Extensive Form Games

(i) X : a (finite) set of nodes, A : a (finite) set of possible actions

N = {1, … , I}: a (finite) set of players

(ii) p : $X \to X \cup \{\varnothing\}$: specify a single <u>predecessor</u>

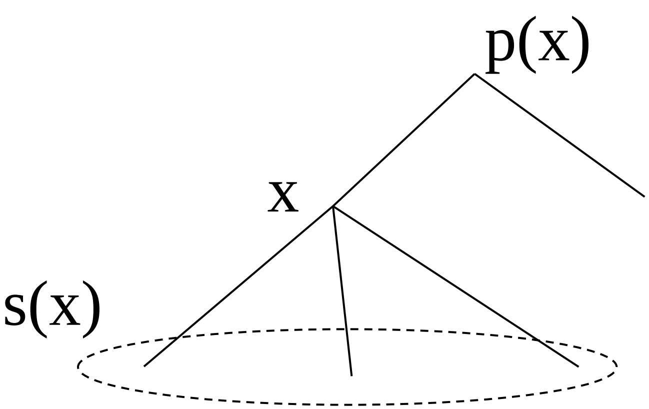
x is the initial node $\to p(x) = \varnothing$, denoted x_0

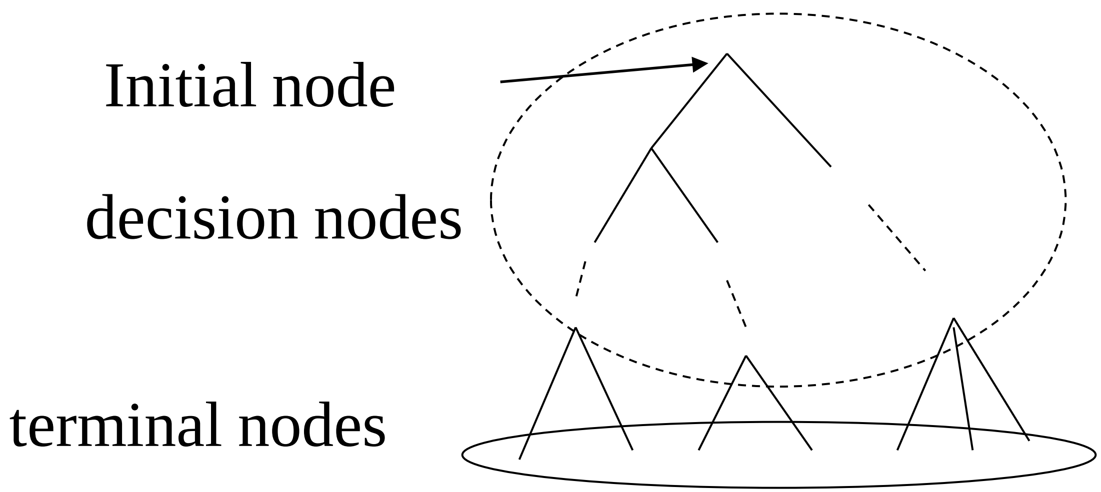
o.w. $\to p(x) \in X$

$s(x) = p^{-1}(x) = \{y \in X \mid p(y) = x\}$: the immediate successors of x

Tree structure $\to \{p(x)\} \cap s(x) = \varnothing$

$T = \{x \in X \mid s(x) = \varnothing\}$: terminal nodes; X-T : decision nodes



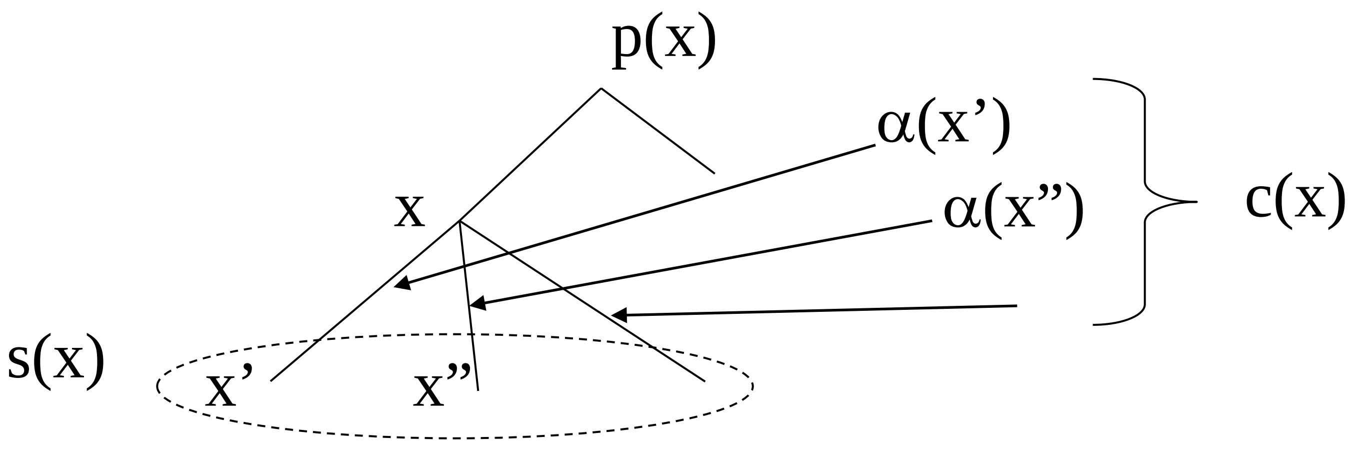


Extensive Form Games

(iii) $\alpha : X - \{x_0\} \rightarrow A$ action leads to x

$$x', x'' \in s(x),\ x' \neq x'' \ \rightarrow\ \alpha(x') \neq \alpha(x'')$$

$$c(x) = \{a \in A \mid a = \alpha(x') \text{ for some } x' \in s(x)\}$$



Extensive Form Games

(iv) $h : X \rightarrow H$ (collection of information sets)

$h(x)$: information set that contains x

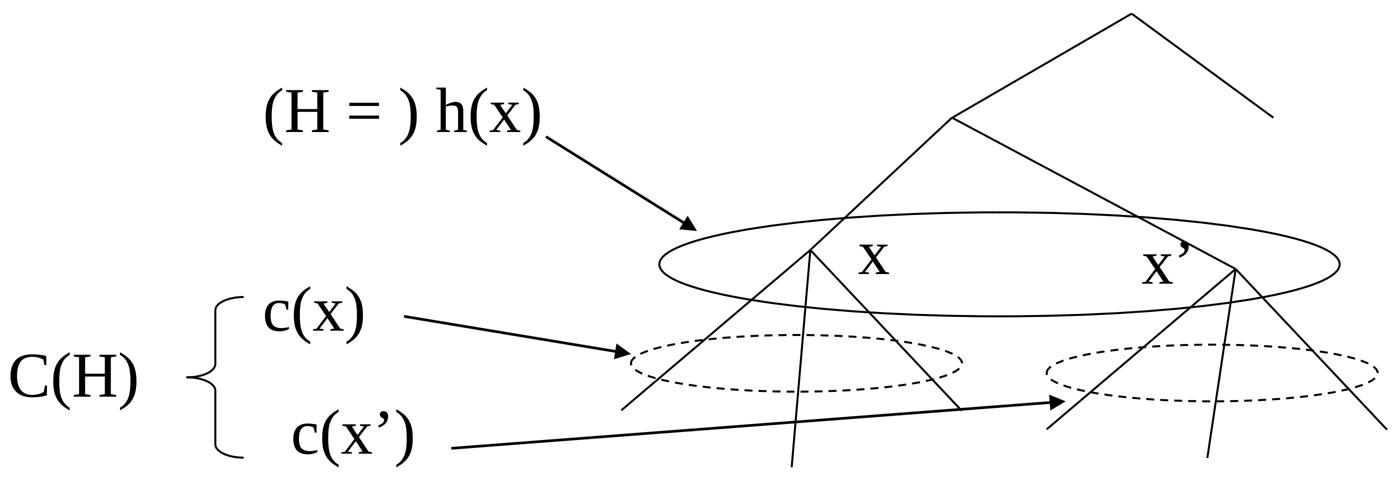
$h(x) = h(x')$ $\Rightarrow$ x, x' belong to the same information set

$\Rightarrow$ $c(x) = c(x')$

(Information sets form a partition of X.)

choices available at an information set H

$$C(H) = \{a \in A \mid a \in c(x) \text{ for some } x \in H\}$$



Extensive Form Games

(v) $\iota : \mathsf{H} \rightarrow \{0, 1, \ldots, I\}$

$\iota(H)$: the player who moves at the decision nodes in H

$\mathsf{H}_i = \{H \in \mathsf{H} \mid i = \iota(H)\}$ collection of i's information sets

H_0 = collection of information sets containing chance moves

(vi) $\rho : \mathsf{H}_0 \times A \rightarrow [0, 1]$ probability assigned to an action

$\rho(H, a) = 0$ if a is not in $C(H)$

$\sum_{a \in C(H)} \rho(H, a) = 1$ for all $H \in \mathsf{H}_0$

(vii) $u = \{u_1, \ldots, u_I\}$ payoff functions

u_i : T (set of terminal nodes) $\rightarrow \Re$

Extensive form game

$\Gamma_E = \{X, A, N = \{0,1,\ldots,I\}, p, \alpha, \mathsf{H}, h, \iota, \rho, u\}$

<u>Finiteness</u>: # of actions, # of moves, # of players

Strategic Form (Normal Form) Games

Definition 7.D.1:

Player i's strategy $s_i : \mathsf{H}_i \rightarrow A$

$s_i(H) \in C(H)$ for all $H \in \mathsf{H}_i$

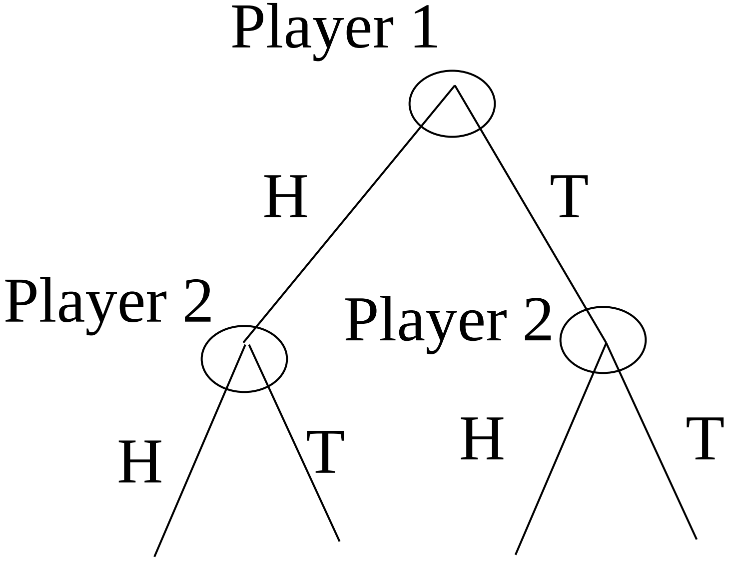
Strategy : complete contingent plan that tells a player to do at each of her information sets if she plays there

Strategy

Definition 7.D.1:

Player i's strategy $s_i : \mathcal{H}_i \to A$, $s_i(H) \in C(H)$ for all $H \in \mathcal{H}_i$

Example 7.D.1 (Matching Pennies Version B)



1 has two strategies (H, T)

2 has four strategies

(HH, HT, TH, TT)

HT $\Rightarrow$ play H if 1 plays H

(left information set)

play T if 1 plays T

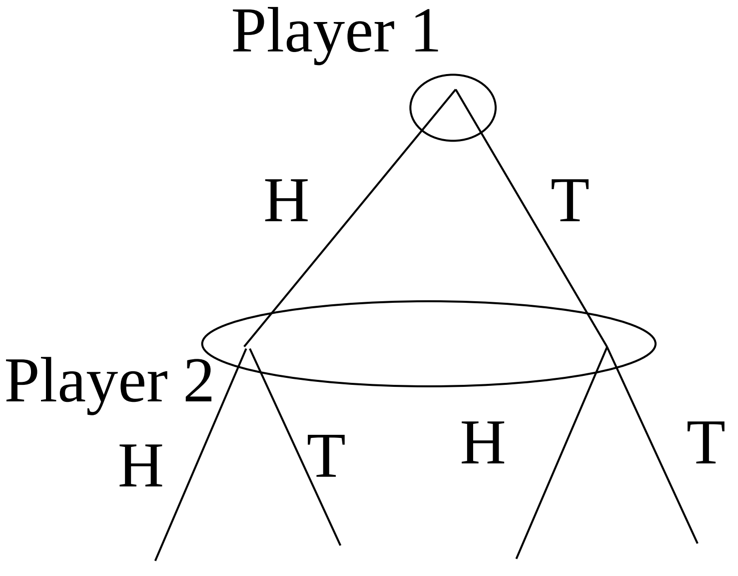
(right information set)

Strategy

Definition 7.D.1:

Player i's strategy $s_i : \mathcal{H}_i \rightarrow A$, $s_i(H) \in C(H)$ for all $H \in \mathcal{H}_i$

Example 7.D.2 (Matching Pennies Version C)



1 has two strategies (H, T)

2 has two strategies (H, T)

Notation: $s = (s_1, \ldots, s_I)$ strategy combination (profile)

$s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_I)$

$s = (s_i, s_{-i})$

Strategic Form (Normal Form) Game

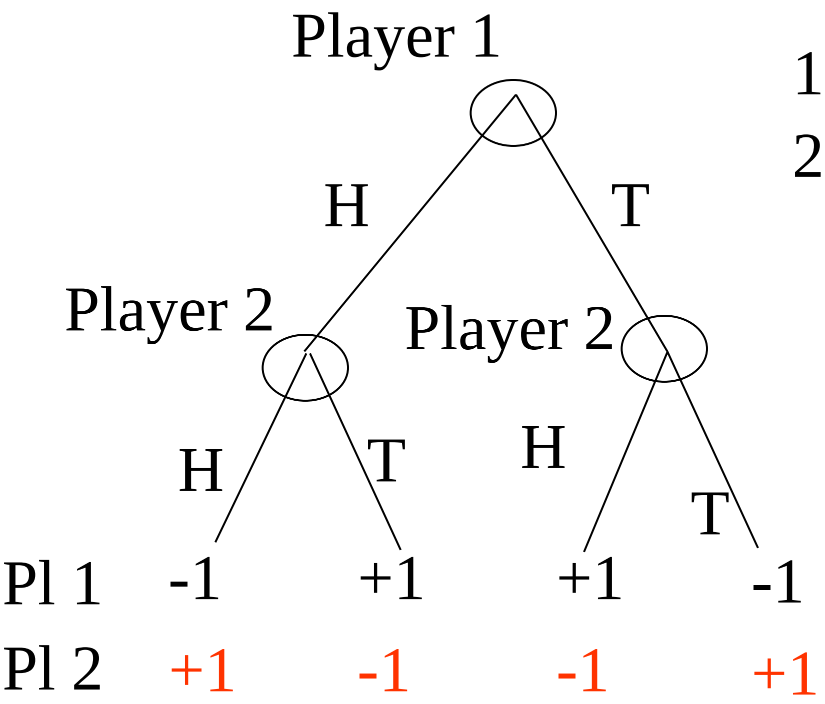
Definition 7.D.2:

Strategic form game $\Gamma_N = [N = \{0,1,\ldots,I\}, \{S_i\}, \{u_i\}]$

$N = \{0,1,\ldots,I\}$: set of players, S_i : player i's strategy set

$u_i : S_1 \times \ldots \times S_I \rightarrow \Re$, i's payoff function

Example 7.D.3 (Matching Pennies Version B)



1 has two strategies (H or T)

2 has four strategies (HH, HT, TH, TT)

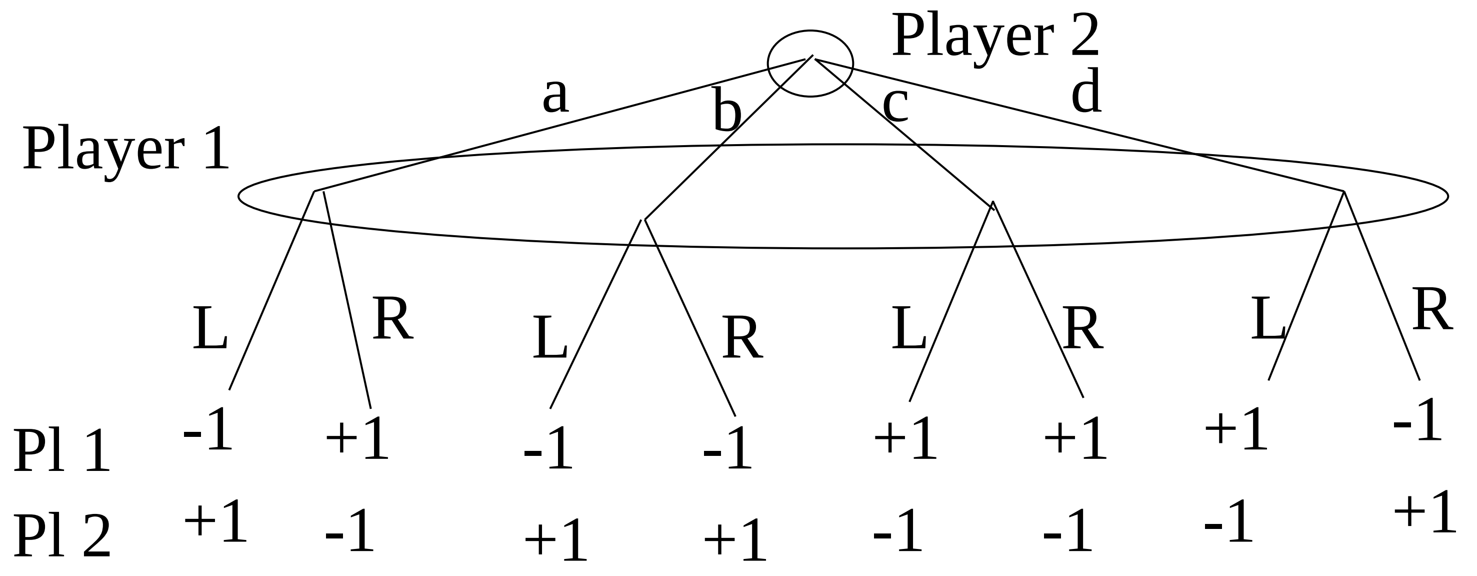
Pl. 2

Pl. 1	HH	HT	TH	TT
H	-1, +1	-1, +1	+1, -1	+1, -1
T	+1, -1	-1, +1	+1, -1	-1, +1

Strategic Form (Normal Form) Game

Note: extensive form game → strategic form game (unique)

<u>not unique</u> ←



Pl. 2

Pl. 1

	a	b	c	d
L	-1, +1	-1, +1	+1, -1	+1, -1
R	+1, -1	-1, +1	+1, -1	-1, +1

Randomized Strategy

Definition 7.E.1: (mixed strategy)

S_i : i's strategy set

$\sigma_i : S_i \rightarrow [0, 1]$ $\sigma_i(s_i) \geq 0$: prob. playing $s_i \in S_i$

$\Sigma_{si \in Si}\, \sigma_i(s_i) = 1$

$S_i = \{s_{1i}, \ldots , s_{Mi}\}$ (player i has M pure strategies)

i's set of mixed strategies

$\Delta(S_i) = \{(\sigma_{1i}, \ldots , \sigma_{Mi}) \mid \Sigma_{m=1}^{M}\, \sigma_{mi} = 1,\ \sigma_{mi} \geq 0\ \forall m = 1,\ldots,M\}$

$\sigma_{mi} = \sigma_i(s_{mi})$ mixed extension of S_i

i's expected payoff under $\sigma = (\sigma_1, \ldots , \sigma_I)$

$\Sigma_{(s1,\ldots,sI) \in S1 \times \ldots \times SI}\, \sigma_1(s_1) \ldots \sigma_I(s_I)\, u_i(s_1, \ldots ,s_I)$

$\Gamma_N = (N = \{0,1,\ldots,I\}, \{\Delta(S_i)\}, \{u_i\})$,

mixed extension of $\Gamma_N = (N = \{0,1,\ldots,I\}, \{S_i\}, \{u_i\})$,

Randomized Strategy

Definition 7.E.2: (behavior strategy)

extensive form game

i's behavior strategy λ assigns

to every information set $H \in \mathsf{H}_i$ and action $a \in C(H)$

probability $\lambda_i(a, H) \geq 0$

with $\Sigma_{a \in C(H)} \lambda_i(a, H) = 1$ for all $H \in \mathsf{H}_i$

Behavior strategy $\Rightarrow$ Mixed strategy

Games with perfect recall

$\rightarrow$ Behavior strategy $\Leftrightarrow$ Mixed strategy

Assignments

Problem Set 2 (due April 29):

Exercises (page 233) : 7.D.1, 7.D.2, 7.E.1

Reading Assignments:

Text Chapter 8, pp.235-245