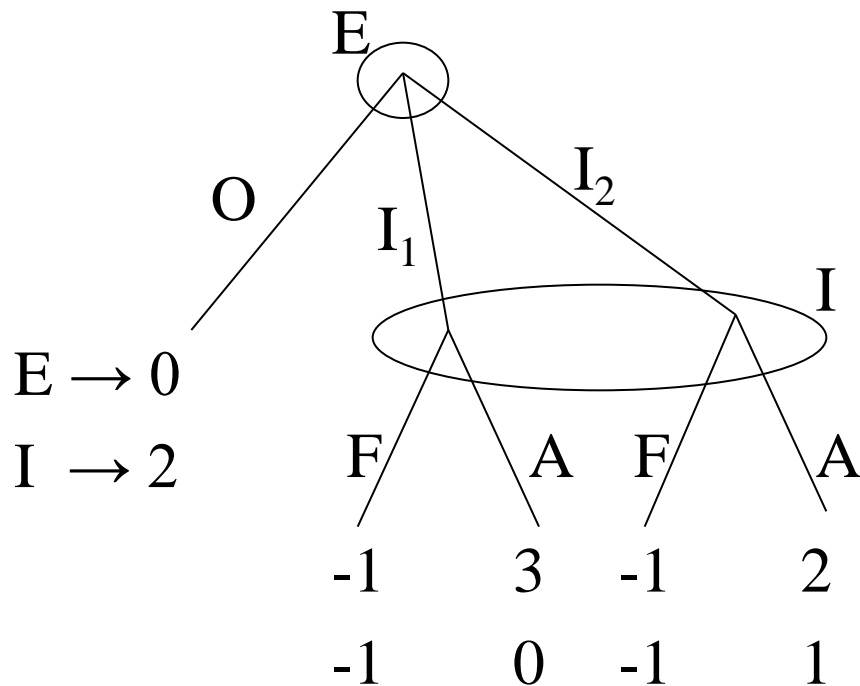


Weak Perfect Bayesian Nash Equilibrium (motivation)



		I	
		F	A
E	O	<u>0</u> , <u>2</u>	0, <u>2</u>
	I ₁	-1, -1	<u>3</u> , <u>0</u>
	I ₂	-1, -1	2, <u>1</u>

Nash eq (SPNE)

→ (O, F), (I₁, A)

For I : in either decision point, $A > F$ ($-1 < 0$, $-1 < 1$)

→ I should play “A”.

→ introduce “belief”

Weak Perfect Bayesian Nash Eq (definition)

Def. 9.C.1: $\mu = (\mu(x))_{x \in X}$ is a system of beliefs (X : set of all nodes)

if $\sum_{x \in H} \mu(x) = 1 \quad \forall \text{ information set } H$

Def. 9.C.2: $\sigma = (\sigma_1, \dots, \sigma_I)$ is sequentially rational at H given μ

if $E(u_{i(H)} \mid H, \mu, \sigma_{i(H)}, \sigma_{-i(H)}) \geq E(u_{i(H)} \mid H, \mu, \sigma^{\wedge}_{i(H)}, \sigma_{-i(H)})$

$\forall \sigma^{\wedge}_{i(H)} \in \Delta(S_{i(H)}) \quad (i(H) : \text{the player who moves at } H)$

$E(u_{i(H)} \mid H, \mu, \sigma_{i(H)}, \sigma_{-i(H)})$: expected payoff to $i(H)$ from H

if he/she is in H according to the prob. given by μ

and he/she plays $\sigma_{i(H)}$, and rivals play $\sigma_{-i(H)}$.

$\sigma = (\sigma_1, \dots, \sigma_I)$ is sequentially rational given μ

if $\forall H, \sigma = (\sigma_1, \dots, \sigma_I)$ is sequential rational at H given μ

Weak Perfect Bayesian Nash Eq (definition)

Def. 9.C.3.: (σ, μ) is a weak perfect Bayesian Eq (WPBE) if

- (i) σ is sequential rational given μ
- (ii) μ is derived from σ by Bayes' rule if possible, i.e.,

$$\forall H \text{ such that } \text{Prob}(H | \sigma) > 0$$

$$\mu(x) = \text{Prob}(x | \sigma) / \text{Prob}(H | \sigma) \quad \forall x \in H$$

WPBE and Nash Equilibrium

Prop. 9.C.1: σ is a Nash Equilibrium

$\Leftrightarrow \exists \mu$ such that

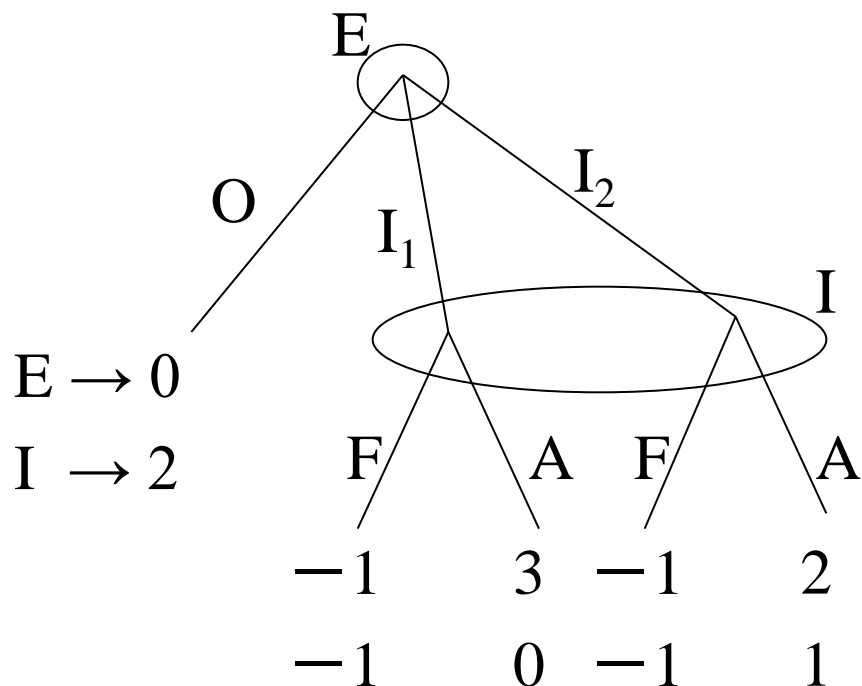
(i) σ is sequentially rational given μ

at H with $\text{Prob}(H \mid \sigma) > 0$.

(ii) μ is derived from σ by Bayes' rule whenever possible.

Cor.: (σ, μ) is a WPBE $\rightarrow \sigma$ is a Nash Equilibrium

WPBE in Ex.9.C.1

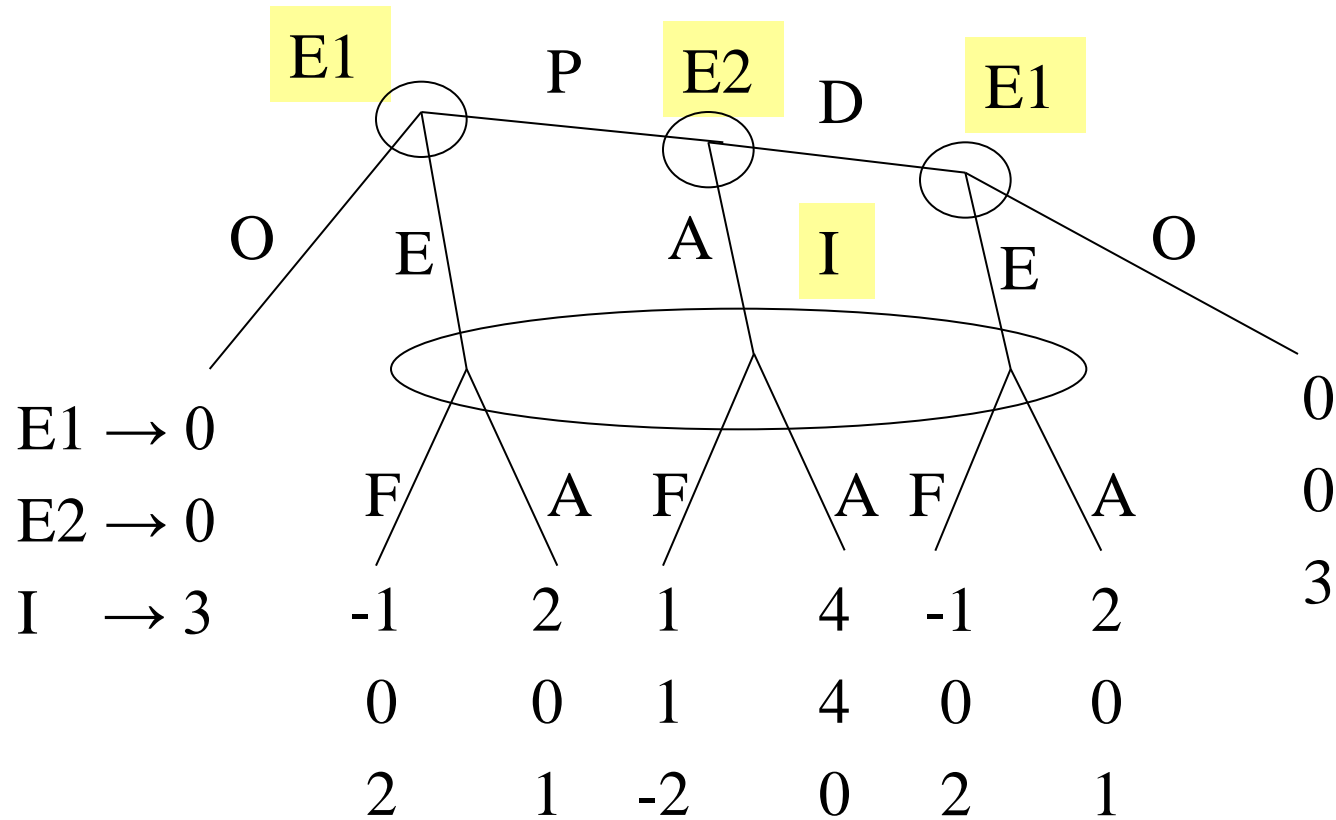


Nash eq (SPNE)
 $\rightarrow (O, F), (I_1, A)$

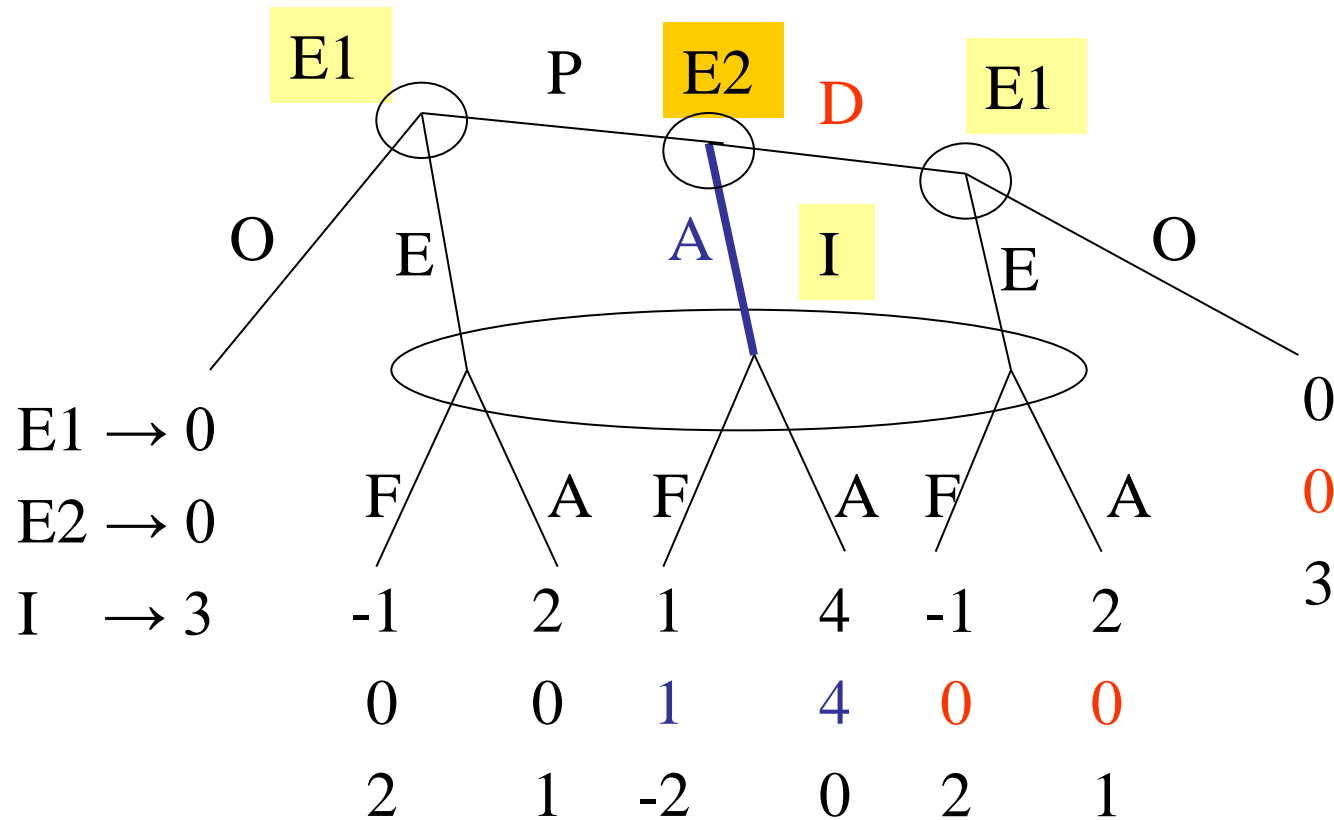
“F” is not sequentially rational
 for any belief
 $-1 < 0, -1 < 1$

WPBE $\rightarrow ((I_1, A), \mu = (1, 0))$

WPBE in Ex.9.C.2

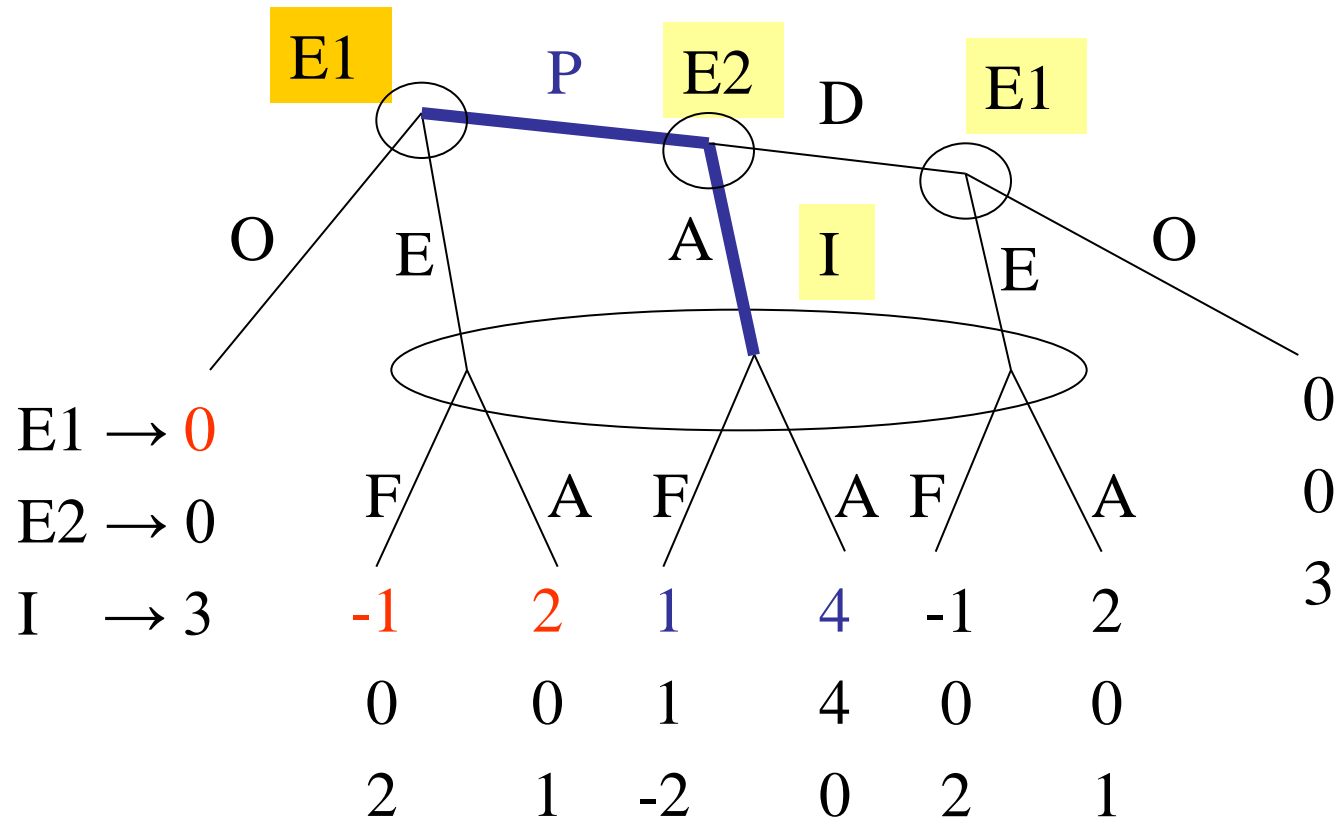


WPBE in Ex.9.C.2



E2 plays “A” since $1, 4 > 0$

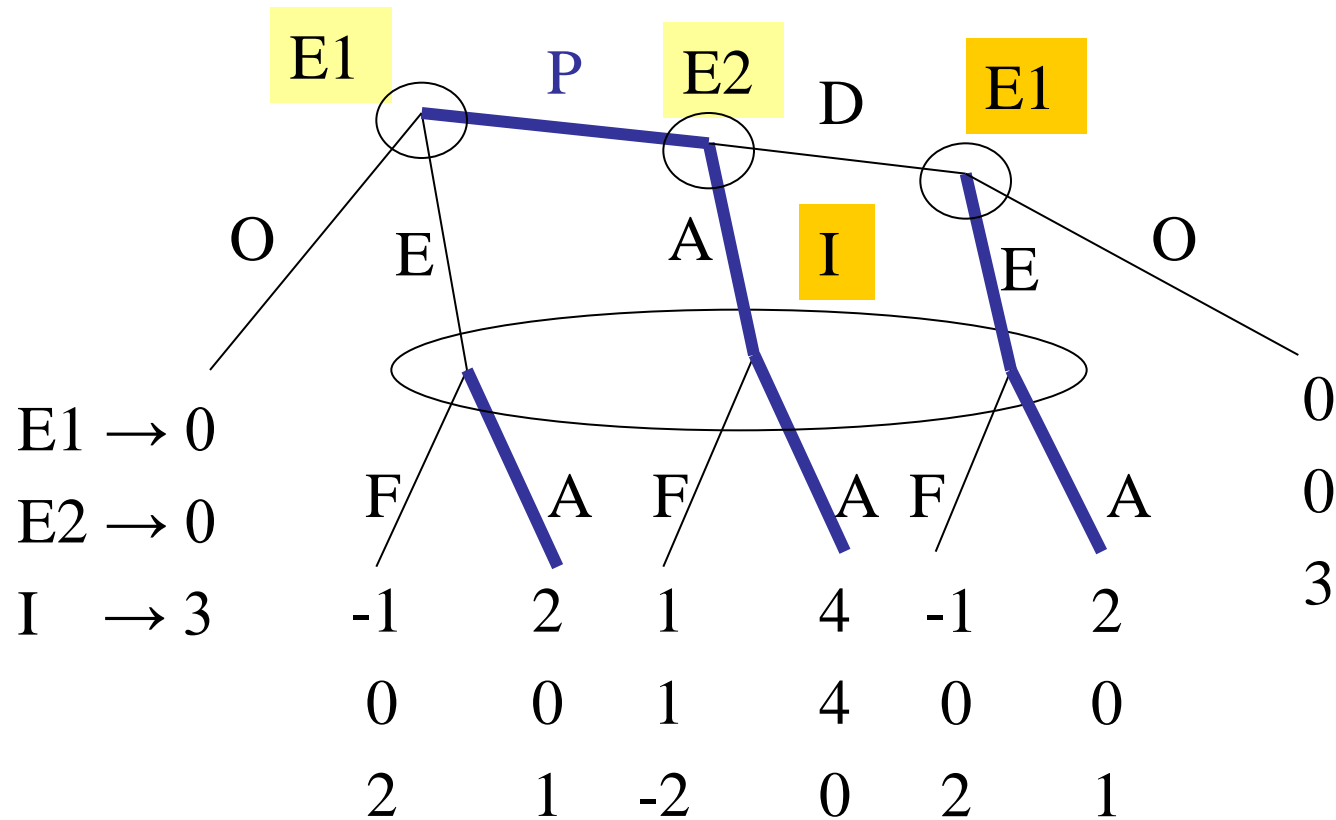
WPBE in Ex.9.C.2



E1 plays "P" since $4 > 2$, $1 > -1 \rightarrow P > E$

$4, 1 > 0 \rightarrow P > O$

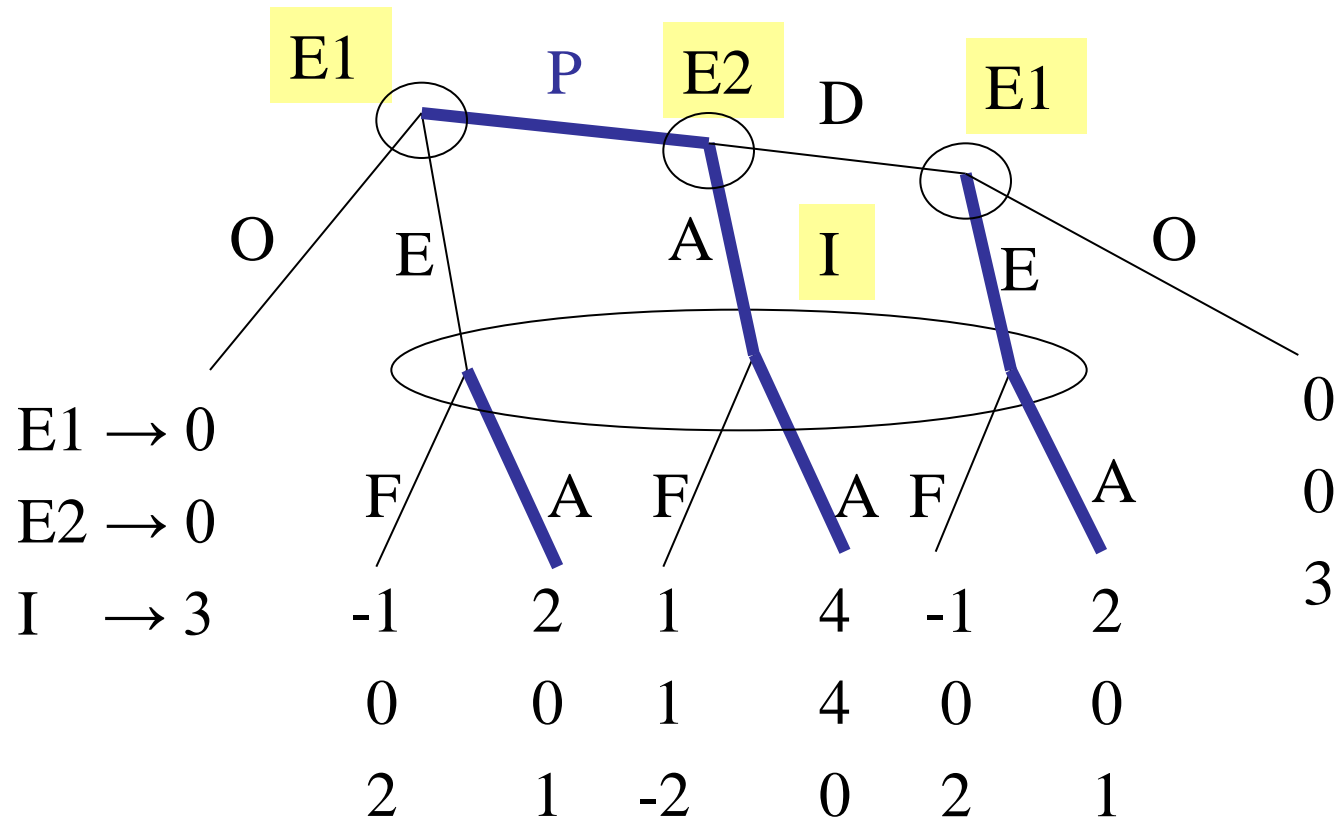
WPBE in Ex.9.C.2



I's belief $(0, 1, 0) \rightarrow$ I plays "A" since $0 > -2$

Then E1 plays "E" since $2 > 0$.

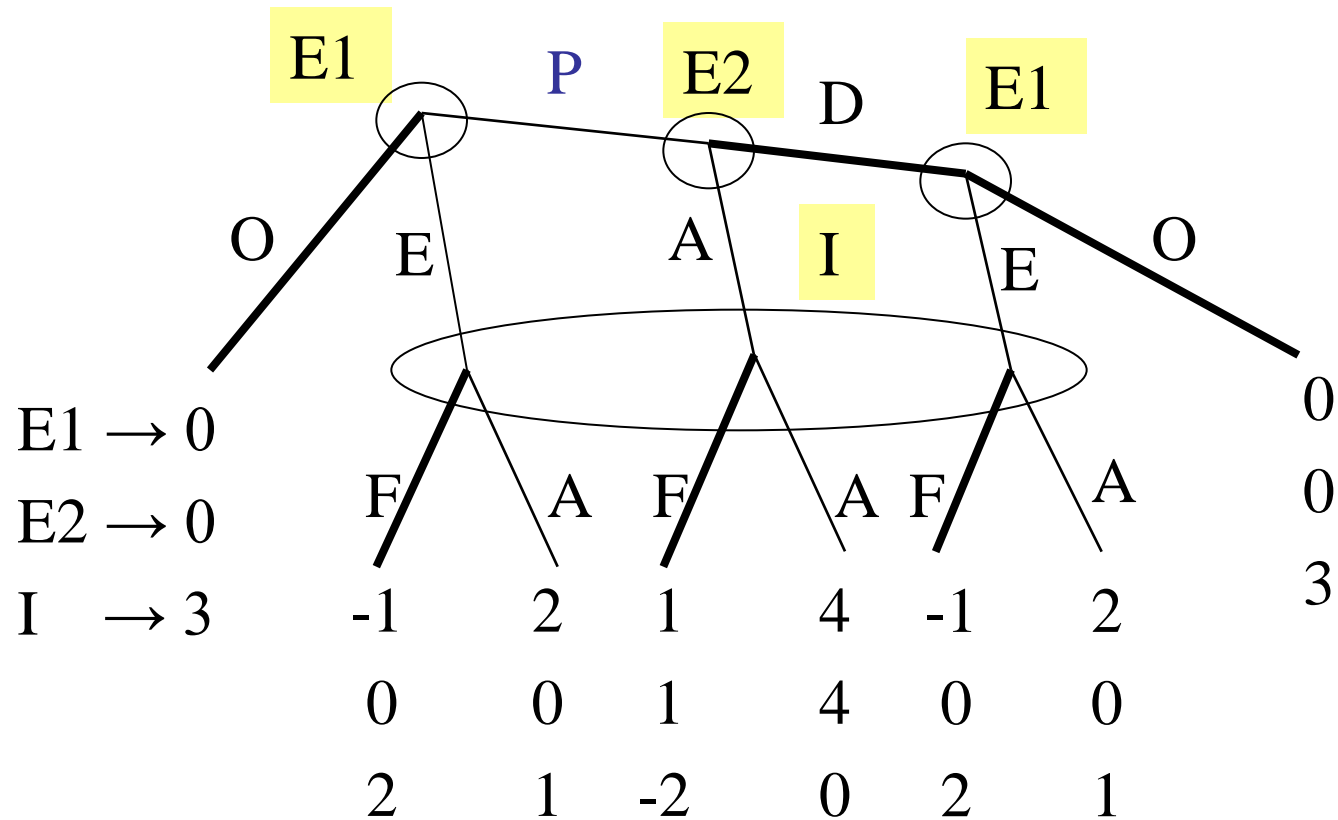
WPBE in Ex.9.C.2



WPBE : ((P, E), (A), (A), (0, 1, 0))

Note: ((O, O), (D), (F)) Nash eq. (SPNE)

WPBE in Ex.9.C.2



((O, O), (D), (F)) Nash eq. (SPNE)

Assignments

Problem Set 9 (due July 16)

Exercises (pp.301-305)

9.C.1

Reading Assignment:

Text, Chapter 9, pp.287-291