

Dominant Strategy

Prisoner's Dilemma

Player 2

Player 1

	DC	C
DC	-2, -2	-10, -1
C	-1, -10	-5, -5

Player 1: “Confess” is the best strategy regardless of what 2 plays.

Player 2: Same. → strictly dominant strategy

Definition 8.B.1: (strictly dominant strategy)

In $\Gamma_N = [N = \{0, 1, \dots, I\}, \{S_i\}, \{u_i\}]$,

$s_i \in S_i$ is a strictly dominant strategy for i

if $u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i}) \quad \forall s'_i \in S_i - \{s_i\}, \quad \forall s_{-i} \in S_{-i}.$

Domination

Definition 8.B.2: (strictly dominated strategy)

Let $s_i, s'_i \in S_i$. s'_i strictly dominates s_i if

$$u_i(s'_i, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i} \in S_{-i}.$$

If there exists at least one s'_i that strictly dominates s_i , s_i is said to be strictly dominated.

Note: s_i is a strictly dominant strategy if it strictly dominates all other strategies in S_i .

Example 8.B.1:

1: U, M strictly dominates D
1 can eliminate D.

2: no domination

Player 1

Player 2

	L	R
U	1, -1	-1, 1
M	-1, 1	1, -1
D	-2, 5	-3, 2

Weakly Dominant Strategy

Definition 8.B.3:

Let $s_i, s'_i \in S_i$. s'_i weakly dominates s_i if

$$u_i(s'_i, s_{-i}) \geq u_i(s_i, s_{-i}) \quad \forall s_{-i} \in S_{-i}$$

$$u_i(s'_i, s_{-i}) > u_i(s_i, s_{-i}) \quad \exists s_{-i} \in S_{-i}$$

If there exists at least one s'_i that weakly dominates s_i ,
 s_i is said to be weakly dominated.

s_i is a weakly dominant strategy

if it weakly dominates all other strategies in S_i .

Example 8.B.2:

1: D weakly dominates U, M

2: no weak domination

1 can eliminate U and M ???

		Player 2	
		L	R
Player 1	U	5, 1	4, 0
	M	6, 0	3, 1
	D	6, 4	4, 4

Iterated Deletion

Example 8.B.3:

1 is DA's brother and
allow 1 to go free if both play DC.

No domination for 1.

2: C strictly dominates DC.

Player 1

Player 2

	DC	C
DC	0, -2	-10, -1
C	-1, -10	-5, -5

Payoffs and rationality of both players are common knowledge

→ 1 believes 2 eliminates DC and plays C

(1 knows 2's payoffs and rationality)

→ 1 plays C since $-5 > -10$. → (C, C)

Further iteration of deletion is possible.

Note: Order of deletion does not affect the final outcome.

Iterated Deletion of Weakly Dominated Strategies

Deletion of weakly dominated strategies

→ other players play all strategies with positive probability

→ C! to iterated deletion

Example 8.B.2:

1: D weakly dominates U, M

2: no weak domination

Delete M → L w-dom R → (D, L)

Delete U → R w-dom L → (D, R)

(Delete M & U → (D, L) or (D, R))

	L	R
U	5, 1	4, 0
M	6, 0	3, 1
D	6, 4	4, 4

	L	R
U	5, 1	4, 0
D	6, 4	4, 4

	L	R
M	6, 0	3, 1
D	6, 4	4, 4

Domination with Mixed Strategies

Definition 8.B.4: (strictly dominated strategy with mixed strategies)

Let $\sigma_i, \sigma'_i \in \Delta(S_i)$. σ'_i strictly dominates σ_i if

$$u_i(\sigma'_i, \sigma_{-i}) > u_i(\sigma_i, \sigma_{-i}) \quad \forall \sigma_{-i} \in \prod_{j \neq i} \Delta(S_j).$$

σ_i is said to be strictly dominated

if there exists at least one σ'_i that strictly dominates σ_i ,

σ_i is a strictly dominant strategy

if it strictly dominates all other strategies in $\Delta(S_i)$.

No domination for 1 and 2
in pure strategies.

$(1/2, 0, 1/2)$ strictly dominates M.

Pl. 1

Pl. 2

	L	R
U	10, 1	0, 4
M	4, 2	4, 3
D	0, 5	10, 2

Domination with Mixed Strategies

Proposition 8.B.1:

$s_i \in S_i$ is strictly dominated in $\Gamma_N = [N = \{0, 1, \dots, I\}, \{\Delta(S_i)\}, \{u_i\}]$ iff there exists $\sigma'_i \in \Delta(S_i)$ such that

$$u_i(\sigma'_i, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i} \in S_{-i} = \prod_{j \neq i} S_j.$$

Note: $u_i(\sigma'_i, \sigma_{-i}) > u_i(\sigma_i, \sigma_{-i}) \quad \forall \sigma_{-i} \in \prod_{j \neq i} \Delta(S_j)$

iff $u_i(\sigma'_i, s_{-i}) > u_i(\sigma_i, s_{-i}) \quad \forall s_{-i} \in \prod_{j \neq i} S_j.$



Delete all strictly dominated pure strategies in Γ_N .



How do we eliminate mixed strategies ?

Domination with Mixed Strategies

Exercise 8.B.6:

$s_i \in S_i$ is strictly dominated in $\Gamma_N = [N = \{0, 1, \dots, I\}, \{\Delta(S_i)\}, \{u_i\}]$

\Rightarrow any strategy that plays s_i with positive probability is also strictly dominated.



Can eliminate some dominated mixed strategies.

Can eliminate further.

Neither U nor D strictly dominated;

But $(1/2, 0, 1/2)$ is strictly dominated

By M.

Pl. 2

	L	R
U	10, 1	0, 4
M	6, 2	6, 3
D	0, 5	10, 2

Pl. 1

Domination with Mixed Strategies

Elimination of dominated strategies in

$$\Gamma_N = [N = \{0, 1, \dots, I\}, \{\Delta(S_i)\}, \{u_i\}]$$

1. Iteratively eliminate strictly dominated pure strategies.
2. Let S_i^u be the remaining pure strategy set of i
3. Eliminate strictly dominated mixed strategies in $\Delta(S_i^u)$

Rationalizable Strategies

Definition 8.C.1:

In $\Gamma_N = [N = \{0, 1, \dots, I\}, \{\Delta(S_i)\}, \{u_i\}]$, $\sigma_i \in \Delta(S_i)$ is a best response for i to σ_{-i} if $u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma'_i, \sigma_{-i}) \quad \forall \sigma'_i \in \Delta(S_i)$.

Strategy σ_i is never a best response if there is no σ_{-i} to which σ_i is a best response.

Note: Strictly dominated \rightarrow never be a best response
never be a best response even if not strictly dominated

Rationalizable Strategies

Pl. 2

	b_1	b_2	b_3	b_4
a_1	0, <u>7</u>	2, 5	<u>7</u> , 0	0, 1
a_2	5, 2	<u>3</u> , <u>3</u>	5, 2	0, 1
a_3	<u>7</u> , 0	2, 5	0, <u>7</u>	0, 1
a_4	0, <u>0</u>	0, -2	0, <u>0</u>	<u>10</u> , -1

— denotes the best response

Pl. 1

b_4 is not strictly dominated.

But b_4 is never the best response.

$a_1 \rightarrow b_1$

$a_2 \rightarrow b_2$

$a_3 \rightarrow b_3$

$a_4 \rightarrow b_1, b_3$

Rationalizable Strategies

Iterated elimination of “never be a best response” strategies

Definition 8.C.2:

In $\Gamma_N = [\{0, 1, \dots, I\}, \{\Delta(S_i)\}, \{u_i\}]$, the strategies in $\Delta(S_i)$ that survives the iterated deletion of strategies that are never be a best response are called i 's rationalizable strategies.

Note: Order of deletion does not affect

Rationalizable Strategies

Pl. 2

— denotes best response

Pl. 1

	b_1	b_2	b_3	b_4
a_1	0, <u>7</u>	2, 5	<u>7</u> , 0	0, 1
a_2	5, 2	<u>3</u> , <u>3</u>	5, 2	0, 1
a_3	<u>7</u> , 0	2, 5	0, <u>7</u>	0, 1
a_4	0, <u>0</u>	0, -2	0, <u>0</u>	<u>10</u> , -1

b_4 is never a best response \rightarrow eliminate b_4

$\rightarrow a_4$ is never a best response \rightarrow eliminate a_4

rationalizable strategies $\rightarrow \{a_1, a_2, a_3\}$ for 1, $\{b_1, b_2, b_3\}$ for 2

Chain of justification:

$(a_2, b_2, a_2, b_2, a_2, \dots), (a_1, b_3, a_3, b_1, a_1, b_3, \dots)$

$(a_4, b_4, \text{nothing})$

Rationalizable Strategies

Existence of rationalizable strategies \leftarrow existence of Nash eq.
many rationalizable strategies.

set of rationalizable str.

\subseteq remaining strategies after iterative deletion of
strictly dominated strategies

strictly dominated \rightarrow never be a best response

Two-person games:

set of rationalizable str.

= remaining strategies after iterative deletion of
strictly dominated strategies

Note: Three or more person games \rightarrow not true
(OK for correlated str.)

Assignments

Problem Set 3 (due May 5):

Exercises (p.262) 8.B.1, 8.B.3, 8.B.6, 8.B.7

Reading Assignments:

Text Chapter 8, pp.246-253