



Chap.4: Concept of Stress



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1.1 Introduction



Analyzing and designing various load-bearing structures
Determination of stresses and deformations

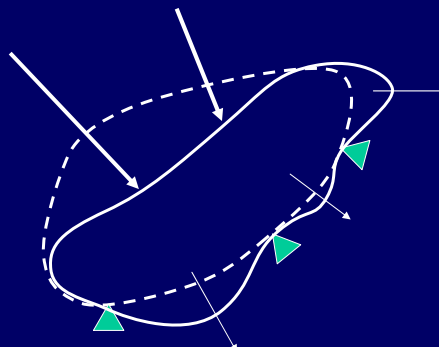
Deflection – Deformation – Strain - Stress

変位 — 変形 — ひずみ — 応力

構造物に力が作用

—
変形

—
つりあい

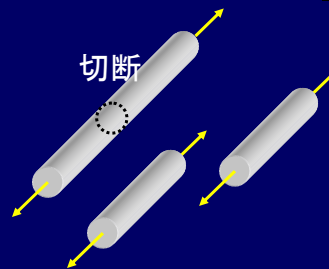


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1.2 Stresses in the member of a structure



Forces and Stresses : 力と応力



The force per unit area : 単位面積あたりの力

$$\sigma = P/A$$

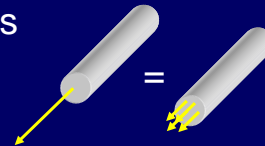
Intensity of the force

Tensile stress + compressive stress

$$1 \text{ kPa} = 10^3 \text{ N/m}^2$$

$$1 \text{ MPa} = 10^6 \text{ N/m}^2$$

$$1 \text{ GPa} = 10^9 \text{ N/m}^2$$

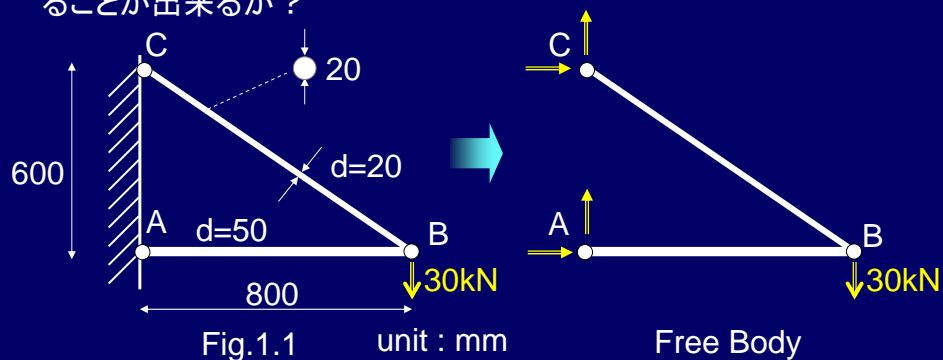


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1.3 Analysis and Design



Fig.1.1 の部材BCは鋼材で出来ている。この鋼材に対する許容応力度は165MPaである。この部材BCは安全に外力を支えることができるか？

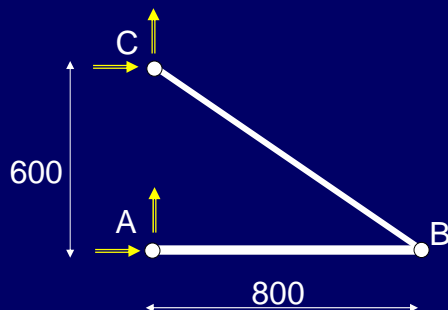


もしもこの部材がアルミ製であるとする、安全に外力を支持するために必要な断面積は？

アルミの許容応力は100MPaである

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1.3 Analysis and Design



$$-T_{BC} \cos \theta - T_{AB} = 0$$

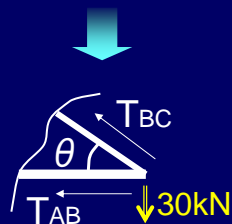
$$T_{BC} \sin \theta - 30 = 0$$

$$T_{BC} \frac{3}{5} = 30$$

$$T_{BC} = 50$$

$$T_{AB} = -50 \cdot \frac{4}{5}$$

$$T_{AB} = -40$$



$$\sigma = \frac{T_{BC}}{A} = \frac{50000}{\pi (10^2)} = 159 \text{ Mpa}$$

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1.3 Analysis and Design



もしもこの部材がアルミ製であるとする、安全に外力を支持するに必要な断面積は？

アルミの許容応力は100MPaである

$$\sigma = \frac{T_{BC}}{A} \quad A = \frac{T_{BC}}{\sigma_{\max}} = \frac{50 \times 10^3 \text{ (N)}}{100 \times 10^6 \text{ (N/m}^2\text{)}} = 500 \times 10^{-6} \text{ (m}^2\text{)} = 500 \text{ (mm}^2\text{)}$$

$$A = \pi r^2$$

$$r = \sqrt{\frac{A}{\pi}} = 12.62 \text{ (mm)} \quad d = w = 25.2 \text{ (mm)}$$

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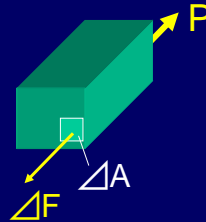
1.4 Axial Loading : Normal Stresses



The normal stress in a member under axial loading

$$\sigma = P/A$$

$$\int dF = \int_A \sigma dA$$



In practice, we shall assume that the distribution of normal stresses in an axially loaded members is uniform, except in the immediate vicinity of loading points

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1.5 Shearing stresses



Shearing stresses are commonly found in bolts, rivets, pins used to connect structural members



ピン結合



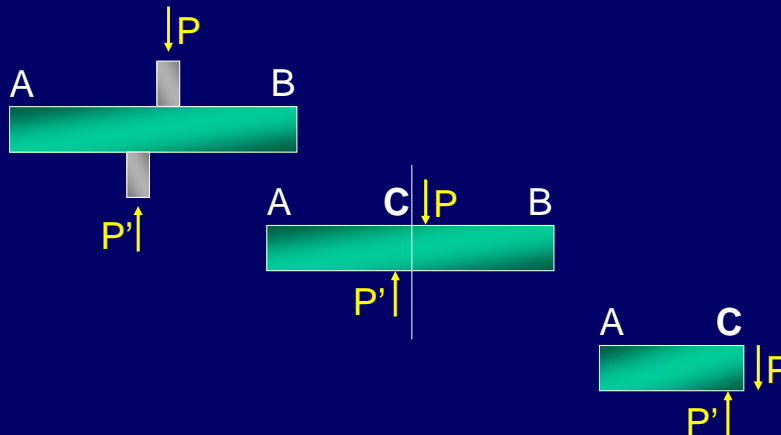
ピン結合で構成された
構造物

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1.5 Shearing stresses



Passing section at **C** between the points of application of the two forces, we conclude that internal forces must exist and their resultant is equal to P



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1.5 Shearing stresses (せん断応力)

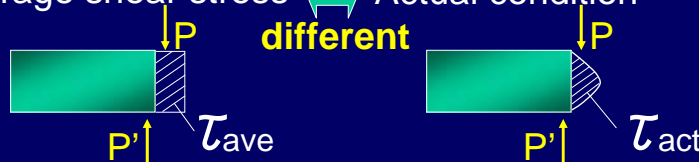


Average shear stress : 平均せん断応力

$$\tau_{ave} = \frac{P}{A}$$

The stress distribution

Average shear stress \longleftrightarrow Actual condition
different



τ_{ave} is an approximately-employed expression for the design of structural members

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The rivet joint

Bolted Joint

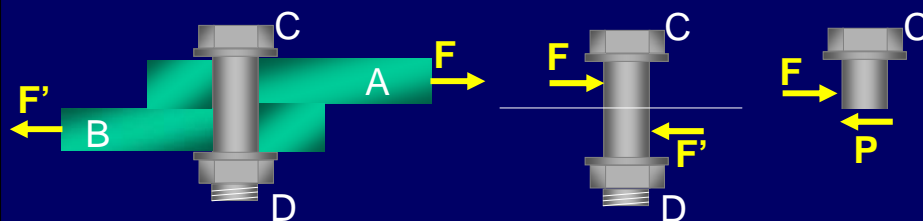
Splice joint : 添接と呼ばれる



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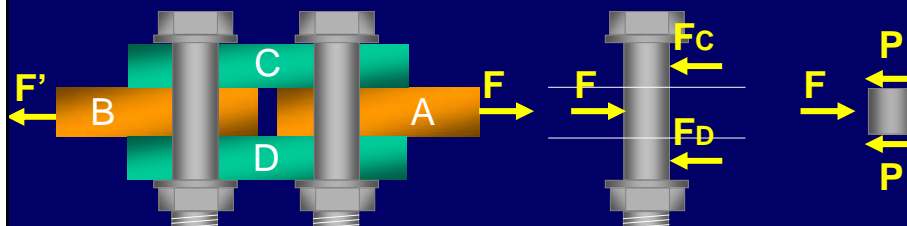
Single shear
片面添接

$$\tau_{ave} = \frac{F}{A}$$



Double shear
重要な継ぎ手 両面添接

$$\tau_{ave} = \frac{F}{2A}$$

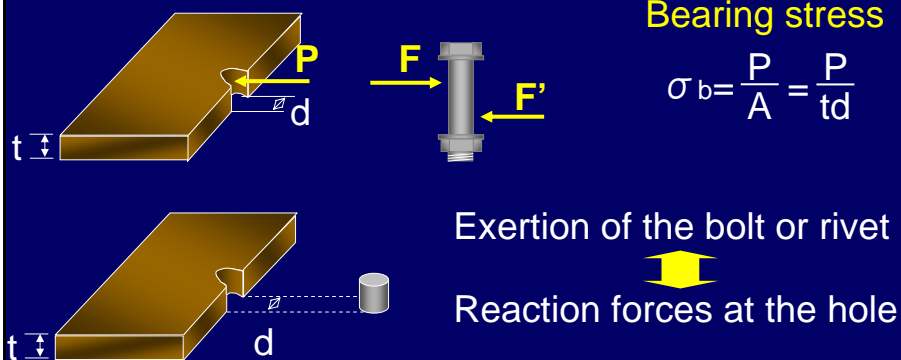


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1.6 Bearing Stress in Connections (継ぎ手部での支圧応力)



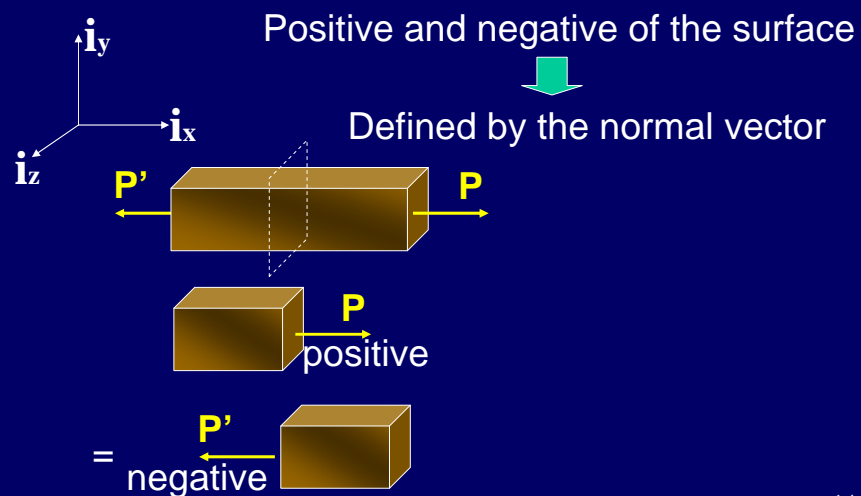
Bolts, rivets, and pins create stresses in the members along the bearing surface or surface of contact



Suppose : The stresses are uniform in the hole wall

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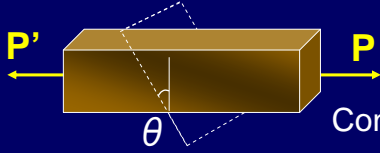
1.7 Stresses on an Oblique Plane under Axial Loading (傾いた面上の応力:軸力作用下)



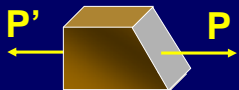
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1.7 Stresses on an Oblique Plane under Axial Loading

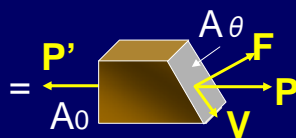
(傾いた面上の応力:軸力作用下)



Consider a section forming an angle θ

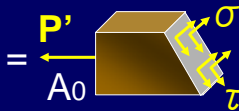


Draw the free-body diagram of a portion of member



Resolving P into F and V , respectively normal and tangential to the section, we have

$$F = P \cos \theta \quad V = P \sin \theta$$

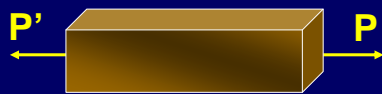


$$\sigma = \frac{F}{A_\theta} \quad \tau = \frac{V}{A_\theta}$$

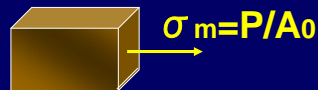
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1.7 Stresses on an Oblique Plane under Axial Loading

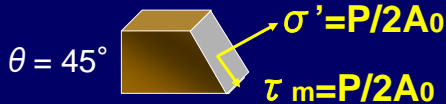
(傾いた面上の応力:軸力作用下)



$$\sigma = \frac{P \cos \theta}{A_0 / \cos \theta} = \frac{P}{A_0} \cos^2 \theta$$

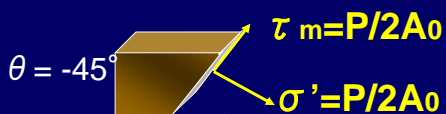


$$\text{when } \theta = 0^\circ \quad \sigma_{\max} = \frac{P}{A_0}$$



$$\tau = \frac{P}{A_0} \sin \theta \cos \theta$$

$$= \frac{1}{2} \frac{P}{A_0} \sin 2\theta$$



$$\text{when } \theta = 45^\circ \quad \tau_{\max} = \frac{P}{2A_0}$$

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