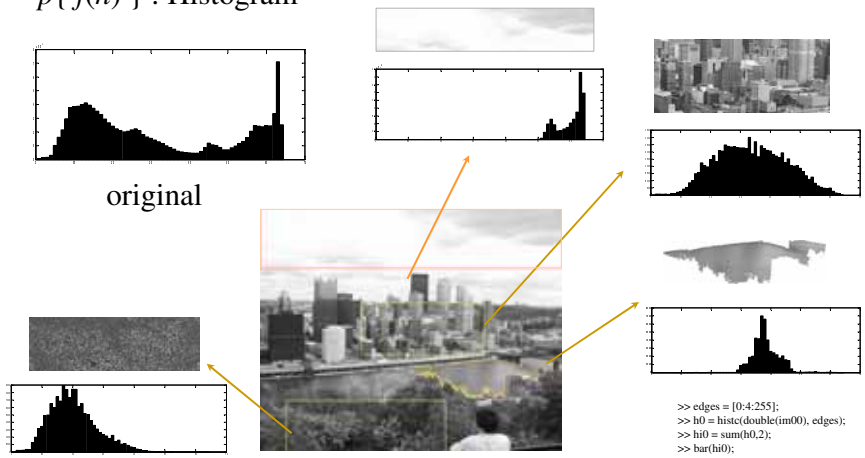
光画像工学
Optical imaging and image processing (II)

$p\{\, f(n) \,\}$: Histogram

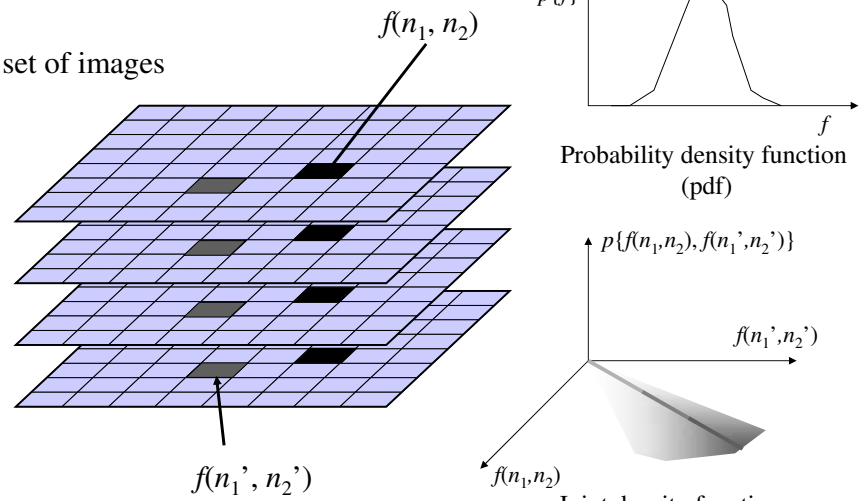



```
im00 = imread('C:¥・・・¥test-histogram-org.png');
im01 = imread('C:¥・・・¥test-histogram-01.png');
im02 = imread('C:¥・・・¥test-histogram-02.png');
im03 = imread('C:¥・・・¥test-histogram-03.png');
im04 = imread('C:¥・・・¥test-histogram-04.png');
```

```
>> edges = [0:4:255];
>> h0 = histc(double(im00), edges);
>> hi0 = sum(h0,2);
>> bar(hi0);
>> h1 = histc(double(im01), edges);
>> hi1 = sum(h1,2);
>> bar(hi1);
>> h2 = histc(double(im02), edges);
>> hi2 = sum(h2,2);
>> bar(hi2);
>> h3 = histc(double(im03), edges);
>> hi3 = sum(h3,2);
>> bar(hi3);
>> h4 = histc(double(im04), edges);
>> hi4 = sum(h4,2);
>> bar(hi4);
```

1.4.5 Statistical characterization of images

A set of images



Additive noise: $g = f + n$

$\langle n^2 \rangle = \sigma^2$

Uncorrelated noise: $\langle n \cdot f \rangle = 0$, $\langle n_k \cdot n_l \rangle = 0$ for $k \neq l$

$$\bar{g}_K = \frac{1}{K}\sum_{k=1}^{K} g_k = f + \frac{1}{K}\sum_{k=1}^{K} n_k$$

$$\langle \bar{n}_K^2 \rangle = \langle (\bar{g}_K - f)^2 \rangle = \left\langle \left(\frac{1}{K}\sum_{k=1}^{K} n_k \right)^2 \right\rangle = \frac{1}{K^2}\sum_{k=1}^{K} \langle (n_k)^2 \rangle = \frac{\sigma^2}{K}$$

$g_k = f + n_k$

Original | Additive noise | Average of two images

Average of 10 images | Average of 20 images

```
bim = imread('C:¥・・・¥Momiji-small.png');
imshow(bim);
im = double(bim);
for i = 1:20
  for k=1:3
    n=randn([256,320]);
    noise(:,:,k)=(n-0.5)*25.5;
    imn(:,:,:,i) = im + noise;
end
end
bimn = uint8(imn);
imshow(bimn(:,:,:,1));
imm2 = (imn(:,:,:,1)+imn(:,:,:,2))/2;
imshow(uint8(imm2));
imm10 = mean( imn(:,:,:,[1:10]), 4 );
imshow(uint8(imm10));
imm20 = mean( imn(:,:,:,[1:20]), 4 );
imshow(uint8(imm20));
```

2. Image detection and digitization
2. 画像の検出とデジタル化

2.1 Image sampling
2.1 画像のサンプリング

- Mathematical expression of image sampling

  $f(x, y)$: Original image

  $f_s(x, y)$: Sampled image

  $f[m, n]$: two-dimensional discrete signal. (m, n : integer)

  – The sampling interval in x and y directions : d_x, d_y

  – Equidistance sampling

  $f[m, n] = f(md_x, nd_y)$

$$f_s(x,y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} f[m,n]\delta(x-md_x, y-nd_y)$$

$$= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} f(md_x, nd_y)\delta(x-md_x, y-nd_y)$$

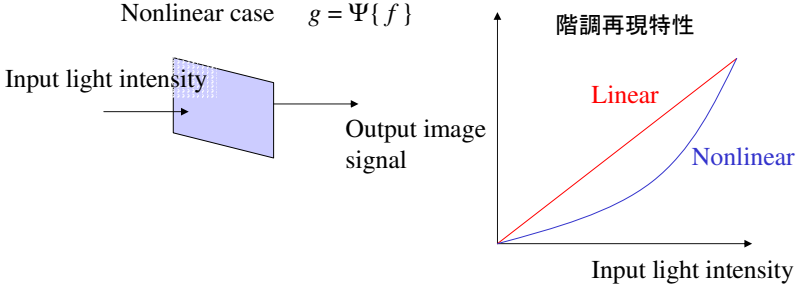
$$= f(x,y)\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(x-md_x, y-nd_y) = f(x,y)\mathrm{comb}(\frac{x}{d_x})\mathrm{comb}(\frac{y}{d_y})$$

2.3 Nonlinearity of image sensors
2.3 センサーの非線形性

- Tone reproduction characteristics of an image sensor

  Linear case:: $g = af + b$

  Nonlinear case $g = \Psi\{f\}$

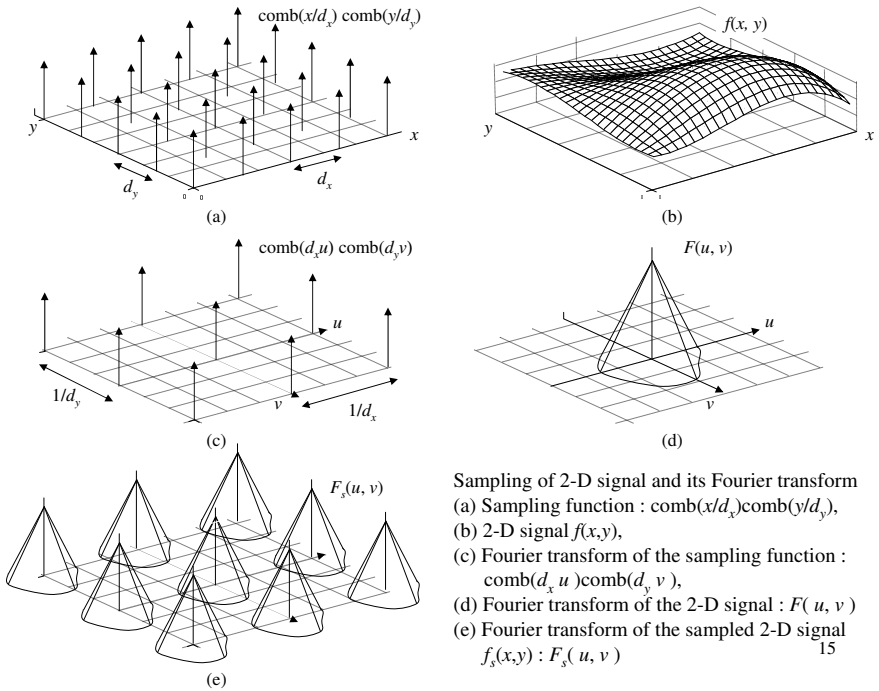


Polynomial expansion of the nonlinear function:

$$g = a_0 + a_1 f + a_2 f^2 + a_3 f^3 + \ldots$$

Its Fourier transform

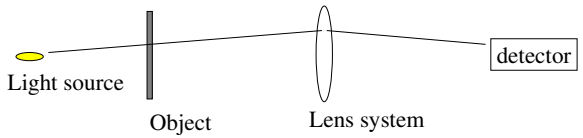
$$G = a_0\,\delta(u,v) + a_1 F + a_2\{F * F\} + a_3\{F * F * F\} + \ldots$$



Sampling of 2-D signal and its Fourier transform
(a) Sampling function : $\mathrm{comb}(x/d_x)\mathrm{comb}(y/d_y)$,
(b) 2-D signal $f(x,y)$,
(c) Fourier transform of the sampling function : $\mathrm{comb}(d_x u)\mathrm{comb}(d_y v)$,
(d) Fourier transform of the 2-D signal : $F(u, v)$
(e) Fourier transform of the sampled 2-D signal $f_s(x,y)$: $F_s(u, v)$

2.5 Sampling in practical imaging systems
2.5 実際のイメージングシステムにおけるサンプリング

- Sampling aperture of the image detector



- Aperture sensitivity function: $r(x, y)$

$$f_s(x,y) = [f(x,y) * r(-x,-y)]\cdot\mathrm{comb}(\frac{x}{d_x})\mathrm{comb}(\frac{y}{d_y})$$

- Its Fourier transform yields

$$F_s(u,v) = [F(u,v)\mathrm{sinc}(a_x u)\mathrm{sinc}(a_y v)] * [d_x d_y \mathrm{comb}(d_x u)\mathrm{comb}(d_y v)]$$

- If the shape of the sampling aperture is rectangular,

$$f_s(x,y) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} f[m,n]\delta(x-md_x)\delta(y-nd_y)$$

$$= [f(x,y) * \{\mathrm{rect}(\frac{x}{a_x})\mathrm{rect}(\frac{y}{a_y})\}]\cdot\mathrm{comb}(\frac{x}{d_x})\mathrm{comb}(\frac{y}{d_y})$$