光画像工学
Optical imaging and image processing (III)

学術国際情報センター
山口雅浩

E-mail: yamaguchi.m.aa@m.titech.ac.jp
http://guchi.gsic.titech.ac.jp

2.6 2D Discrete Fourier transform

2D DFT

$$F[k,l] = \boldsymbol{DFT}\{f[m,n]\} = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f[m,n]\exp\{-j2\pi(\frac{mk}{M}+\frac{nl}{N})\}$$

Inverse 2D DFT

$$f[m,n] = \boldsymbol{DFT}^{-1}\{F[k,l]\} = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} F[k,l]\exp\{j2\pi(\frac{mk}{M}+\frac{nl}{N})\}$$

2D Fourier transform in continuous space

$$F(u,v) = \iint f(x,y)\exp\{-j2\pi(ux+vy)\}dxdy$$

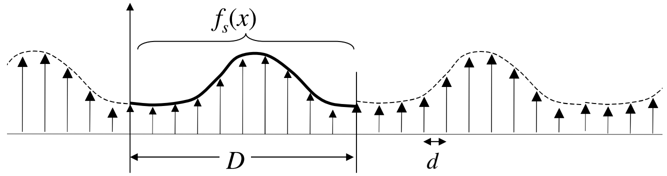
Inverse 2D Fourier transform in continuous space

$$f(x,y) = \iint F(u,v)\exp\{j2\pi(ux+vy)\}dudv$$

Sampling and periodicity in DFT, Number of pixels = N

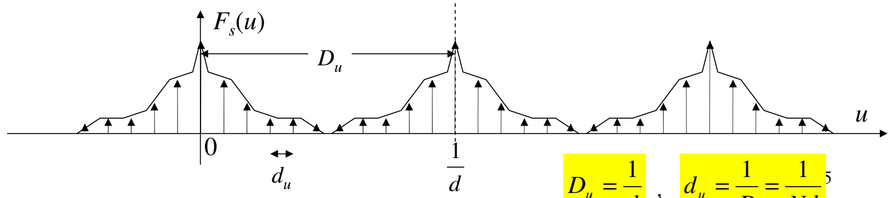
Spatial domain:

Sampling pitch (intervals of delta functions) = d,
Image size (period) = D $\longrightarrow$ $N = \frac{D}{d}$



Fourier domain (Frequency domain):

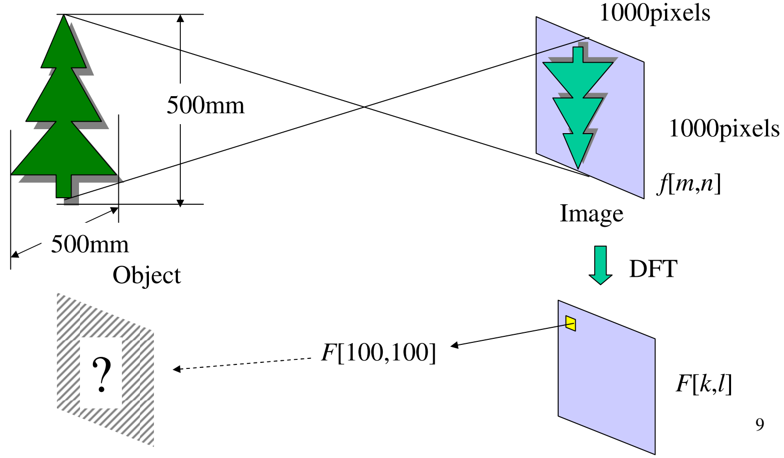
Sampling pitch (intervals of delta functions) = d_u,
Period in frequency domain = D_u $\longrightarrow$ $N = \frac{D_u}{d_u}$



$D_u = \frac{1}{d}$, $d_u = \frac{1}{D} = \frac{1}{Nd}$

練習問題4

Consider a sampled image $f[m,n]$ in 1000×1000 pixels. The image size corresponds to 500×500 [mm] of the object. Let $F[k,l]$ be the 2D DFT of $f[m,n]$. Draw the shape of the sinusoidal component on the object domain, corresponding to the Fourier coefficient $F[100,100]$. Derive the periods and the spatial frequencies with the units [mm] and [cycles/mm] as well.



2.7 Fourier analysis of linear shift-invariant imaging system

- 2-D linear system in continuous space

$$g(x,y)=\iint h(x,y;x',y')f(x',y')dx'dy'$$

- Shift-invariant (space-invariant)

$$g(x,y)=\iint h(x-x',y-y')f(x',y')dx'dy'$$
$$=f(x,y)*h(x,y)$$

→ Convolution

$h(x, y)$: Impulse response, point spread function (PSF)　インパルス応答 点像分布関数

- 2-D linear shift-invariant imaging system with additive noise

$$g(x,y)=\iint_{-\infty}^{\infty} h(x-x',y-y')f(x'y')dx'dy'+n(x,y)$$

- Fourier transform of 2-D shift-invariant imaging system

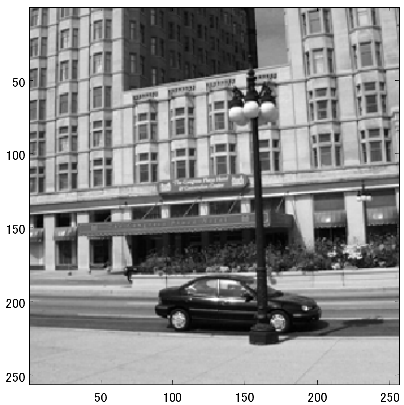
$$G(u,v)=H(u,v)F(u,v)$$
$$G(u,v)=H(u,v)F(u,v)+N(u,v)$$

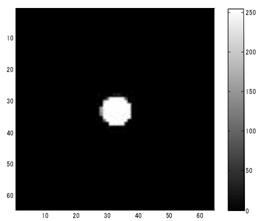
$H(u,v)$　: Transfer function　伝達関数、周波数特性

2-D Linear, shift-invariant system in discrete space

Discrete convolution　離散たたみ込み

$$g[m,n]=\boldsymbol{S}\{f[m,n]\}=\sum_{m',n'}h[m-m',n-n']f[m',n']$$
$$=h[m,n]*f[m,n]$$





$h_D[m,n]$

$M_h \times N_h$ pixels

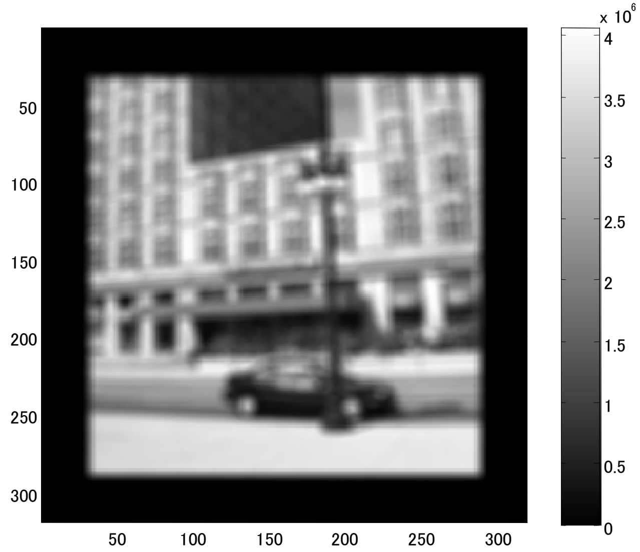
Discrete signals within a finite region

$f_D[m,n]$

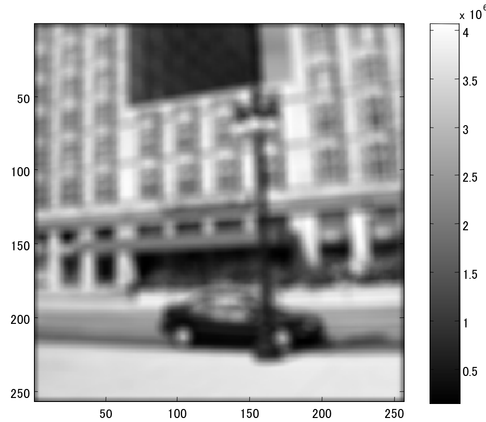
$M \times N$ pixels

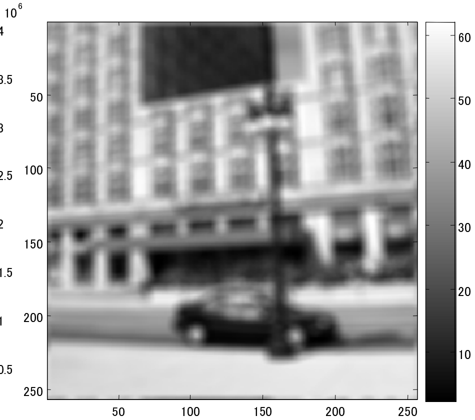
imagesc(img);
imagesc(ci);

- cres = conv2(double(img), double(ci));
- imagesc(cres);



- cres = conv2(double(img), double(ci), 'same');
- imagesc(cres);
- imagesc(real(res));





Modulation transfer function

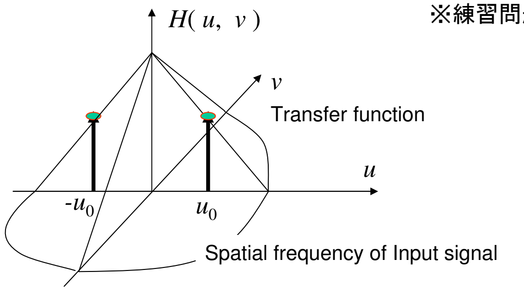
Transfer function : H
Modulation transfer function : | H |
Phase transfer function: arg{ H }

MTFの求め方（1）

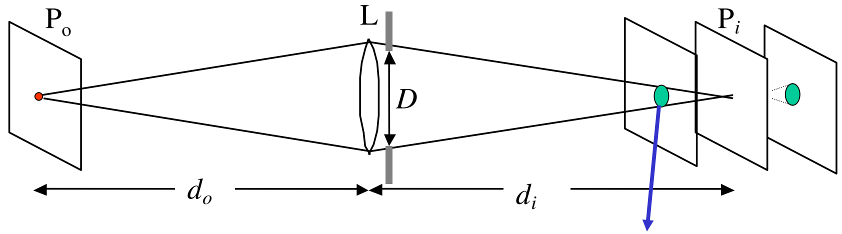
入力信号 $A_I + A_I \cos 2\pi u_0 x$
出力信号 $A_I' + A_O \cos(2\pi u_0 x + \phi)$

$$H(u_0,\ 0) = \frac{A_O}{A_I}$$

※練習問題：上式が正しいことを確かめよ

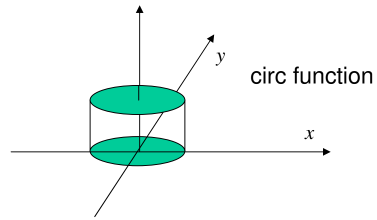


Impulse response of a defocused optical imaging system



- Incoherent model
  (Geometrical model)

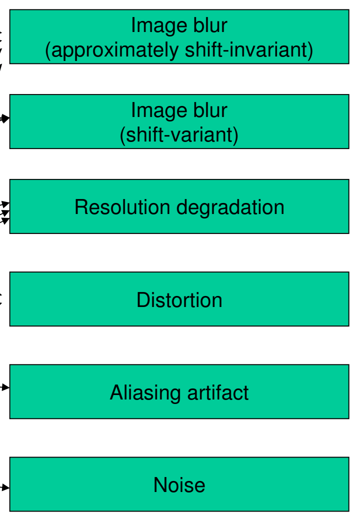
  2D object: (Approximately) shift-invariant
  3D object: Shift-variant



Coherent (wave optics model) case: see “diffraction and image formation.”

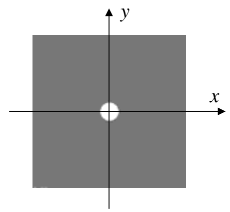
Causes of image degradation
画像劣化とその原因

- Defocus 焦点はずれ
- Lens aberration 収差
  - Spherical aberration
  - Coma
  - Astigmatism
  - Curvature of field
  - Distortion
- Diffraction limit 回折限界
- Sampling aperture サンプリング開口
- Sampling サンプリング
- Distortion 歪み
- Noise ノイズ

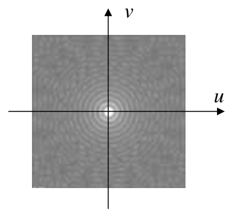


Transfer function of a defocused optical imaging system

- $\boldsymbol{F}\{ \mathrm{circ}(r) \} = J_1(2\pi\rho) / \rho$
  (Fourier-Bessel transform)
  - J_1 : Bessel function of the first kind, order 1.



Impulse response
Point spread function



Transfer function

Size	Large	Small
	Small	Large