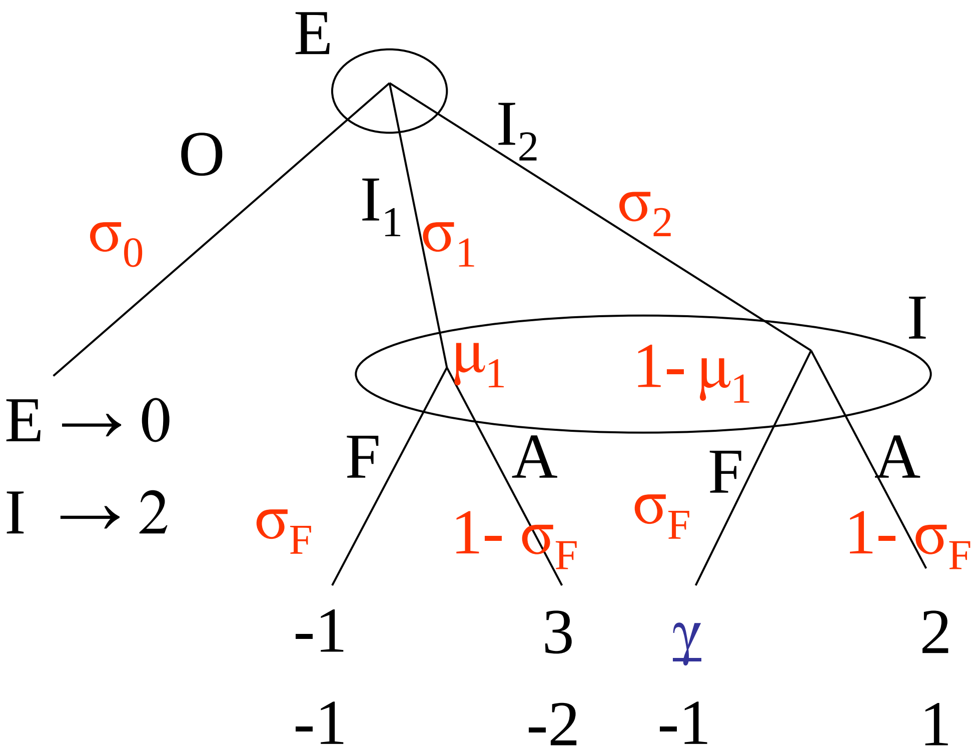
WPBE in Ex.9.C.3



$\gamma \geq 0$

$(-1 < \gamma < 0 \rightarrow$ Ex.9.C.2)

E's strategy: $(\sigma_0, \sigma_1, \sigma_2)$

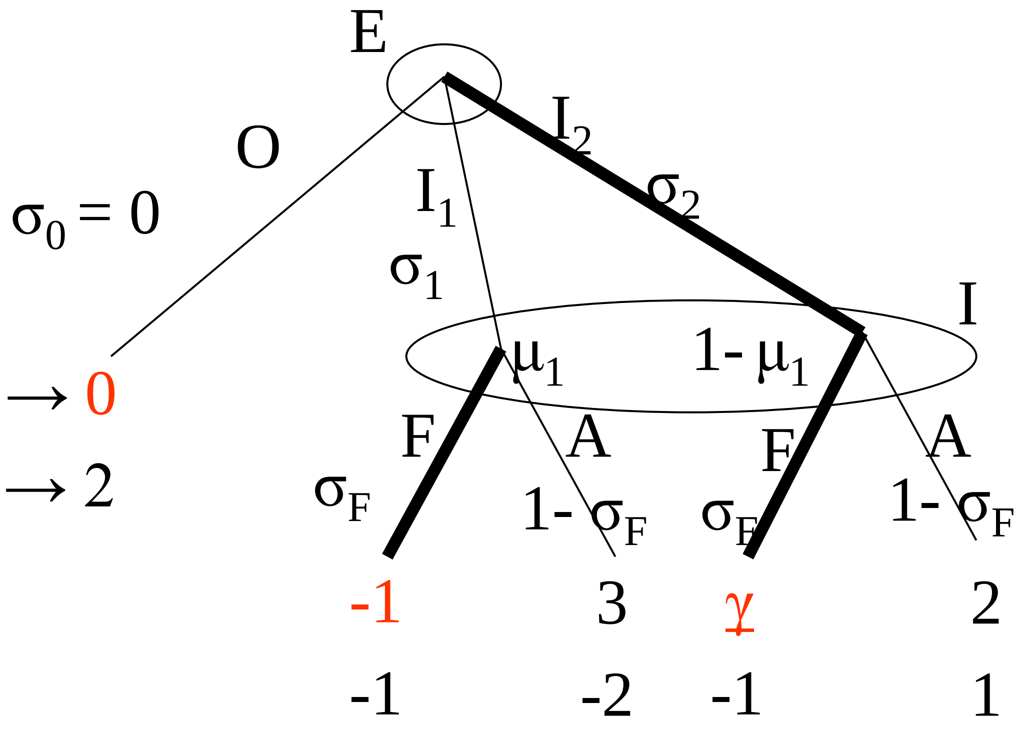
I's strategy: $(\sigma_F, 1- \sigma_F)$

I's belief: $(\mu_1, 1- \mu_1)$

I_2 dominates O $\rightarrow \sigma_0 = 0$

	F	A
O	0, 2	0, 2
I_1	-1, -1	3, -2
I_2	γ, 1	2, 1

WPBE in Ex.9.C.3



$E \rightarrow 0$

$I \rightarrow 2$

$\gamma \geq 0$

$\mu_1 > 2/3 \rightarrow F$
$\mu_1 < 2/3 \rightarrow A$
$\mu_1 = 2/3 \rightarrow F$ or A

$F \rightarrow -1$

$A \rightarrow -2\mu_1+(+1)(1-\mu_1)$

$= 1-3\mu_1$

$-1>1-3\mu_1 \leftrightarrow \mu_1>2/3$

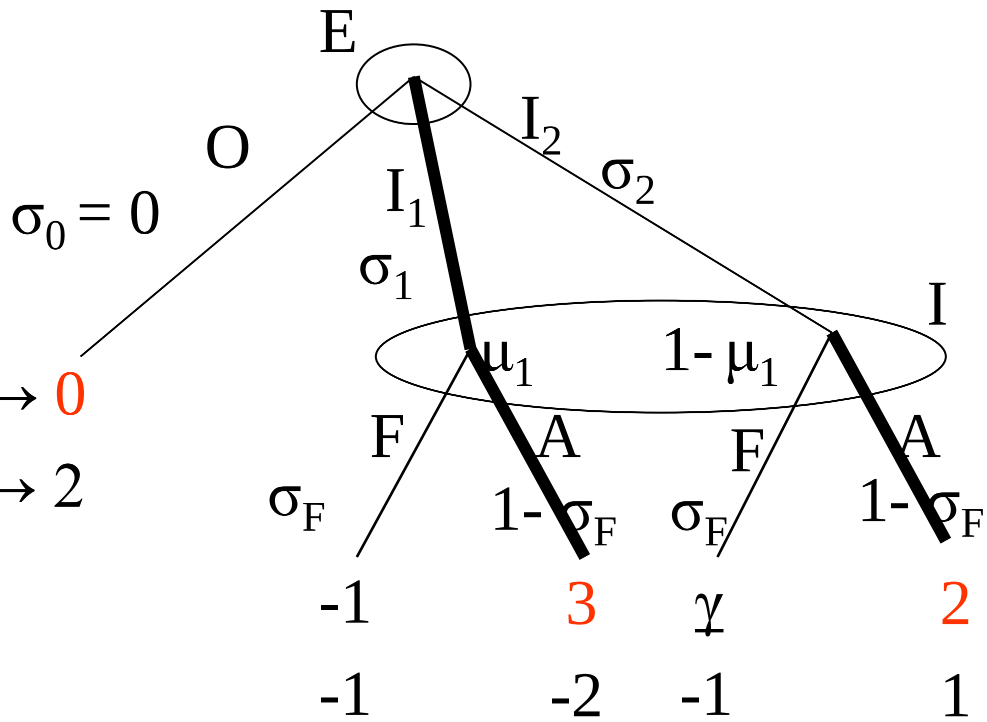
$\mu_1 \geq 2/3$

I plays F ($\sigma_F = 1$)

$\rightarrow$ E plays I_2 since $\gamma > 0 > -1$

$\rightarrow \mu = (0, 1)$ C! to $\mu_1 > 2/3$

WPBE in Ex.9.C.3



$E \rightarrow 0$

$I \rightarrow 2$

$\underline{\gamma} \geq 0$

$\mu_1 > 2/3 \rightarrow$ F

$\mu_1 < 2/3 \rightarrow$ A

$\mu_1 = 2/3 \rightarrow$ F or A

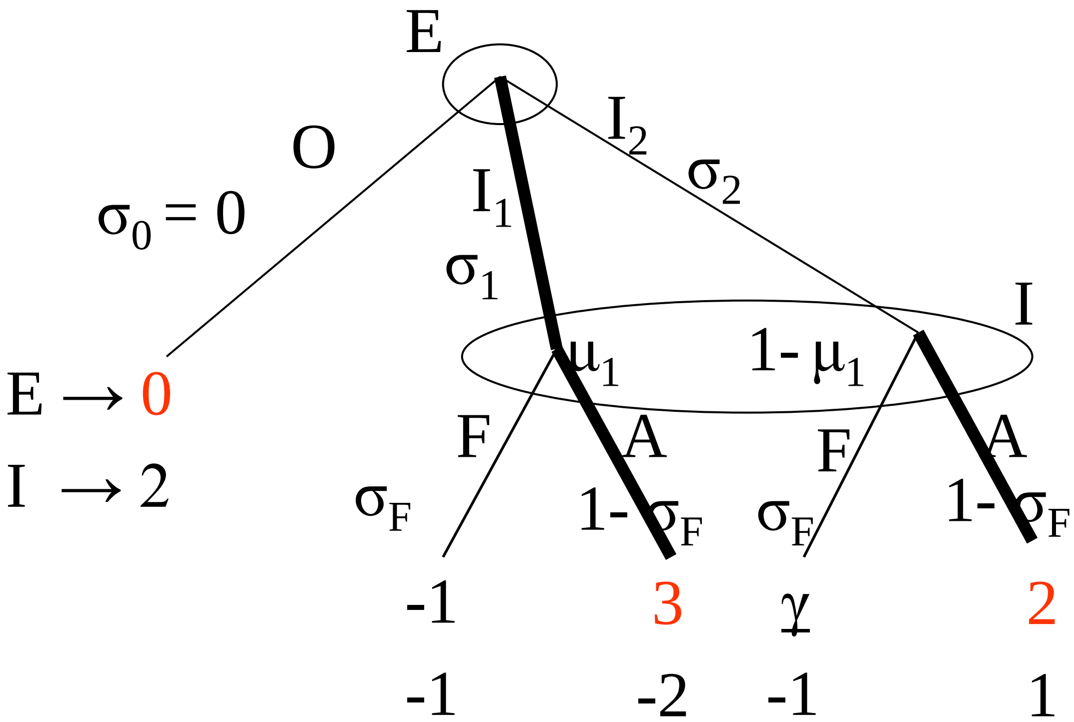
$\underline{\mu_1 < 2/3}$

I plays A ($\sigma_F = 0$)

$\rightarrow$ E plays I_1 since 3 > 2 > 0

$\rightarrow$ $\mu = (1, 0)$ C! to $\mu_1 < 2/3$

WPBE in Ex.9.C.3



$\gamma \geq 0$

$\mu_1 > 2/3 \rightarrow$ F
$\mu_1 < 2/3 \rightarrow$ A
$\mu_1 = 2/3 \rightarrow$ F or A

$\mu_1 = 2/3$

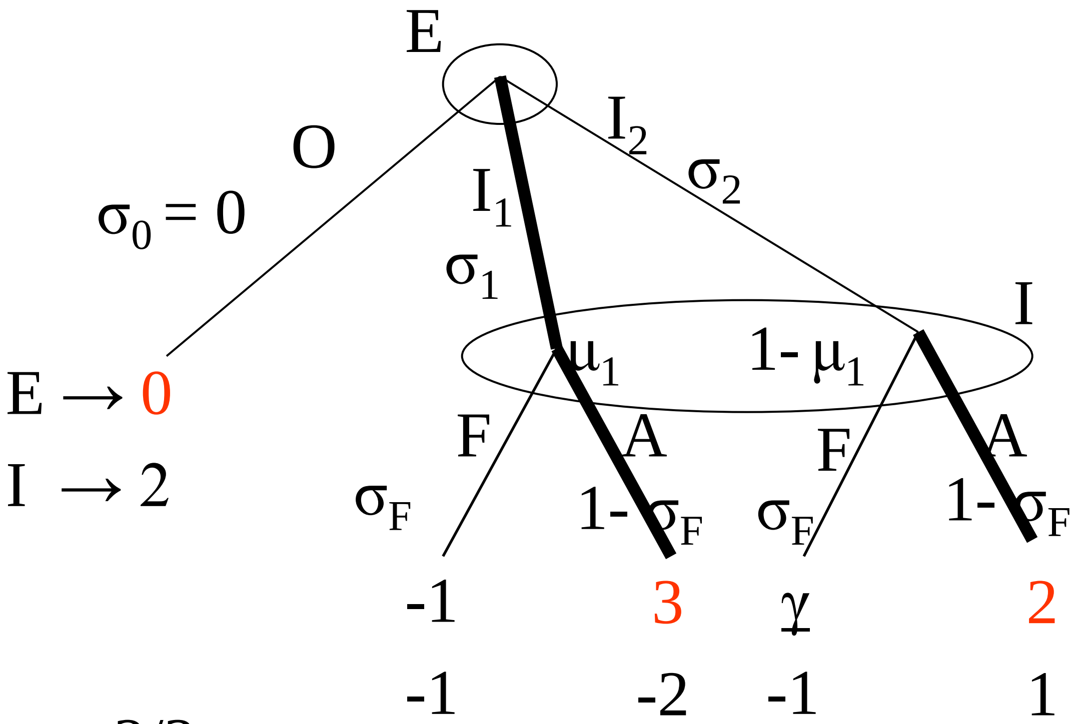
E : $\sigma_1 = 2/3, \ \sigma_2 = 1/3$

since $\sigma_0 = 0$, $\mu_1 = 2/3$ and $1 - \mu_1 = 1/3$

→ E : I_1 and I_2 are indifferent under $(\sigma_F, \ 1 - \sigma_F)$

since $\sigma_1, \ \sigma_2 > 0$

WPBE in Ex.9.C.3



$\gamma \geq 0$

$\mu_1 > 2/3 \rightarrow$ F
$\mu_1 < 2/3 \rightarrow$ A
$\mu_1 = 2/3 \rightarrow$ F or A

$\mu_1 = 2/3$

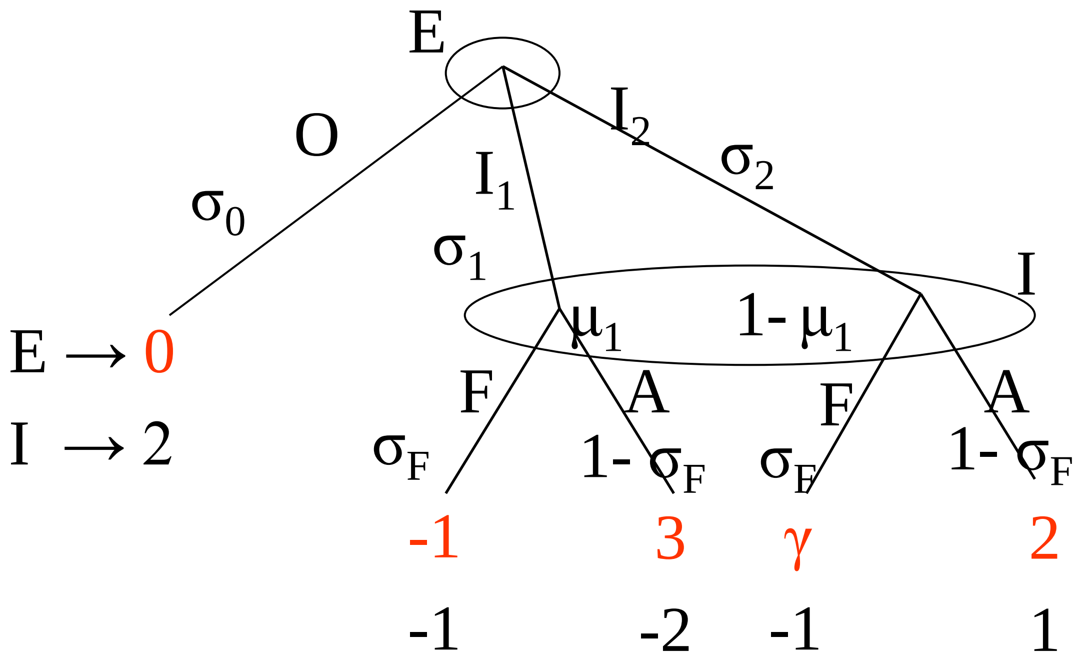
E : I_1 and I_2 are indifferent under $(\sigma_F, \ 1 - \sigma_F)$ since $\sigma_1, \ \sigma_2 > 0$.

E's payoff : $I_1 \rightarrow - \sigma_F + 3(1-\sigma_F)$, $\quad I_2 \rightarrow \gamma\sigma_F + 2(1-\sigma_F)$

$$- \sigma_F + 3(1-\sigma_F) = \gamma\sigma_F + 2(1-\sigma_F) \rightarrow \sigma_F = 1/(\gamma + 2)$$

I's strategy : $(\ 1/(\gamma + 2), \ (\gamma + 1)/(\gamma + 2))$

WPBE in Ex.9.C.3

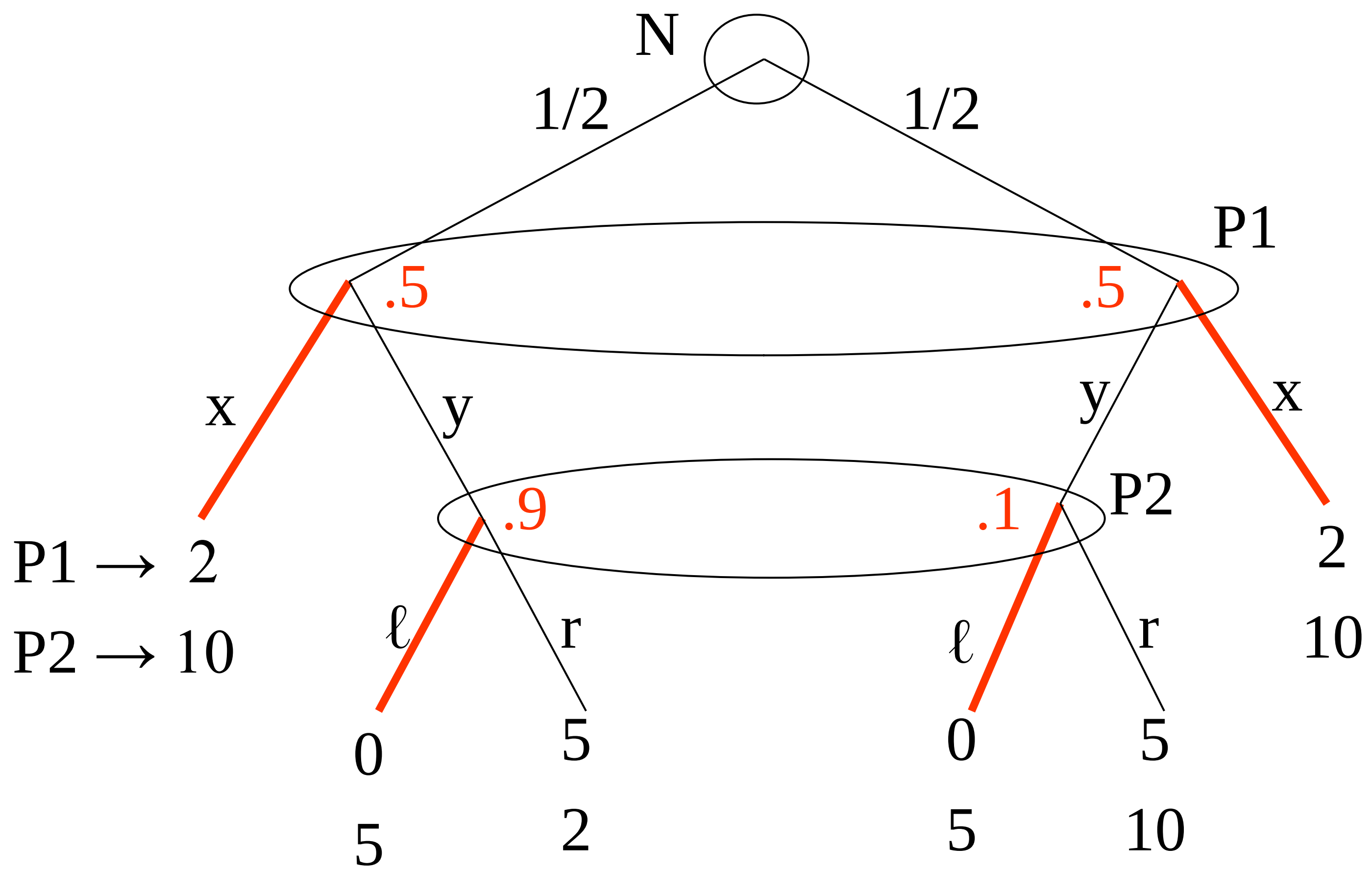


$\gamma \geq 0$

WPBE

((0, 2/3, 1/3), (1/(γ+2), (γ+1)/(γ+2)), μ =(2/3, 1/3))

Sequential Equilibrium (motivation, Ex.9.C.4)



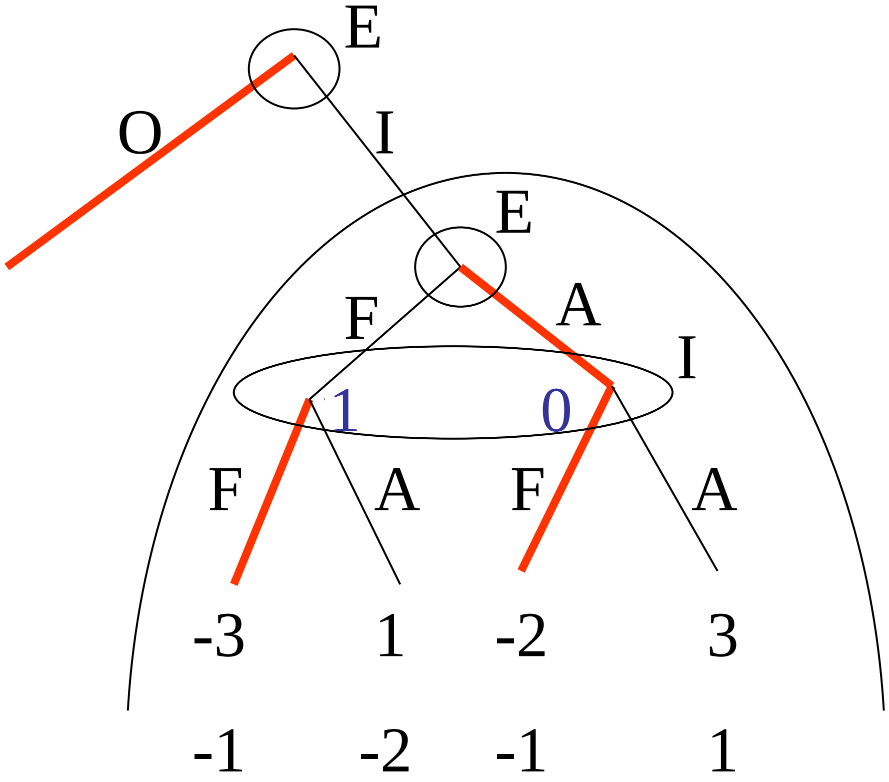
(x, ℓ, (.5, .5), (.9, .1)) → WPBE

P2 has an <u>arbitrary</u> belief since his information set is <u>not</u> reached in equilibrium. ? ? ?

Sequential Equilibrium (motivation, Ex.9.C.5)

E → 0

I → 2



((O,A), F, (1,0))

→ WPBE

E \ I	F	A
F	-3, <u>-1</u>	1,-2
A	<u>-2</u>, -1	<u>3</u>, <u>1</u>

Nash eq → (A, A)

((O,A),F) is <u>not</u> SPNE

Sequential Equilibrium (definition)

Def. 9.C.4: (σ, μ) is a sequential equilibrium (SE) if

(i) σ is sequentially rational given μ ;

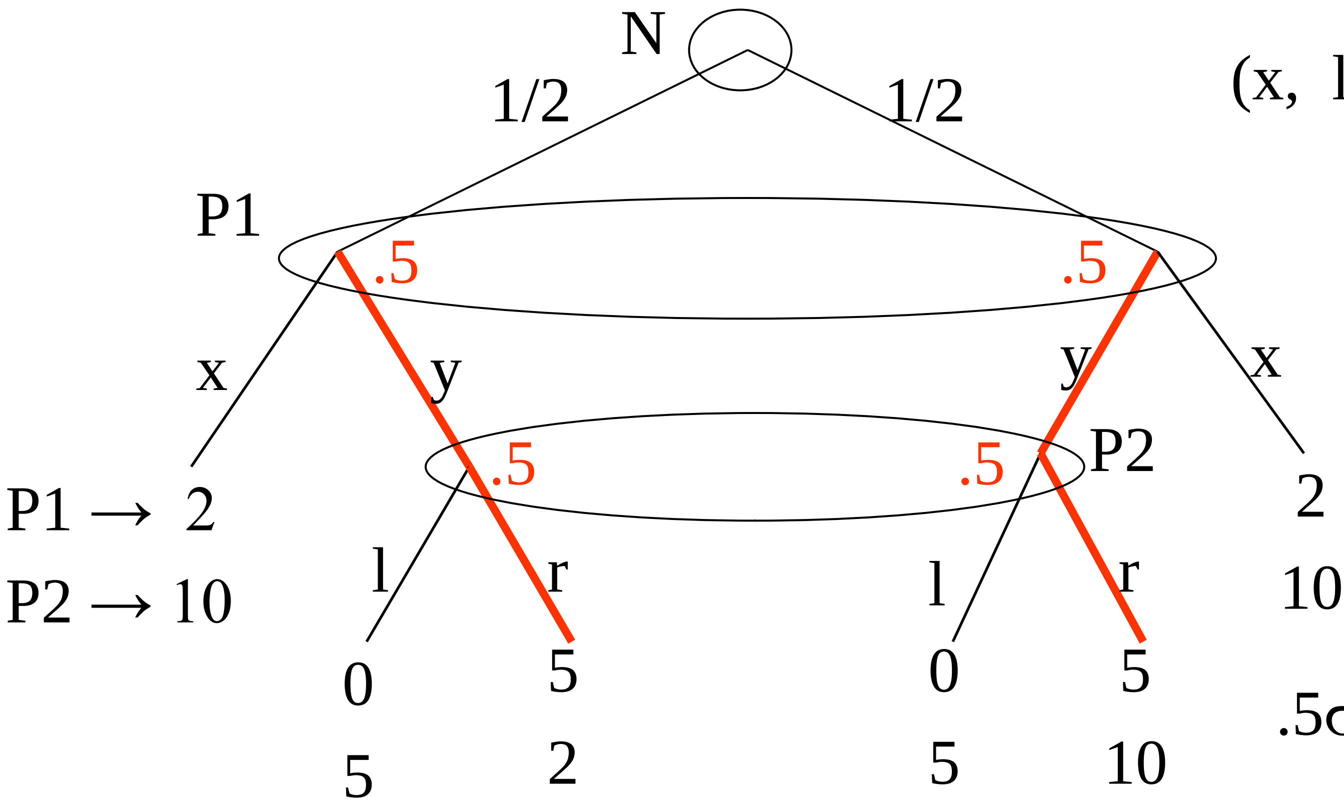
(ii) $\exists$ a sequence of completely mixed strategies $\{\sigma^k\}_{k=1}^{\infty}$

with $\lim_{k\to\infty}\sigma^k = \sigma$ such that $\mu = \lim_{k\to\infty} \mu^k$

where μ^k is the set of beliefs derived from σ^k

using Bayes' rule.

Sequential Equilibrium (Ex. 9.C.4)



(x, l, (.5, .5), (.9, .1))

→ WPBE

$.5\sigma_y/(.5\sigma_y + .5\sigma_y) = .5$

↓

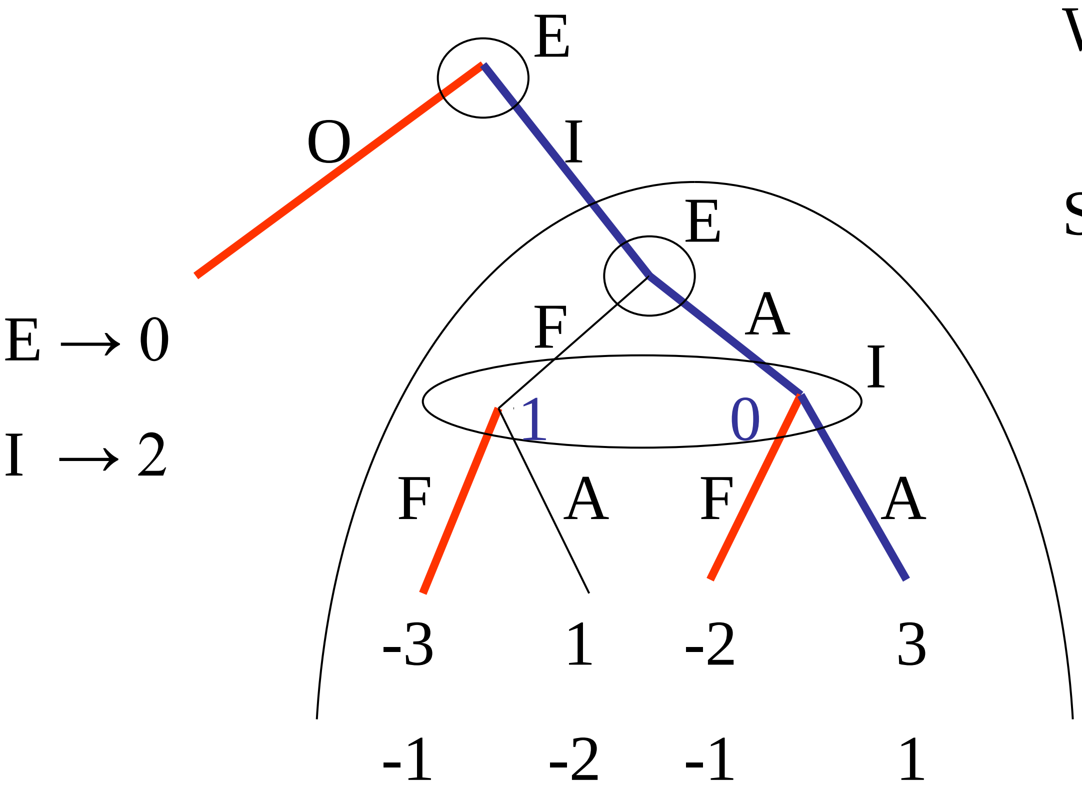
For any comp. mixed strategy (σ_x, σ_y), P2's belief = (.5, .5)

P2's choice must be "r" since 5 < $2\times.5 + 10\times.5 = 6$

P1's choice must be "y" since 2 < 5

SE → (y, r, (.5, .5), (.5, .5))

Sequential Equilibrium (Ex. 9.C.5)



WPBE → ((O,A), F, (1,0))

((I, A), A, (0,1))

SPNE → ((I,A), A)

E \ I	F	A
F	-3, -1	1,-2
A	-2, -1	3, 1

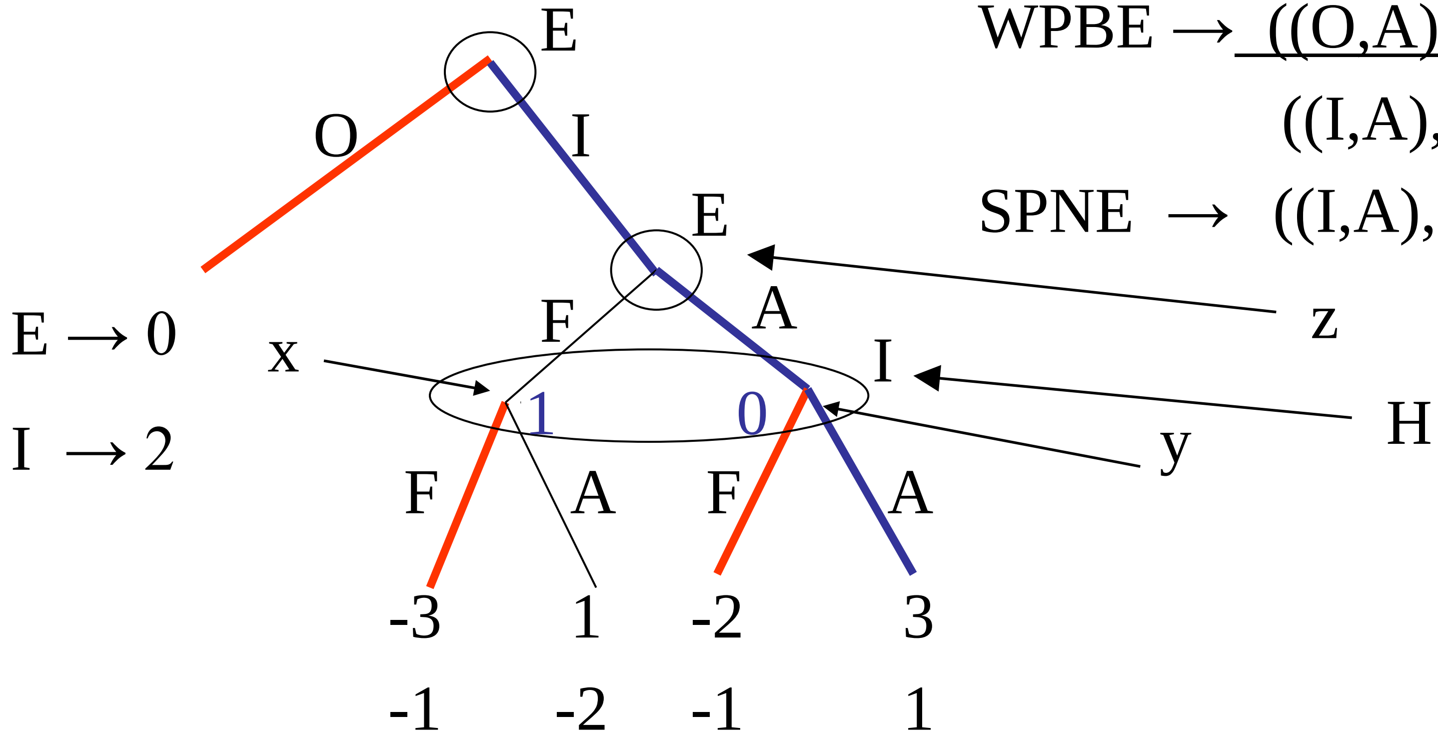
SE must contain (A, A). (→ next slide)

Sequential Equilibrium (Ex. 9.C.5)

WPBE → ((O,A), F, (1,0))

((I,A),A,(0,1))

SPNE → ((I,A), A)



$\underline{\sigma_E(O) = 1,\ \sigma_E(I) = 0,\ \sigma_E(F) = 0,\ \sigma_E(A) = 1,\ \sigma_I(F) = 1,\ \sigma_I(A) = 0}$

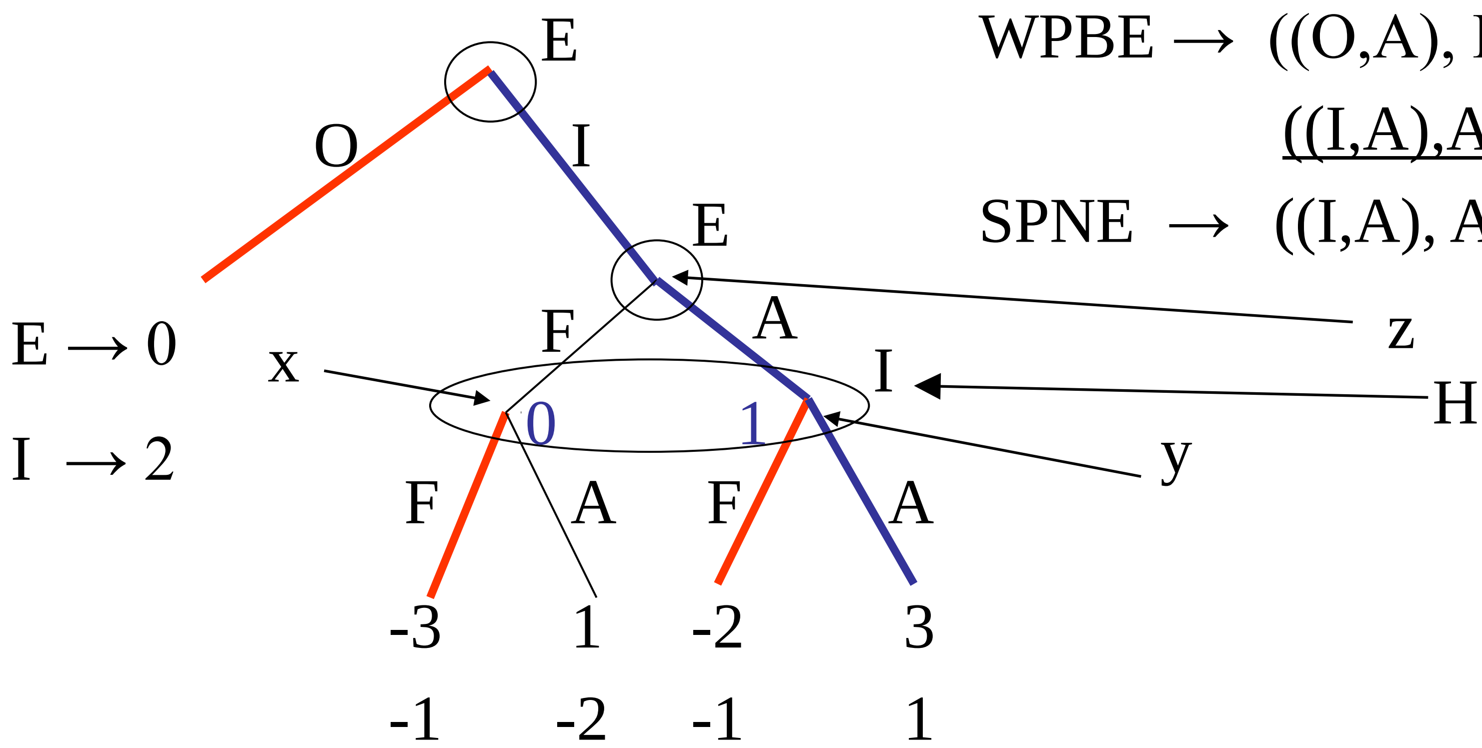
$\rightarrow\ \sigma^k_E(O) = 1-\varepsilon,\ \sigma^k_E(I) = \varepsilon,\ \sigma^k_E(F) = \varepsilon',\ \sigma^k_E(A) = 1-\varepsilon',$

$\sigma^k_I(F) = 1-\varepsilon'',\ \sigma^k_I(A) = \varepsilon''$

$\text{Prob}(H \mid \sigma^k) = \sigma^k_E(I) = \varepsilon,\ \text{Prob}(x \mid \sigma^k) = \sigma^k_E(I) \times \sigma^k_E(F) = \varepsilon \times \varepsilon'$

$\mu^k(x) = \varepsilon' \rightarrow \underline{\mu(x) = 0}$ $\qquad \mu^k(y) = 1-\varepsilon' \rightarrow \underline{\mu(y) = 1}$

Sequential Equilibrium (Ex. 9.C.5)



WPBE → ((O,A), F, (1,0))

((I,A),A,(0,1))

SPNE → ((I,A), A)

$\sigma_E(O) = 0, \sigma_E(I) = 1, \sigma_E(F) = 0, \sigma_E(A) = 1, \sigma_I(F) = 0, \sigma_I(A) = 1$

$\rightarrow \sigma^k_E(O) = \varepsilon, \sigma^k_E(I) = 1 - \varepsilon, \sigma^k_E(F) = \varepsilon', \sigma^k_E(A) = 1 - \varepsilon',$

$\sigma^k_I(F) = \varepsilon'', \sigma^k_I(A) = 1 - \varepsilon''$

$\text{Prob}(H \mid \sigma^k) = \sigma^k_E(I) = 1 - \varepsilon, \ \text{Prob}(x \mid \sigma^k) = \sigma^k_E(I) \times \sigma^k_E(F) = (1-\varepsilon)\varepsilon'$

$\mu^k(x) = \varepsilon' \rightarrow \underline{\mu(x) = 0} \qquad \mu^k(y) = 1 - \varepsilon' \rightarrow \underline{\mu(y) = 1}$

Sequential Equilibrium and SPNE

Prop. 9.C.2: In every SE (σ, μ), σ is an SPNE.

Assignments

Problem Set 10 (due June 23)

Exercises (pp.301-305)

9.C.2, 9.C.6（only 9.C.3 part)

Reading Assignment:

Text, Chapter 9, pp.292-300