Structural Analysis II
構造力学第二
(1)

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Structures that support human activities
人間の生活と生産活動を支える構造物

Beam or Lintel(**梁（はり）**)

Parthenon

Column or post (**柱**)

Stone Hange, UK
A structure at the early days

Timber truss bridge at Cambridge University

Truss structure(**トラス構造**)

Rainbow Bridge under construction

Suspension bridge
吊橋 (つり橋)

Tower (**主塔**)

Cables (**ケーブル**)

Girder (**桁**)

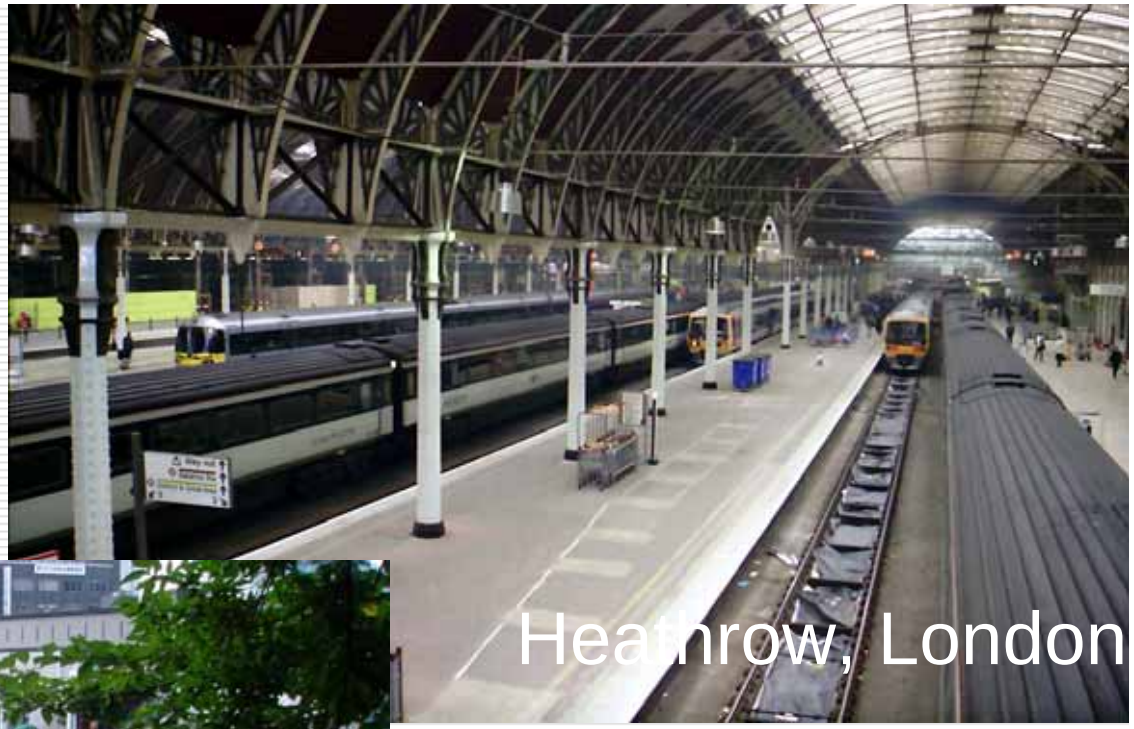
Arch Structures

Seine River

Venezia, Italy

Pavia, Italy

Eiffel Tower



Heathrow, London

Yotsuya Bridge, Tokyo


Fuji River Bridge


City which is formed by various civil engineering structures

SCHEDULE

- 1st: May 23 (M)
- 2nd: June 6 (M)
- 3rd: June 11 (Sat)
- 4th: June 13 (M)
- 5th: June 20 (M)
- 6th: June 27 (M)-----Mid-Exam
- 7th: July 4 (M)
- Final Exam: July 11 (M)

All classes are provided at 13:20-15:50.

Text Book

Harry, H. W. and Louis, F. G., Professors, The Pennsylvania State University

Fundamentals of Structural Analysis, John Wiley & Sons

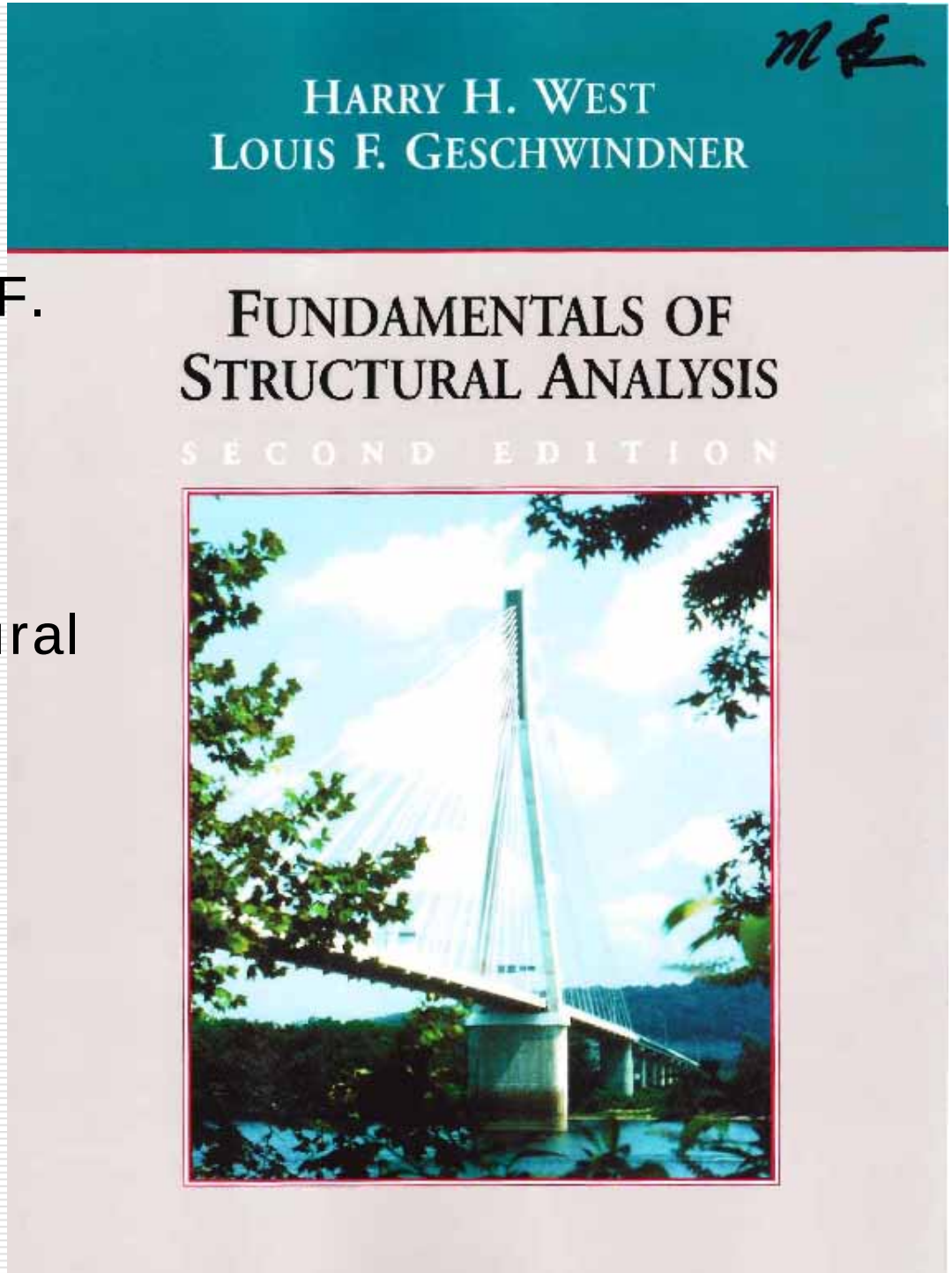


Table of Contents of the Text Book

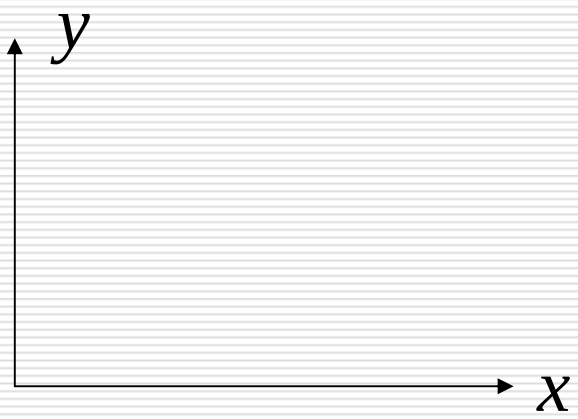
- Part I Orientation
- Part II Analysis of Statically Determinate Structures (**静定構造**)
- Part III Elastic Deflections of Structures (**弾性たわみ**) ＜－＞ (**非弾性たわみ**)
- Part IV Analysis of Statically Indeterminate Structures(**不静定構造**)
- Part V Matrix Methods of Analysis (**マトリックス構造解析**)

CHAPTER 9 MORE BASIC CONCEPTS OF STRUCTURAL ANALYSIS

9.1 REQUIREMENTS AND LIMITATIONS OF EQUILIBRIUM(**つり合い**)

- Statically determinate structures (**静的構造物**)

  For planar structures(2**次元平面構造**) , equilibrium is ensured by satisfying the three equations of static equilibrium (**静的つり合い式**) as



$$\sum P_x = 0$$

$$\sum P_y = 0$$

$$\sum M_z = 0$$

- According to Section 3.3, representing the total number of unknown reaction components of a planar structure as r_a, the structure can be classified into three categories;
  - ✓$r_a < 3$; structure is statically unstable externally (**外的静的に不安定**)
  - ✓$r_a = 3$; structure is statically determinate externally (**外的静的に静定**)
  - ✓$r_a > 3$; structure is statically indeterminate externally (**外的静的に不静定**)
- In the Structural Analysis II, analytical methods of externally statically indeterminate structures are studied.

Consider the beam shown in Fig. 9.1.

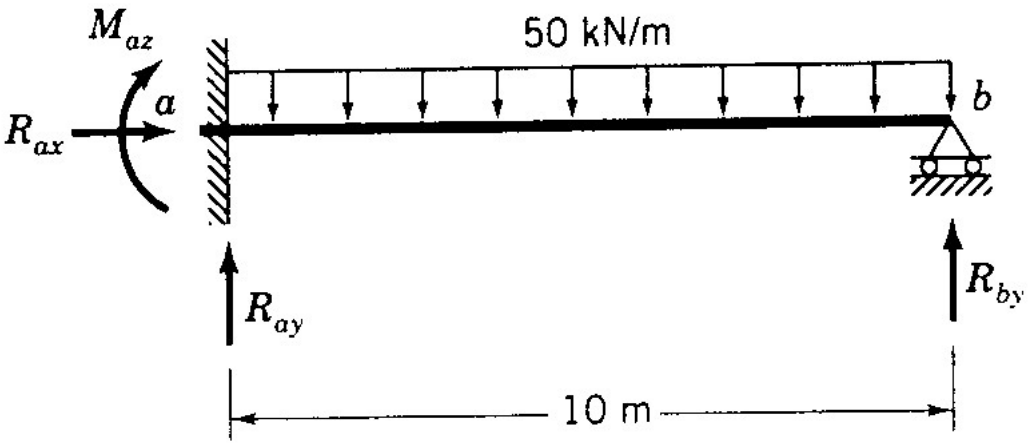


Fig. 9.1

- this beam is statically indeterminate externally because $r_a = 4$.
- A unique solution for the reactions is not possible in this case because there are four unknown reactions while we have only three equilibrium equations as

$$R_{ax} = 0$$

$$R_{ay} + R_{by} = 500 \qquad (9.1)$$

$$M_{az} - 10R_{by} = -2500$$

- There is an infinite number of solutions. Each results in values for R_{ay} and M_{az} that satisfy the equilibrium requirements along with assigned value of R_{by}.
- There is a limitation that equilibrium considerations alone give no clue regarding which one of the infinite array of possible equilibrium solutions is the correct one

9.2 STATIC INDETERMINANCIES (静的不静定) ; REDUNDANCIES (静的不静定次数)

1) External and internal indeterminacy

- All criteria involve a comparison between the number of independent unknown force components (**未知力の数**) and the number of independent equations of equilibrium (**つり合い方程式の数**) that are available for the solution of these unknowns.

- The criteria always take the following form:

  - ✓If there are more equations than there are unknowns, the structure is statically unstable.

  - ✓If there is the same number of equations as unknowns, the structure is statically determinate.

  - ✓If there are fewer equations than unknowns, the structure is statically indeterminate.

- The unknown force quantities must be arranged so as to ensure the stability of the structure
- A structure is statically indeterminate when there are more reaction force components available and/or member forces present than are necessary for stability of the structure.
- The degree of external indeterminacy is equal to the number of reaction components that are available in excess of the number that is required for external stability. These excess reaction components are called redundants (**不静定次数**) because they are unnecessary for the stability of the structure.

- The degree of internal indeterminacy is given by the number of internal force components that are present in excess of those that are needed for internal stability. These are also called redundants（**不静定次数**） since they are not required for a stable structure.

2) Primary structure (基本構造)

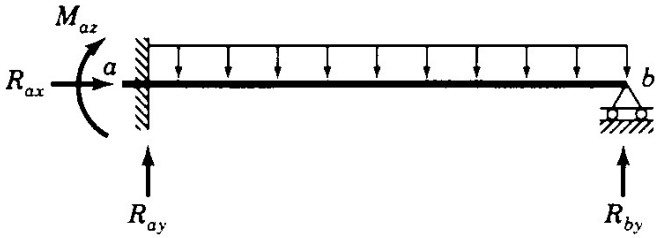
- In certain methods of statically indeterminate analysis, it is necessary to identify explicitly the reaction components or internal force components that they wish to select as the redundants.

- These are then conceptually removed from the structure, and the statically determinate structure that remains is called the primary structure.

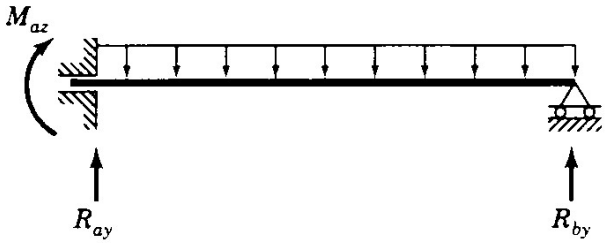
- It is essential that the redundant be selected so that the primary structure is stable.

3) Example of Primary Structure

- Consider the beam which is statically indeterminate externally to the first degree（1**次の不静定はり**）
- There is one redundant, or unnecessary, reaction component.
- If R_{ax} is selected as the redundant, the primary structure becomes as shown in (b).
- This structure is both unstable and indeterminate.



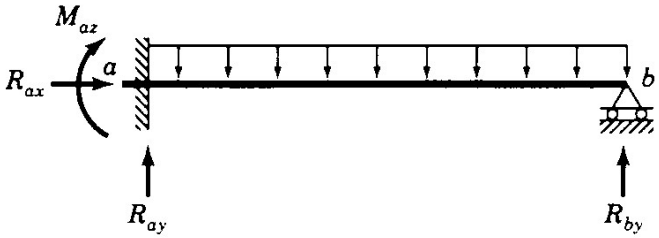
(a) statically indeterminate structure



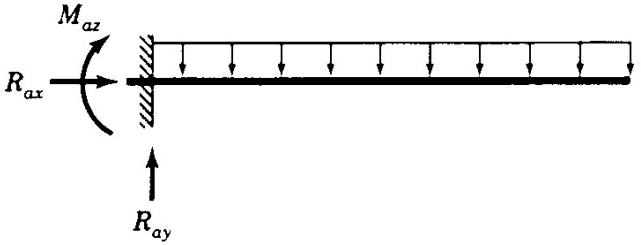
(b) Unstable and statically primary structure

Fig. 9.3 Selection of redundant reaction

- However, if R_{by} is taken as the redundant as shown in (c)
- then the primary structure is stable and statically determinate.



(a) statically indeterminate structure



(c) Stable and statically determinate primary structure

Fig. 9.3 Selection of redundant reaction

9.3 REQUIREMENTS AND LIMITATIONS OF COMPATIBILITY (**適合性、適合条件**)

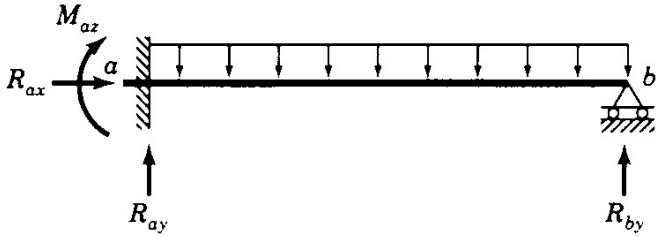
1) What is compatibility?

- Compatibility is constraints (**拘束**) on the displacements of a structure to ensure that its individual elements fit together properly and that the structure conforms to the displacement boundary conditions prescribed at the supports (**支点における変位の境界条件**).
- Compatibility is a requirement (**要件**) that must be satisfied.

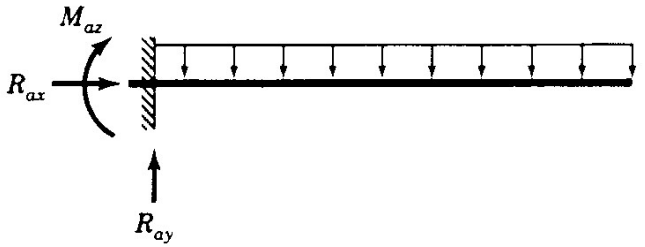
2) Example of primary structure (主構造)

- Analysis of a structure as shown in Fig. 9.3(a) could not be completed by statics alone because of statically indeterminate nature of the structure.

- If the reaction component R_{by} is removed as shown in Fig. 9.3(b), the resulting primary structure is statically determinate.



(a) statically indeterminate structure



(c) Stable and statically determinate primary structure

Fig. 9.3 Selection of redundant reaction

- The reactions R_{ay} and M_{az} can now be determined from statics

- The deflection, which include a deflection of point b, Δ_{b1}, is

$$\Delta_{b1} = -\frac{wl^4}{8EI}$$

$$= -\frac{62500 kN \cdot m^3}{EI}$$

Fig. 9.3(c) Stable and statically determinate primary structure

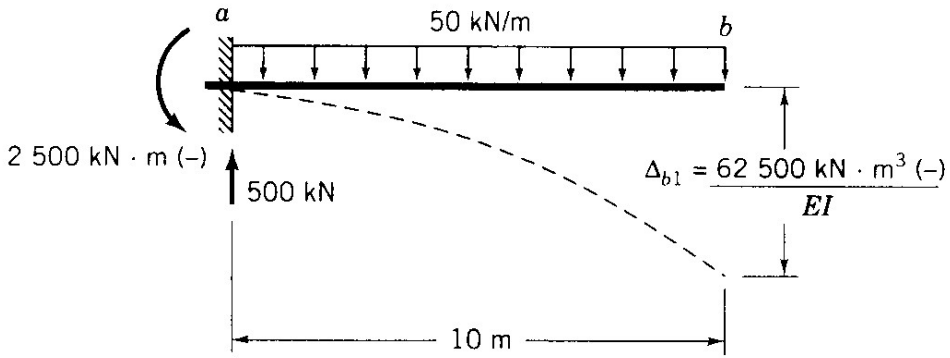
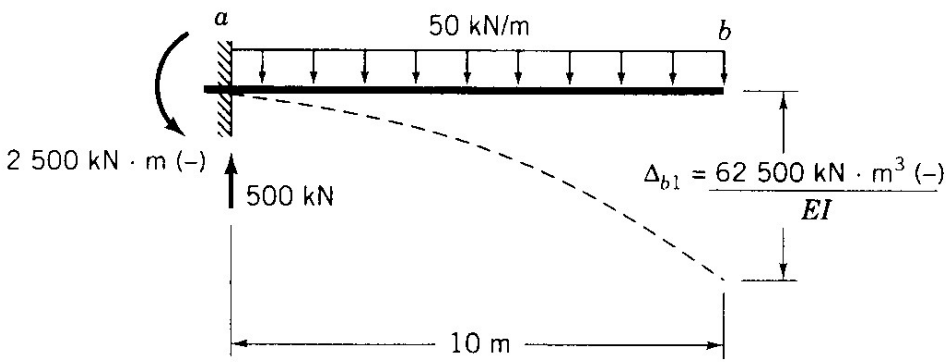


Fig. 9.4 (a) Primary structure subjected to given loading

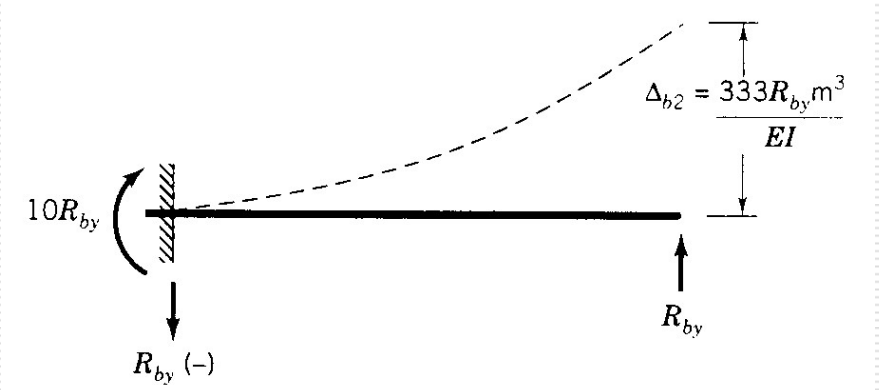
●However the deformation of the structure does not satisfy the boundary condition (境界条件) at point b.

● That is, the vertical restraint required by the support point b is not maintained.

●To remedy this problem, we allow the primary structure to be acted upon by the redundant reaction R_{by} as shown in Fig. 9.4 (b).



(a) Primary structure subjected to given loading



(b) Primary structure subjected to redundant reaction

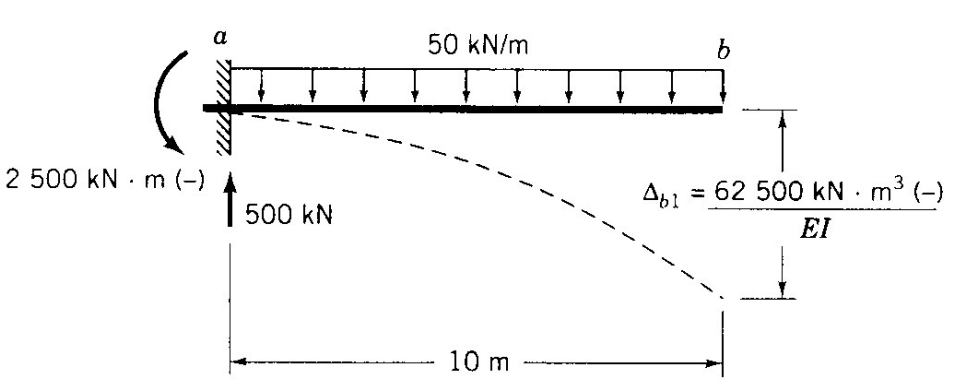
Fig. 9.4 Primary structure of Fig. 9.3

- The deflection of point b due to a point load of R_{by} is

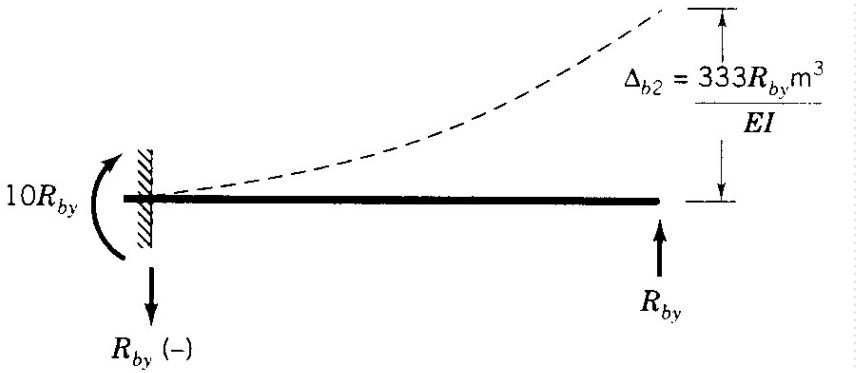
$$\Delta_{b2} = \frac{R_{by} l^3}{3EI}$$

$$= \frac{333 R_{by}}{EI}$$

- For a solution that includes the proper loading and also satisfies the designated boundary condition, the solutions shown in Fig. 9.4(a) and 9.4(b) must be superposed.



(a) Primary structure subjected to given loading



(b) Primary structure subjected to redundant reaction

Fig. 9.4 Primary structure of Fig. 9.3

- Thus, we have the compatibility condition (**变位适合条件**)

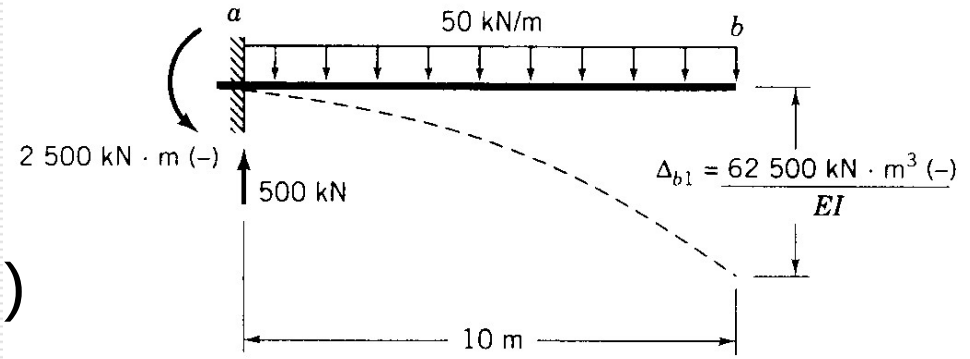
$$\Delta_b = \Delta_{b1} + \Delta_{b2} \qquad (9.3)$$

- Upon substitution and rearrangement,

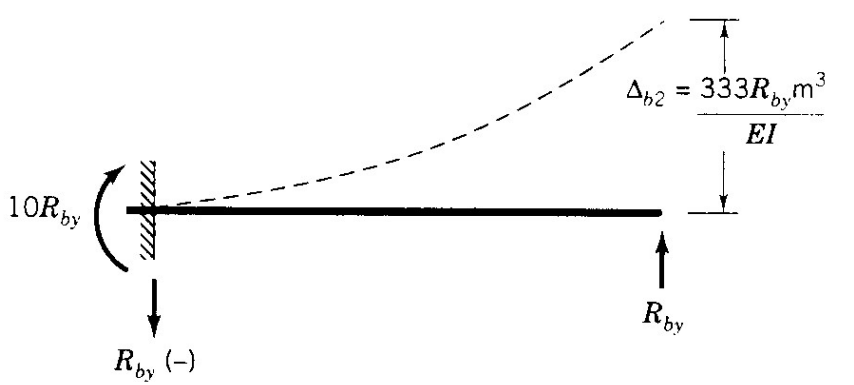
$$\frac{333R_{by}}{EI} = \frac{62500}{EI} \qquad (9.4)$$

- From which

$$R_{by} = 187.5kN \qquad (9.5)$$



(a) Primary structure subjected to given loading



(b) Primary structure subjected to redundant reaction

Fig. 9.4 Primary structure of Fig. 9.3