}_{12}=-4, \bar{c}_{14}=-10, \bar{c}_{21}=2, \bar{c}_{24}=-4, \bar{c}_{31}=5, \bar{c}_{32}=-2$

3

	7	-1	2	5					
0	**4**	7		3	**14**	2		10	18
4		9	**20**	3	**2**	6		8	22
1	**8**	8		7		8	**18**	6	26
	12	20	16	18					

$\bar{c}_{12}=-4, \bar{c}_{14}=-5, \bar{c}_{21}=2, \bar{c}_{24}=1, \bar{c}_{32}=-7, \bar{c}_{33}=-5$

4

	7	1	2	5					
0	**2**	7		3	**16**	2		10	18
2	**2**	9	**20**	3		6		8	22
1	**8**	8		7		8	**18**	6	26
	12	20	16	18					

$\bar{c}_{12}=-2, \bar{c}_{14}=-5, \bar{c}_{23}=-2, \bar{c}_{24}=-1, \bar{c}_{32}=-5, \bar{c}_{33}=-5$

All negative

$$z = 7x_{11} + 3x_{12} + 2x_{13} + 10x_{14} + 9x_{21} + 3x_{22} + 6x_{23} + 8x_{24}$$
$$+ 8x_{31} + 7x_{32} + 8x_{33} + 6x_{34}$$
$$= 7(2) + 2(16) + 9(2) + 3(20) + 8(8) + 6(18) = 296$$

7.5 Assignment Problems

Machine/Job	1	2	3	4
1	14	5	8	7
2	2	12	6	5
3	7	8	3	9
4	2	4	6	10

$x_{ij} = 1$: if machine i is assigned to meet the demand of job j

$x_{ij} = 0$: if machine i is not assigned to meet the demand of job j

$$\min z = 14x_{11} + 5x_{12} + 8x_{13} + 7x_{14} + 2x_{21} + 12x_{22} + 6x_{23} + 5x_{24}$$
$$+ 7x_{31} + 8x_{32} + 3x_{33} + 9x_{34} + 2x_{41} + 4x_{42} + 6x_{43} + 10x_{44}$$

s.t. $x_{11} + x_{12} + x_{13} + x_{14} = 1$ Machine constraints

$\ldots$

$x_{11} + x_{21} + x_{31} + x_{41} = 1$ Job constraints

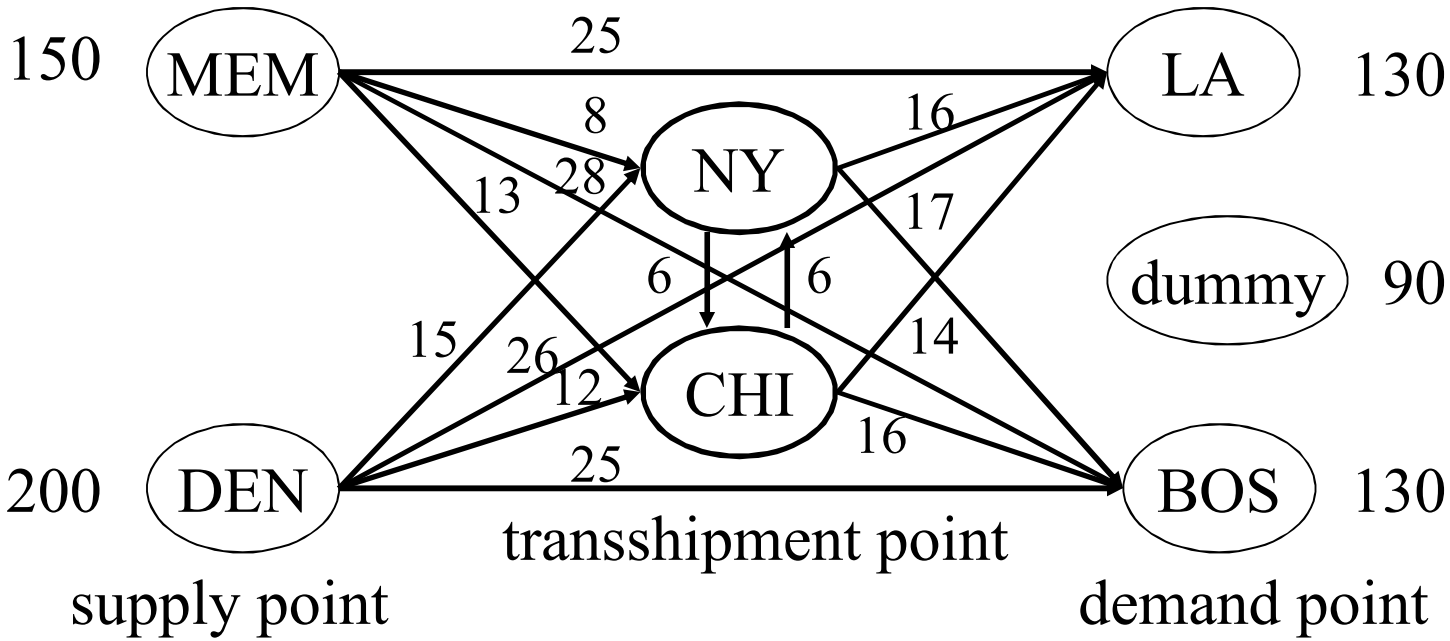
$\ldots$

$x_{ij} = 0$ or $x_{ij} = 1$

<u>Assignment Problem</u>

All supplies and demands = 1

7.6 Transshipment Problems



Total Supply excess total demand

$150 + 200 \geq 130 + 130$

dummy demand point

Optimal Tableau

	NY		CHI		LA		BOS		Dum		
MEM	130	8		13		25		28	20	0	150
DEN		15		12		26	130	25	70	0	200
NY	220	0		6	130	16		17		0	350
CHI		6	350	0		14		16		0	350
	350		350		130		130		90		

Find bfs Minimum cost method

	14		5		8		7	1
	2		12		6		5	1
	7		8		3		9	1
1	2		4		6		10	+
+		1		1		1		

	14		5		8		7	1
	2		12		6		5	1
	7		8	**1**	3		9	+
1	2		4		6		10	+
+		1		+		1		

	14		5		8		7	1
	2		12		6	**1**	5	+
	7		8	**1**	3		9	+
1	2		4		6		10	+
+		1		+		+		

	14	**1**	5		8		7	+
	2		12		6	**1**	5	+
	7		8	**1**	3		9	+
1	2		4		6		10	+
+		+		+		+		

Determine whether a bfs is optimal

	3		5		7		7
0		14	**1**	5		8		7
-2		2		12		6	**1**	5
-4		7		8	**1**	3		9
-1	**1**	2		4		6		10

Z =5+5+3+2=15

Combinatorial Optimization Problems

One of Integer Programming