低雑音増幅回路

通信機器システムの要

雑音の評価

雑音の定式化

瞬時値での評価→困難（不可）

↓

統計的な評価

2乗平均値：$\overline{v_n^2} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} v_n^2(t)dt$

複数の雑音源の取り扱い

$$\lim_{T\to\infty}\left[\frac{1}{T}\int_{-T/2}^{T/2}\{v_{n1}(t)+v_{n2}(t)\}^2 dt\right]$$

$$=\lim_{T\to\infty}\left[\frac{1}{T}\left\{\int_{-T/2}^{T/2}v_{n1}{}^2(t)+2v_{n1}(t)v_{n2}(t)+v_{n2}{}^2(t)dt\right\}\right]$$

$$=\overline{v_{n1}{}^2}+\lim_{T\to\infty}\frac{2}{T}\int_{-T/2}^{T/2}v_{n1}(t)v_{n2}(t)dt+\overline{v_{n2}{}^2}$$

$v_{n1}(t)$と$v_{n2}(t)$が全く独立：無相関

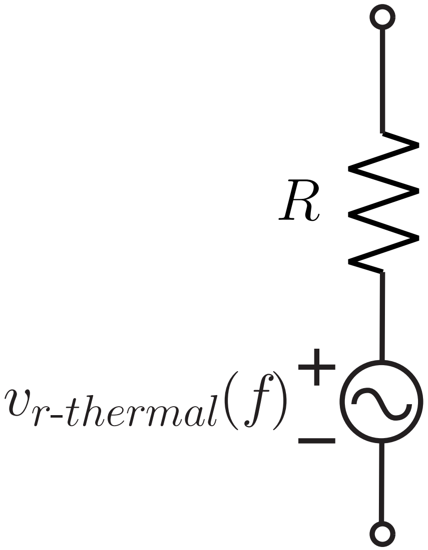
$$\int_0^{\tau}v_{n1}(t)v_{n2}(t)dt=0$$

雑音源を考慮した回路素子モデル

熱雑音, 1/f雑音, ショット雑音

熱雑音：キャリアが移動する際のランダムな動きによって生じる雑音

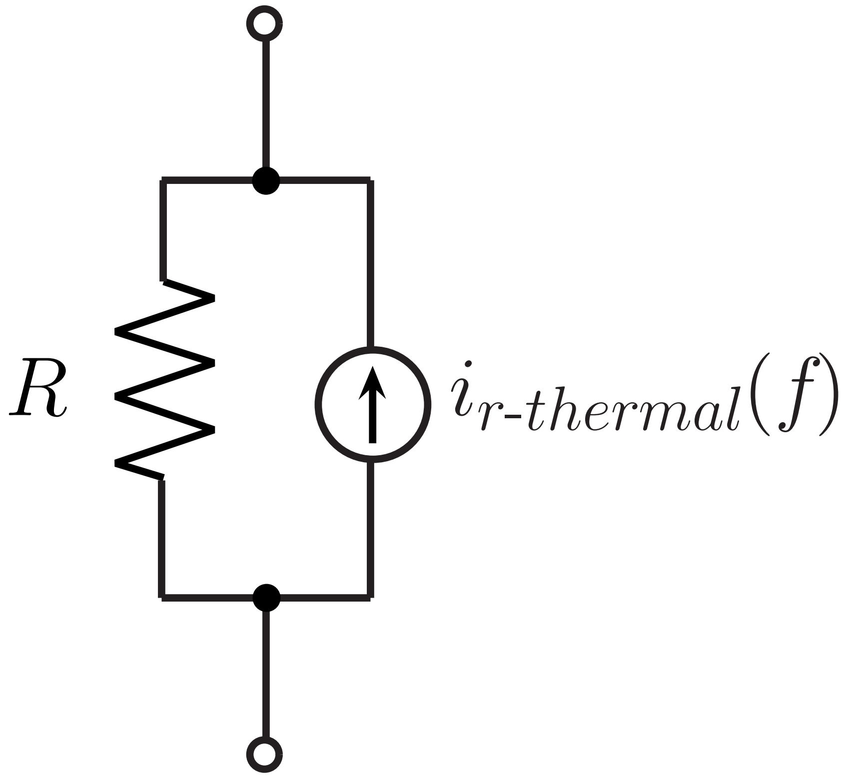
抵抗の熱雑音



(a)

$$\overline{v_{r-thermal}{}^2(f)} = 4kTR\,[V^2/Hz]$$

電源の等価性($v_{r-thermal}=R\,i_{r-thermal}$)から



(b)

$$\overline{i_{r-thermal}{}^2(f)}=4kT\frac{1}{R}[A^2/Hz]$$

MOSトランジスタの熱雑音

キャリアがチャネルを通過する際に発生

$$\overline{v_{mos-thermal}^{\;2}} = 4kT\gamma_n \frac{1}{g_m}$$

$$\overline{i_{mos-thermal}^{\;2}} = 4kT\gamma_n g_m$$

$\gamma_n \approx \frac{2}{3}$　（最小線幅が短くなると増加）

バイポーラトランジスタの熱雑音

ベース広がり抵抗：$\overline{v_{b-thermal}(f)^2} = 4kTr_b$

エミッタ抵抗，コレクタ･エミッタ間抵抗：仮想の抵抗

↓

熱雑音の発生無し

スペクトラム強度が一定：白色雑音

1/f雑音：シリコンの汚れや結晶欠陥によりキャリアが捕らわれたり，捕らわれたキャリアが離されたりを繰り返すというランダムな過程によって生じる雑音

直流電流がないと発生しない

$$\overline{v_{mos-1/f}^{2}(f)} = \frac{\alpha_{1/f}}{C_{OX} W L f}$$

$$\overline{i_{b-1/f}^{2}(f)} = \frac{K_{1/f} I_B^{a}}{f}$$

ショット雑音：

電流＝キャリアによる電流パルスの和の平均値

電流の平均値からの揺らぎ

$$\overline{i_{b-shot}^{2}(f)}=2qI_B$$

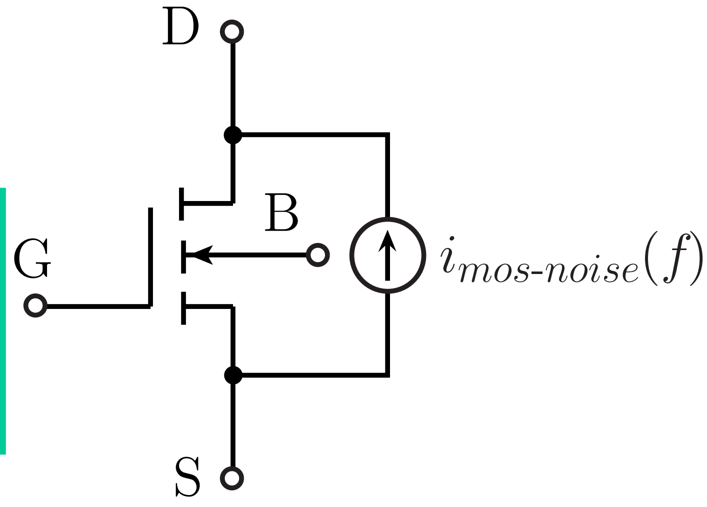
$$\overline{i_{c-shot}^{2}(f)}=2qI_C$$

$$1nA@1GHz \rightarrow 1.6\times10^{-19}C\times6.25\text{個}$$

雑音源を含むトランジスタモデル

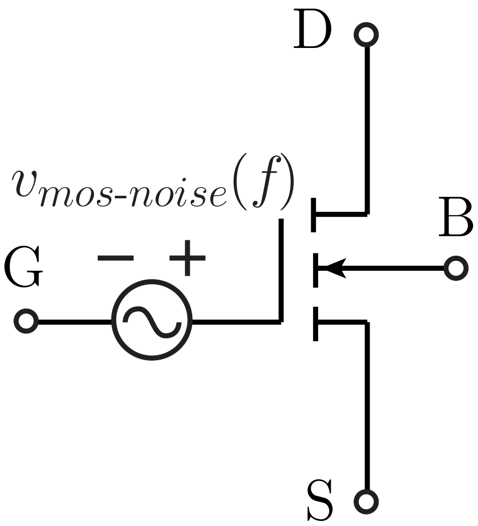
MOSトランジスタモデル

$$\overline{i_{mos-noise}{}^2(f)} = 4kT\gamma_n g_m + \frac{\alpha_{1/f}\, g_m{}^2}{C_{OX} W L f}$$



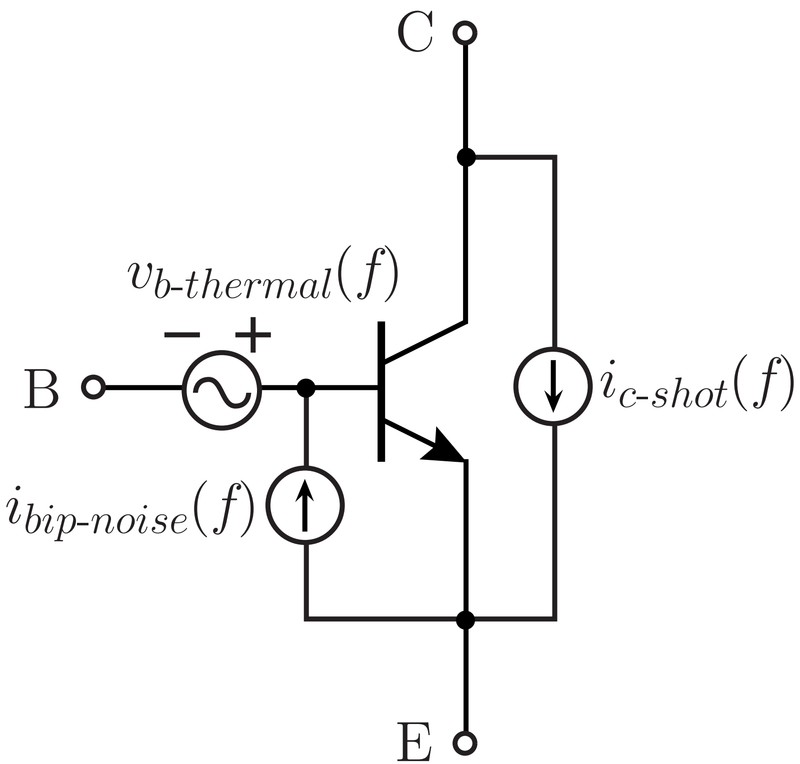
(a)

$$\overline{v_{mos-noise}{}^2(f)} = 4kT\gamma_n \frac{1}{g_m} + \frac{\alpha_{1/f}}{C_{OX} W L f}$$



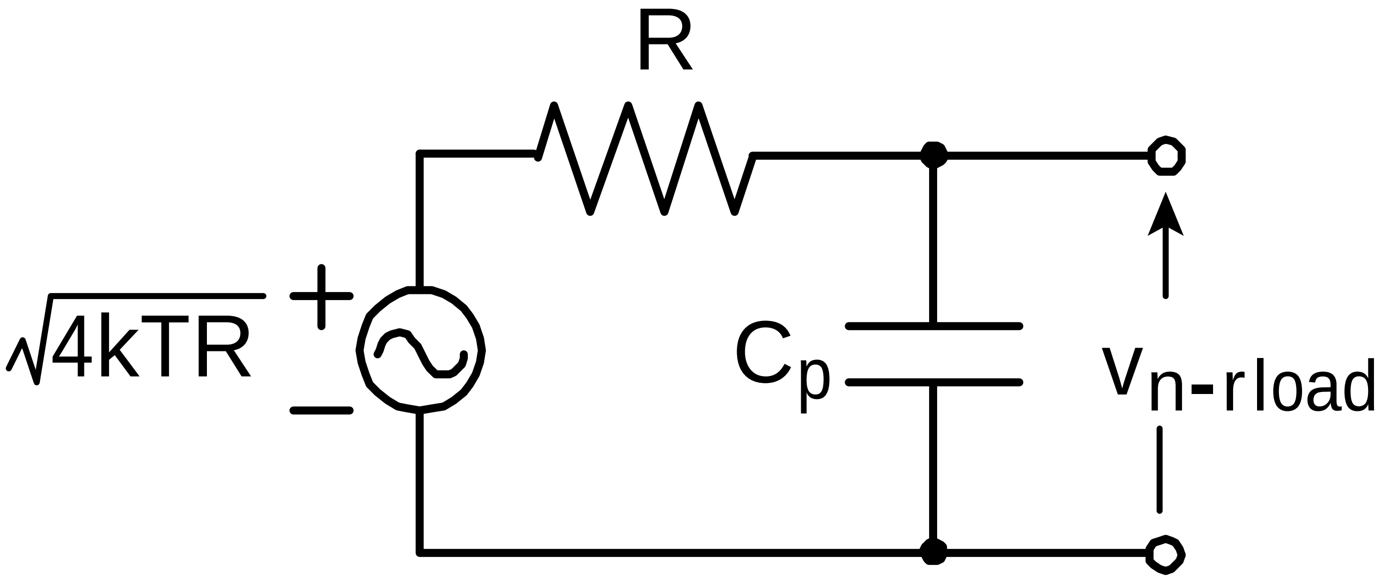
(b)

バイポーラトランジスタモデル



$$\overline{i_{bip-noise}{}^2(f)} = \frac{K_{1/f}\, I_B{}^a}{f} + 2qI_B$$

雑音解析の例(抵抗が発生する雑音)



$$T_{rload}(s) = \frac{1}{1+sC_pR} \Rightarrow \left|T_{rload}(j\omega)\right|^2 = \frac{1}{1+\omega^2 C_p{}^2 R^2}$$

$$T_{rload}(s)=\frac{1}{1+sC_pR}\Rightarrow\left|T_{rload}(j\omega)\right|^2=\frac{1}{1+\omega^2C_p{}^2R^2}$$

$$\overline{v_{n-rload}{}^2}=\int_0^\infty\left|T_{rload}(j\omega)\right|^2\frac{4kTR}{2\pi}d\omega$$

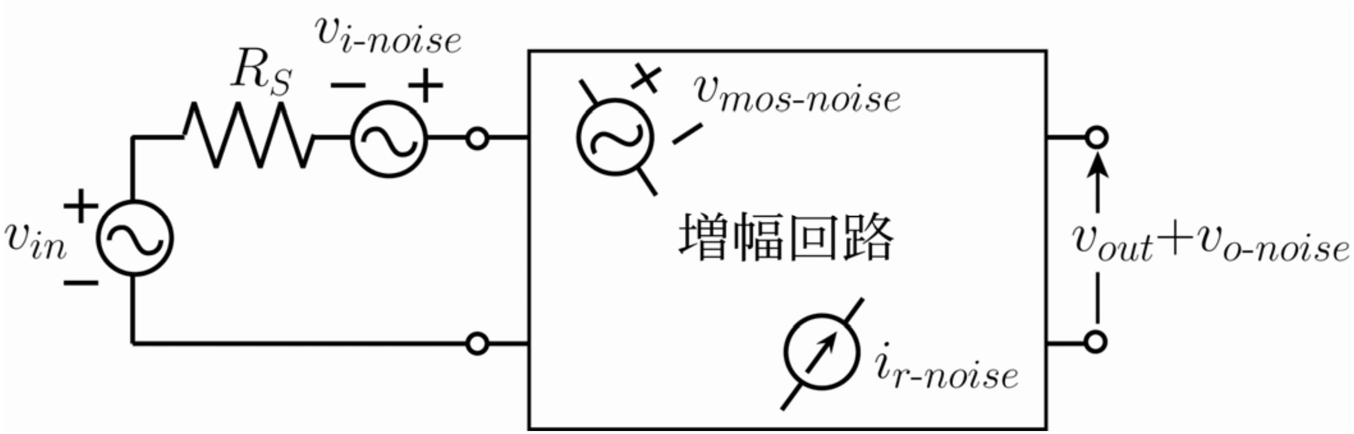
$$=\int_0^\infty\frac{1}{1+\omega^2C_p{}^2R^2}\frac{4kTR}{2\pi}d\omega$$

$$=\frac{2kT}{C_p{}^2R\pi}\int_0^\infty\frac{1}{\omega^2+\frac{1}{C_p{}^2R^2}}d\omega=\frac{2kT}{C_p{}^2R\pi}\left[C_pR\tan^{-1}(C_pR\omega)\right]_0^\infty$$

$$=\frac{2kT}{C_p{}^2R\pi}\cdot C_pR\left(\frac{\pi}{2}-0\right)=\frac{kT}{C_p}$$

増幅回路と雑音

雑音係数と雑音指数

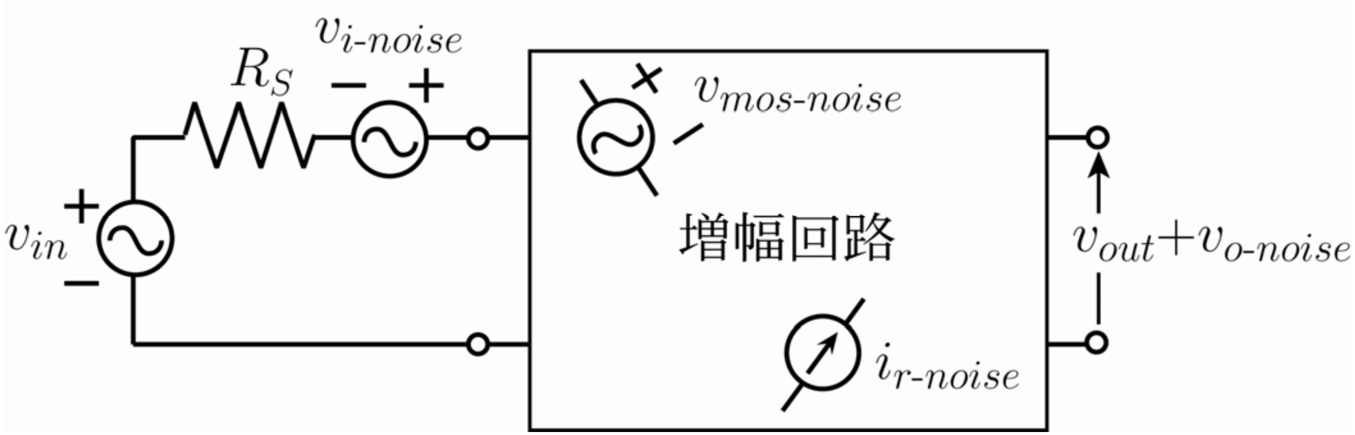


$$SNR_{in}=\frac{\overline{v_{in}^{\,2}}}{v_{i\text{–}noise}^{\,2}}$$

$$SNR_{out}=\frac{A^2\overline{v_{in}^{\,2}}}{v_{o\text{–}noise}^{\,2}}$$

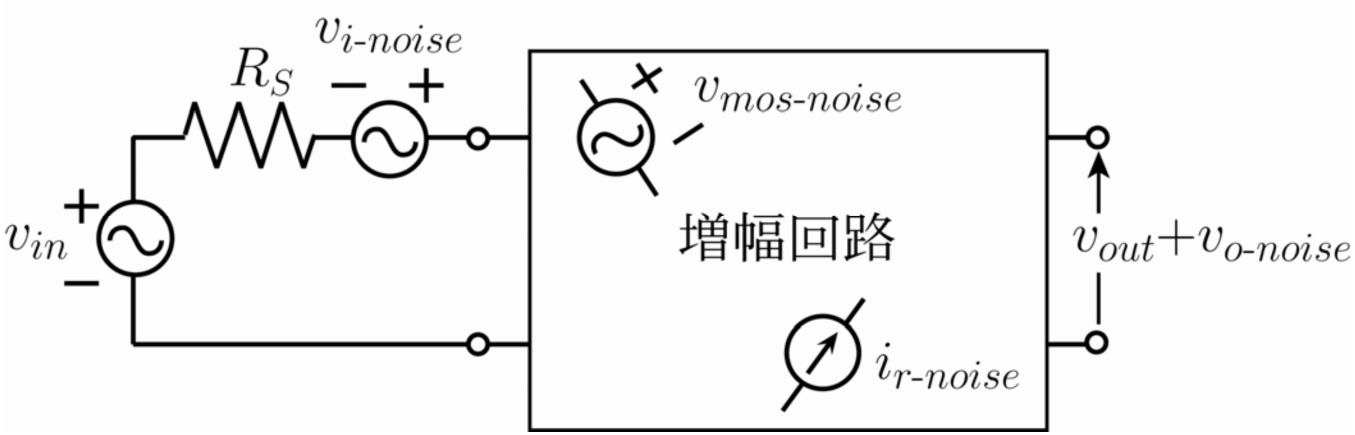
$$F=\frac{SNR_{in}}{SNR_{out}}$$ ：雑音係数

$NF=10\log F$ ：雑音指数



$$\overline{v_{o-noise}^{\ 2}} = A^2\,\overline{v_{i-noise}^{\ 2}} + \overline{v_{inner-noise}^{\ 2}}$$

$v_{mos-noise}$や$i_{r-noise}$などの増幅回路内部の雑音に起因する成分



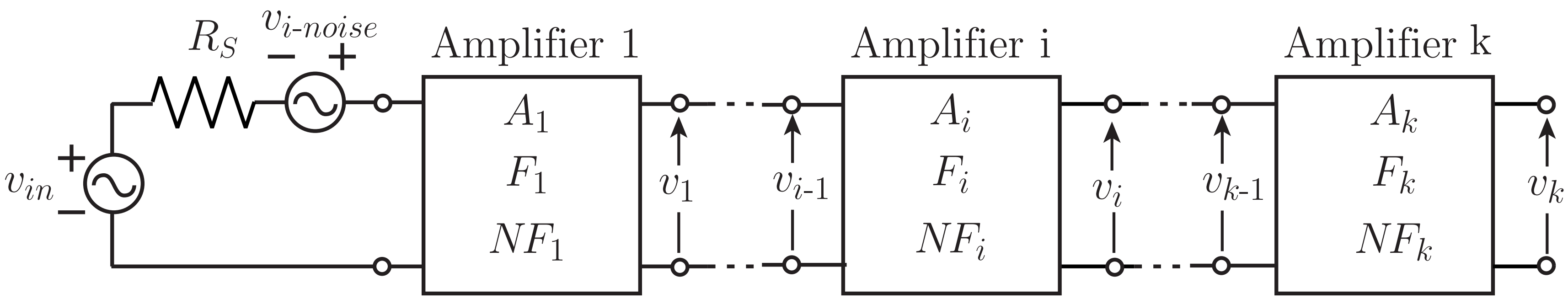
$$\overline{v_{o-noise}^2} = A^2\overline{v_{i-noise}^2} + \overline{v_{inner-noise}^2}$$

$$SNR_{out} = \frac{A^2\overline{v_{in}^2}}{\overline{v_{o-noise}^2}} = \frac{\overline{v_{in}^2}}{\overline{v_{i-noise}^2} + \overline{v_{inner-noise}^2}/A^2}$$

$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{v_{i-noise}^2}} \geq SNR_{out}$$

$$F = 1 + \frac{\overline{v_{inner-noise}^2}}{A^2\overline{v_{i-noise}^2}}$$

縦続接続型システムの評価

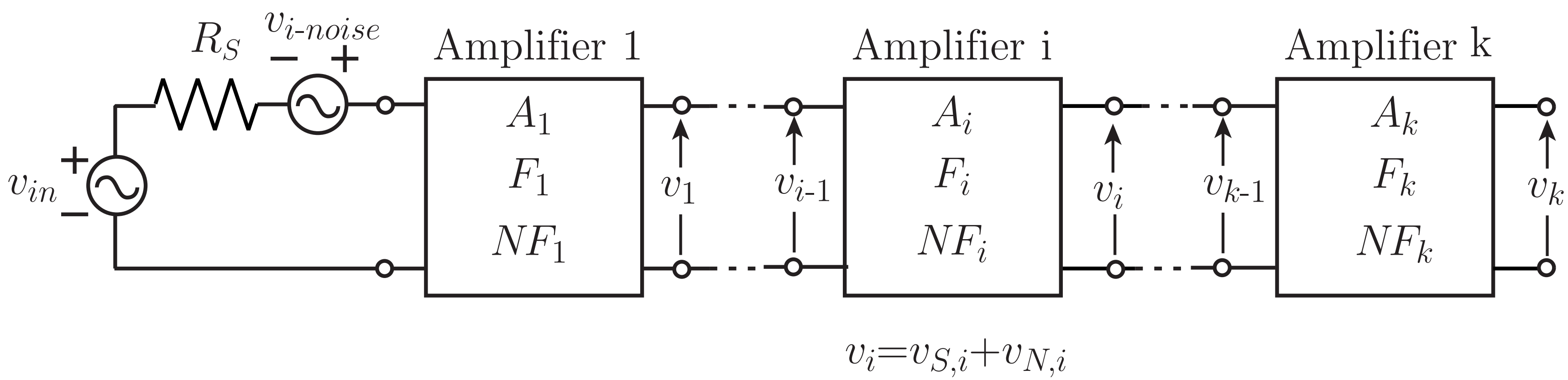


$v_i = v_{S,i} + v_{N,i}$

$$\overline{v_{o-noise}^{\,2}} = A^2\,\overline{v_{i-noise}^{\,2}} + \overline{v_{inner-noise}^{\,2}}$$

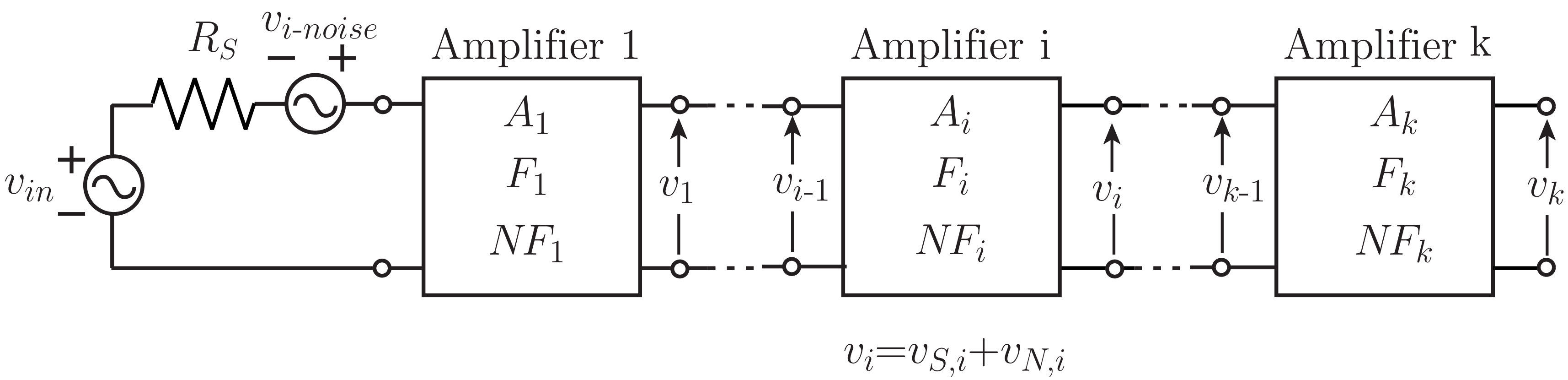
$$\overline{v_{N,i}^{\,2}} = A_i^{\,2} F_i\,\overline{v_{N,i-1}^{\,2}}$$

$$F = 1 + \frac{\overline{v_{inner-noise}^{\,2}}}{A^2\,\overline{v_{i-noise}^{\,2}}}$$



$$\overline{v_{N,i}^{2}} = A_i^2 F_i \overline{v_{N,i-1}^{2}} \qquad \overline{v_{S,i}^{2}} = A_i^2 \overline{v_{S,i-1}^{2}}$$

$$F_{total} = \frac{\dfrac{\overline{v_{in}^{2}}}{\overline{v_{i-noise}^{2}}}}{\dfrac{\overline{v_{S,k}^{2}}}{\overline{v_{N,k}^{2}}}} = \frac{\dfrac{\overline{v_{in}^{2}}}{\overline{v_{i-noise}^{2}}}}{\dfrac{A_1^2 A_2^2 \cdots A_k^2 \overline{v_{in}^{2}}}{A_1^2 F_1 A_2^2 F_2 \cdots A_k^2 F_k \overline{v_{i-noise}^{2}}}} = F_1 F_2 \cdots F_k$$



$$\overline{v_{N,i}^2} = A_i^2 \overline{v_{N,i-1}^2} + \overline{v_{inner-noise,i}^2}$$

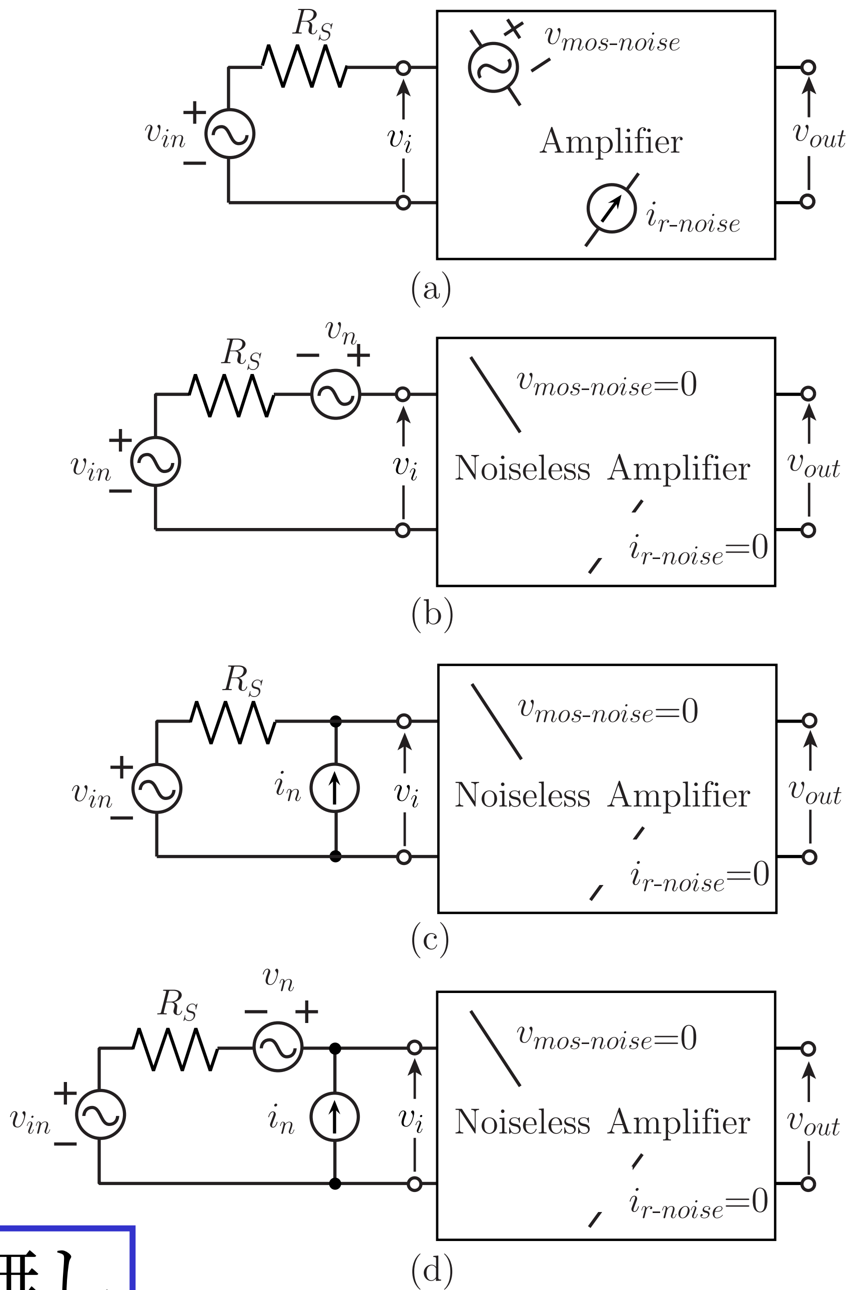
$$\begin{aligned}\overline{v_{N,k}^2} &= A_k^2 A_{k-1}^2 \cdots A_1^2 \overline{v_{i-noise}^2} \\ &+ A_k^2 A_{k-1}^2 \cdots A_2^2 \overline{v_{inner-noise,1}^2} \\ &+ A_k^2 A_{k-1}^2 \cdots A_3^2 \overline{v_{inner-noise,2}^2} + \cdots \\ &+ A_k^2 \overline{v_{inner-noise,k-1}^2} + \overline{v_{inner-noise,k}^2}\end{aligned}$$

$$\overline{v_{N,k}^2} = A_k^2 A_{k-1}^2 \cdots A_1^2 \overline{v_{i-noise}^2}$$
$$+ A_k^2 A_{k-1}^2 \cdots A_2^2 \overline{v_{inner-noise,1}^2}$$
$$+ A_k^2 A_{k-1}^2 \cdots A_3^2 \overline{v_{inner-noise,2}^2} + \cdots$$
$$+ A_k^2 \overline{v_{inner-noise,k-1}^2} + \overline{v_{inner-noise,k}^2}$$

$$F_{total} = \frac{\overline{v_{in}^2}}{\overline{v_{i-noise}^2}} \cdot \frac{\overline{v_{N,k}^2}}{\overline{v_{S,k}^2}} = 1 + \frac{\overline{v_{inner-noise,1}^2}}{A_1^2 \overline{v_{i-noise}^2}} + \frac{\overline{v_{inner-noise,2}^2}}{A_1^2 A_2^2 \overline{v_{i-noise}^2}}$$
$$+ \cdots + \frac{\overline{v_{inner-noise,k-1}^2}}{A_1^2 A_2^2 \cdots A_{k-1}^2 \overline{v_{i-noise}^2}} + \frac{\overline{v_{inner-noise,k}^2}}{A_1^2 A_2^2 \cdots A_k^2 \overline{v_{i-noise}^2}}$$

低雑音増幅回路の設計

増幅回路内部で発生する雑音の等価表現



(b): R_Sが無限大のとき矛盾

(c): R_Sが零のとき矛盾

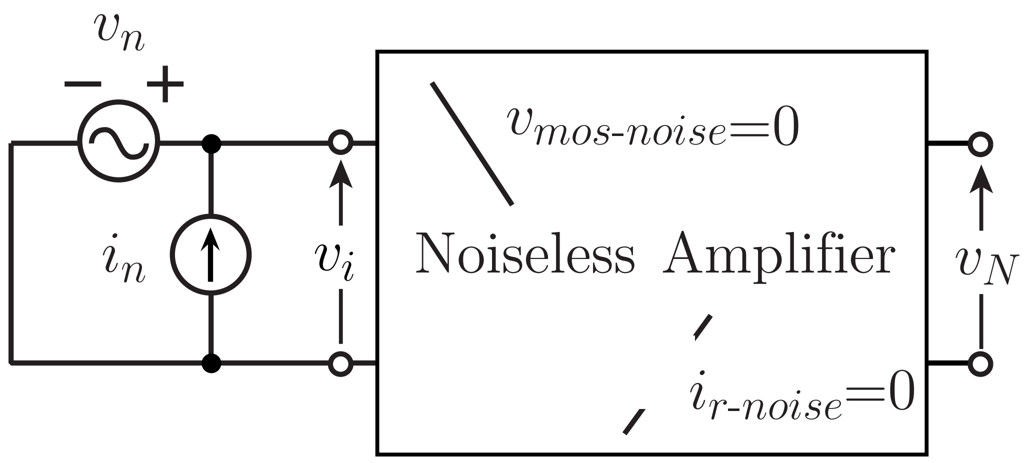
(d): 任意のR_Sについて矛盾無し

増幅回路の出力雑音のみ考慮

v_nの求め方

$$v_n = \frac{v_N}{A}$$

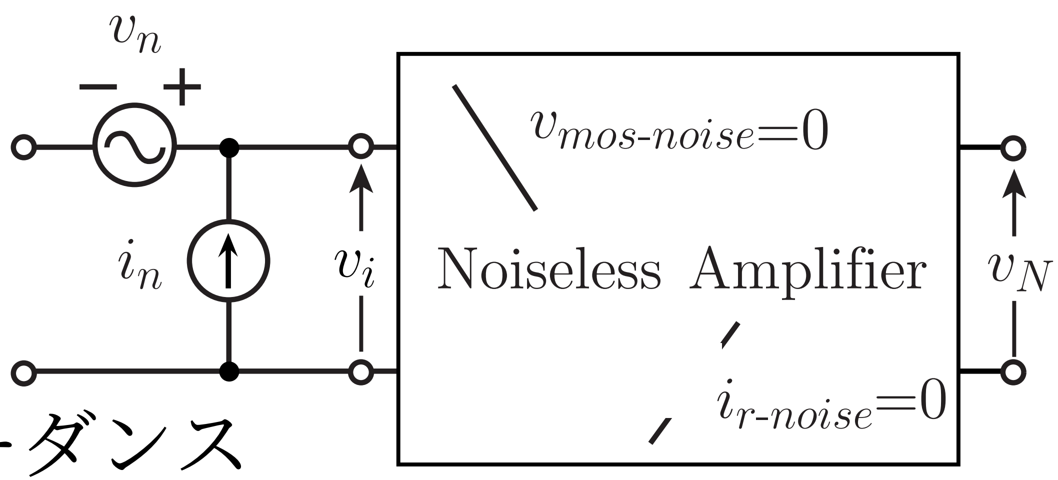
A：電圧利得



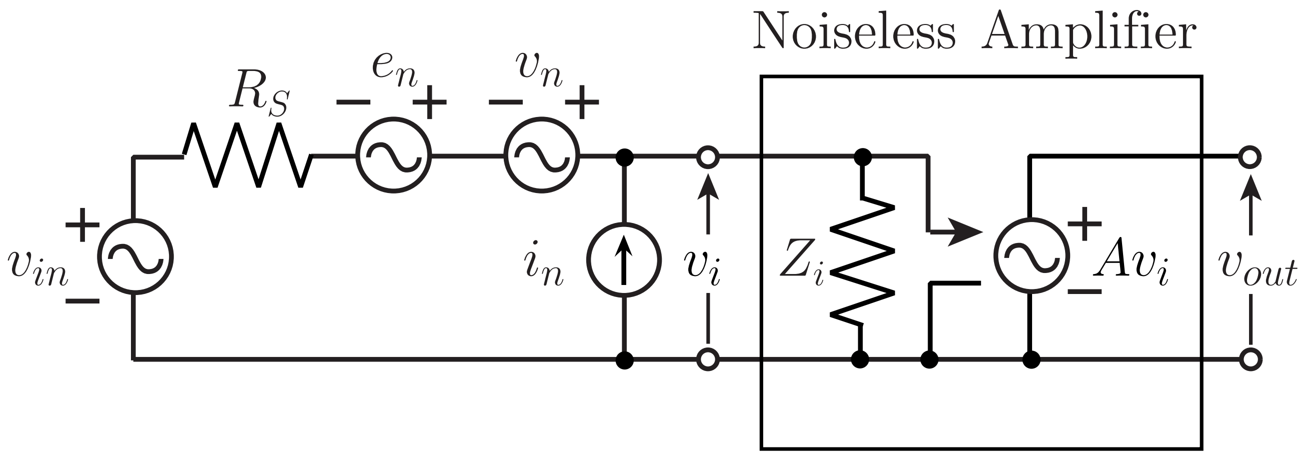
i_nの求め方

$$i_n = \frac{v_N}{Z_T}$$

Z_T: 伝達インピーダンス

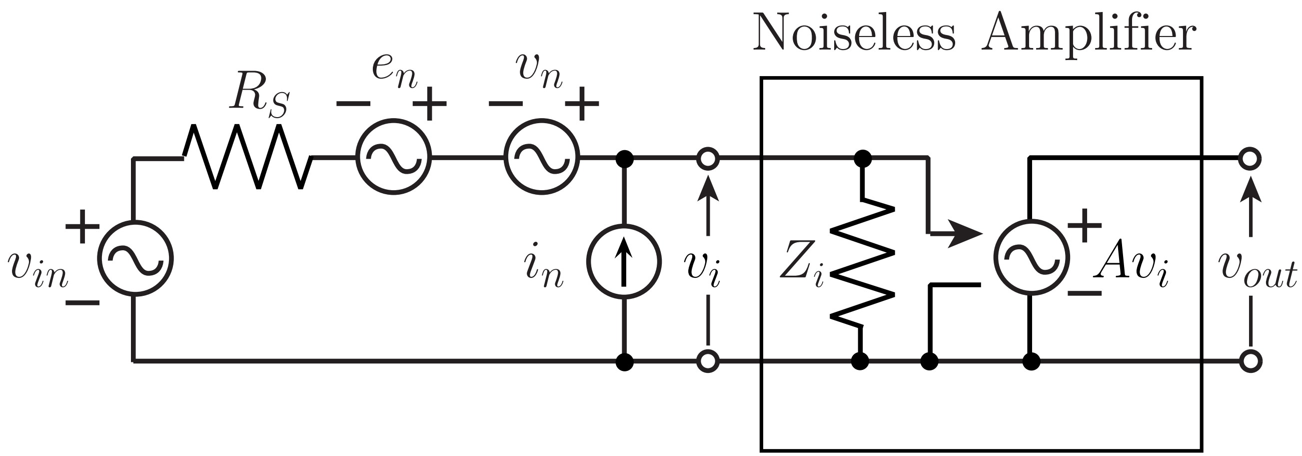


雑音整合



$$\overline{e_n{}^2} = 4kTR_S$$

$$v_S = \frac{AZ_i}{R_S + Z_i} v_{in}$$



e_nとv_n，e_nとi_nは無相関

v_nとi_nも無相関と仮定

$$\overline{v_N{}^2}=\frac{A^2 Z_i{}^2}{|R_S+Z_i|^2}\left(\overline{e_n{}^2}+\overline{v_n{}^2}+R_S{}^2\,\overline{i_n{}^2}\right)$$

$$\overline{e_n^2} = 4kTR_S \qquad v_S = \frac{AZ_{in}}{R_S + Z_{in}} v_{in}$$

$$\overline{v_N^2} = \frac{A^2 Z_{in}^2}{|R_S + Z_{in}|^2} \left(\overline{e_n^2} + \overline{v_n^2} + R_S^2 \overline{i_n^2} \right)$$

$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{e_n^2}}$$

$$SNR_{out} = \frac{\overline{v_S^2}}{\overline{v_N^2}} = \frac{\overline{v_{in}^2}}{\overline{e_n^2} + \overline{v_n^2} + R_S^2 \overline{i_n^2}}$$

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{\overline{e_n^2} + \overline{v_n^2} + R_S^2 \overline{i_n^2}}{\overline{e_n^2}} = 1 + \frac{\overline{v_n^2} + R_S^2 \overline{i_n^2}}{4kTR_S}$$

$$F = 1 + \frac{\overline{v_n^2} + R_S^2\,\overline{i_n^2}}{4kTR_S}$$

相加・相乗平均の定理

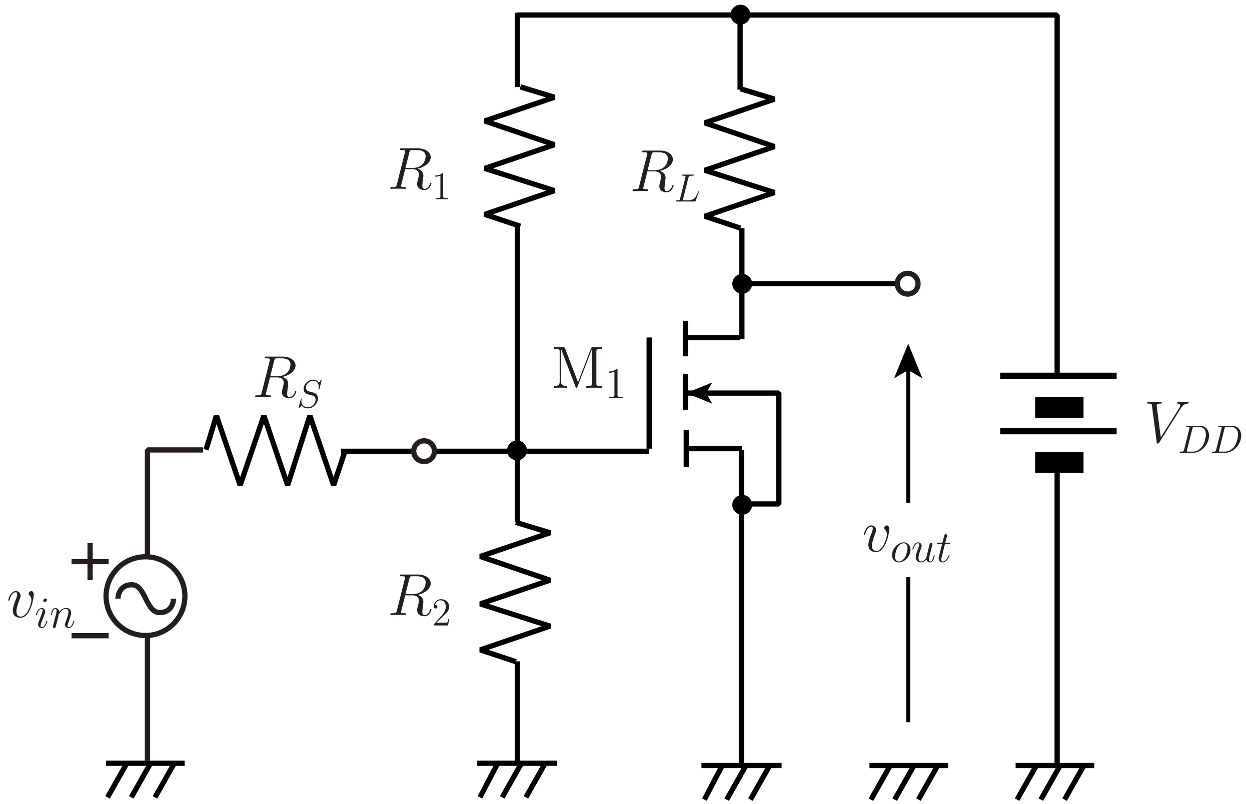
$$a + b \geq 2\sqrt{ab}$$

$$\frac{\overline{v_n^2}}{R_S} = R_S\,\overline{i_n^2}$$ のときFが最小

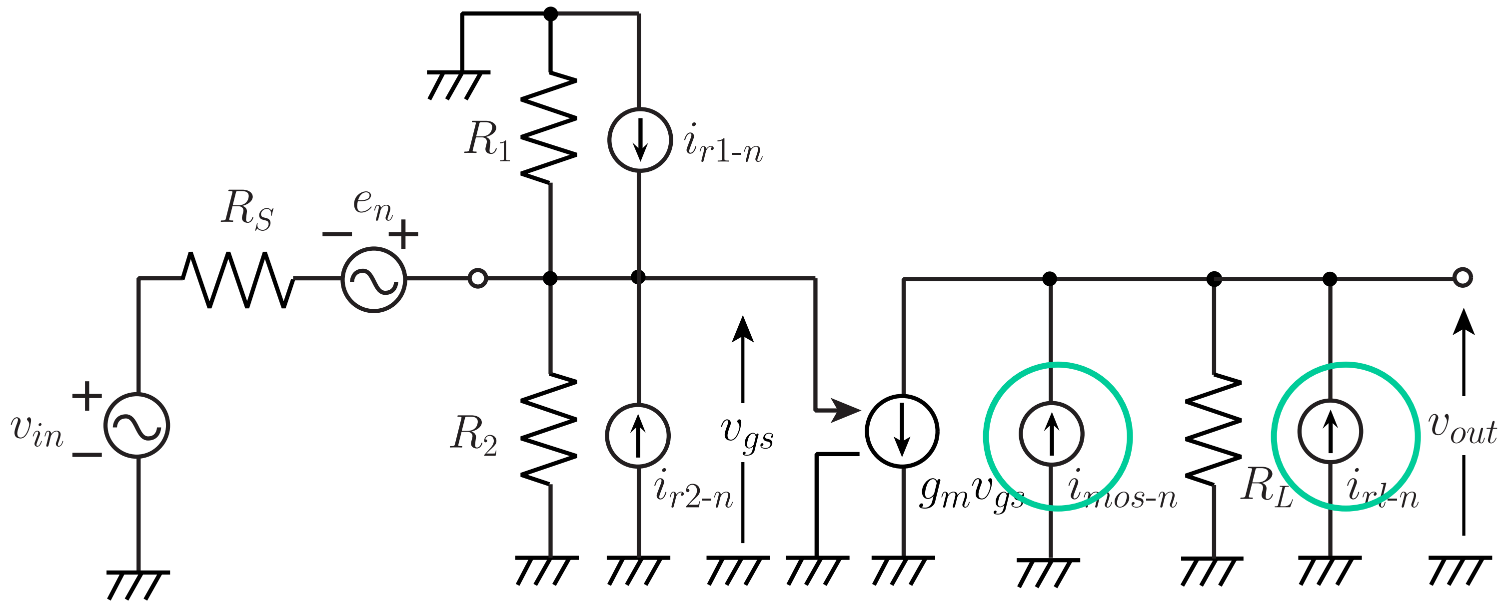
$$R_S = \sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}}$$

実際の増幅回路の雑音解析例

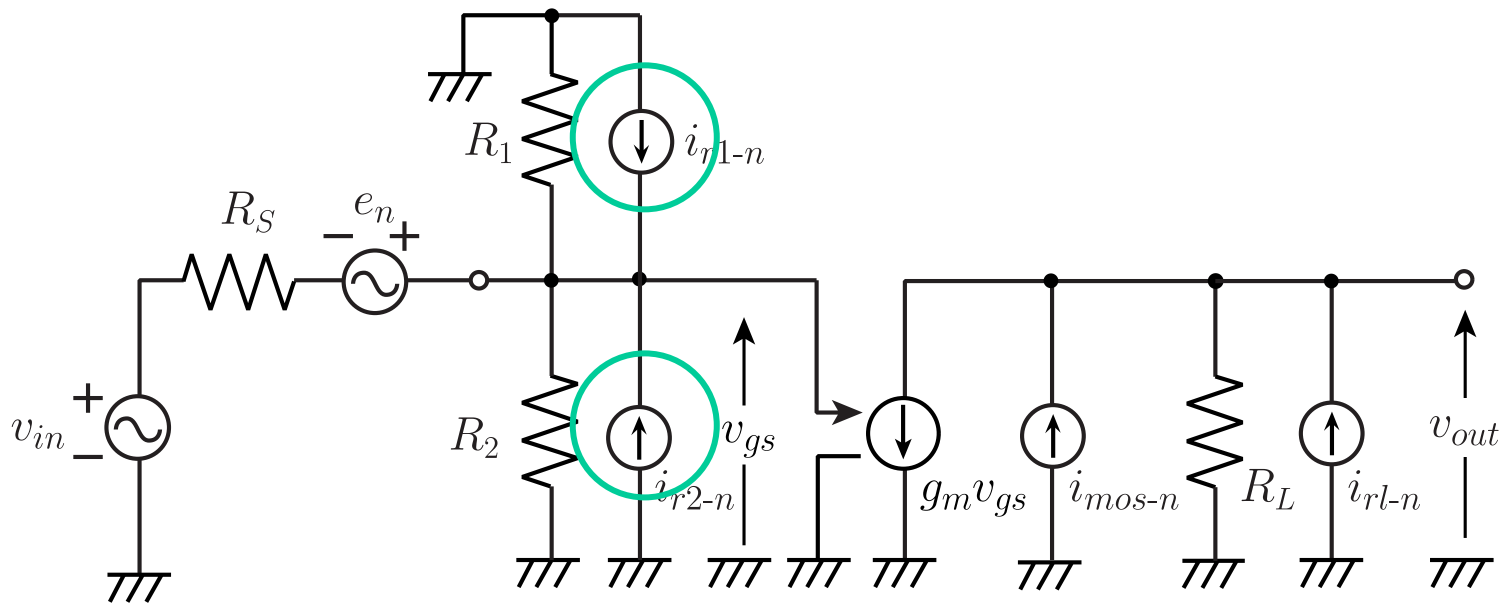
ソース接地増幅回路



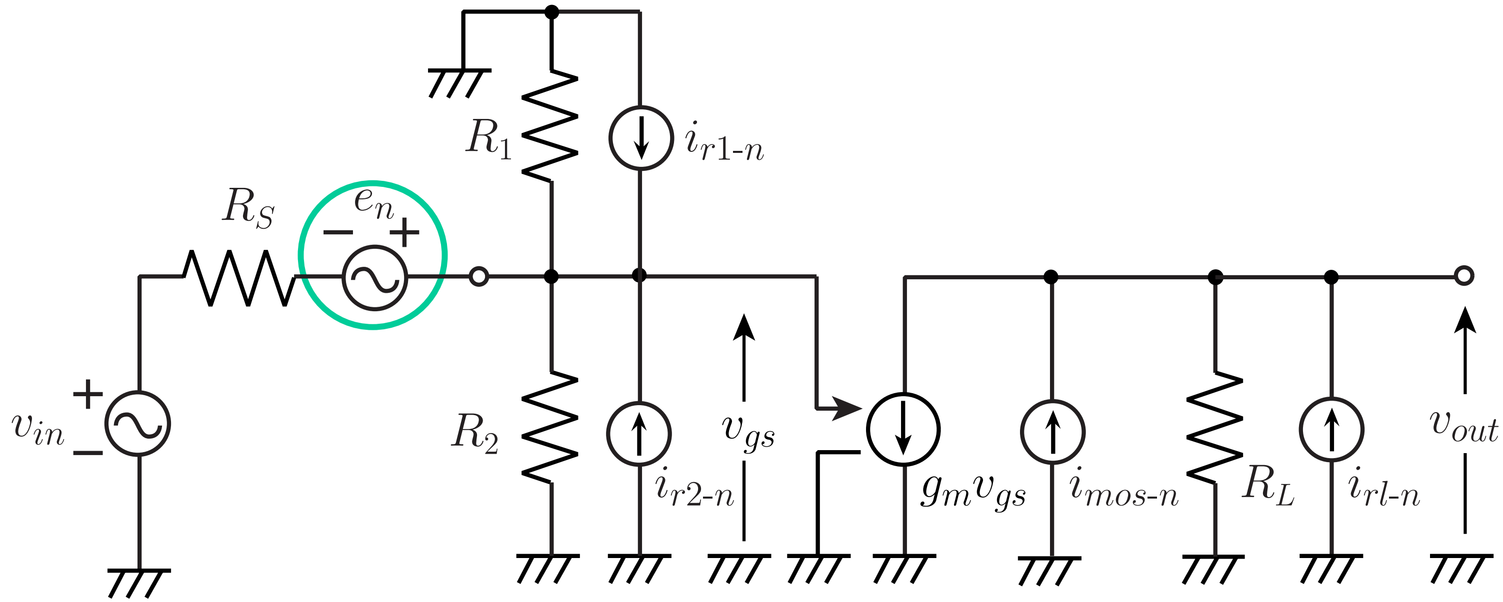
ソース接地増幅回路の小信号モデル



$$\overline{v_{o\text{-}n1}^2} = R_L^2\left(\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2}\right)$$



$$\overline{v_{o\text{-}n}{}_2{}^2} = \left| g_m R_L \frac{R_1 R_2 R_S}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \left(\overline{i_{r1\text{-}n}{}^2} + \overline{i_{r\,2\text{-}n}{}^2} \right)$$



$$\overline{v_{o\text{-}n3}^{\,2}} = \left| g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \overline{e_n^{\,2}}$$

$$\overline{v_N^2} = \overline{v_{o-n1}^2} + \overline{v_{o-n2}^2} + \overline{v_{o-n3}^2}$$

$$= R_L^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$

$$+ \left| g_m R_L \frac{R_1 R_2 R_S}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 (\overline{i_{r1-n}^2} + \overline{i_{r2-n}^2})$$

$$+ \left| g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \overline{e_n^2}$$

$$\overline{v_S^2} = \left| g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right|^2 \overline{v_{in}^2}$$

$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{e_n^2}}$$

$$\overline{v_N^2}=\overline{v_{o\text{-}n1}^2}+\overline{v_{o\text{-}n2}^2}+\overline{v_{o\text{-}n3}^2}$$

$$\overline{v_S^2}=\left|g_m R_L \frac{R_1R_2}{R_1R_2+R_2R_S+R_SR_1}\right|^2 \overline{v_{in}^2}$$

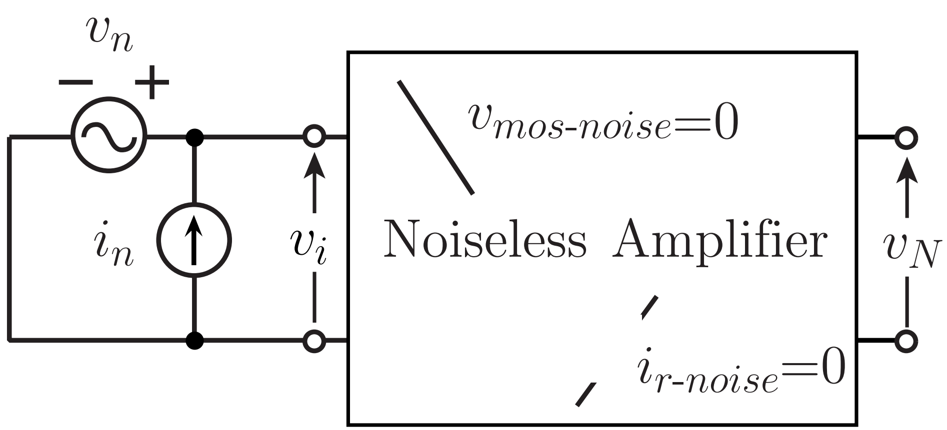
$$SNR_{in}=\frac{\overline{v_{in}^2}}{\overline{e_n^2}} \qquad e_n^2=4kTR_S$$

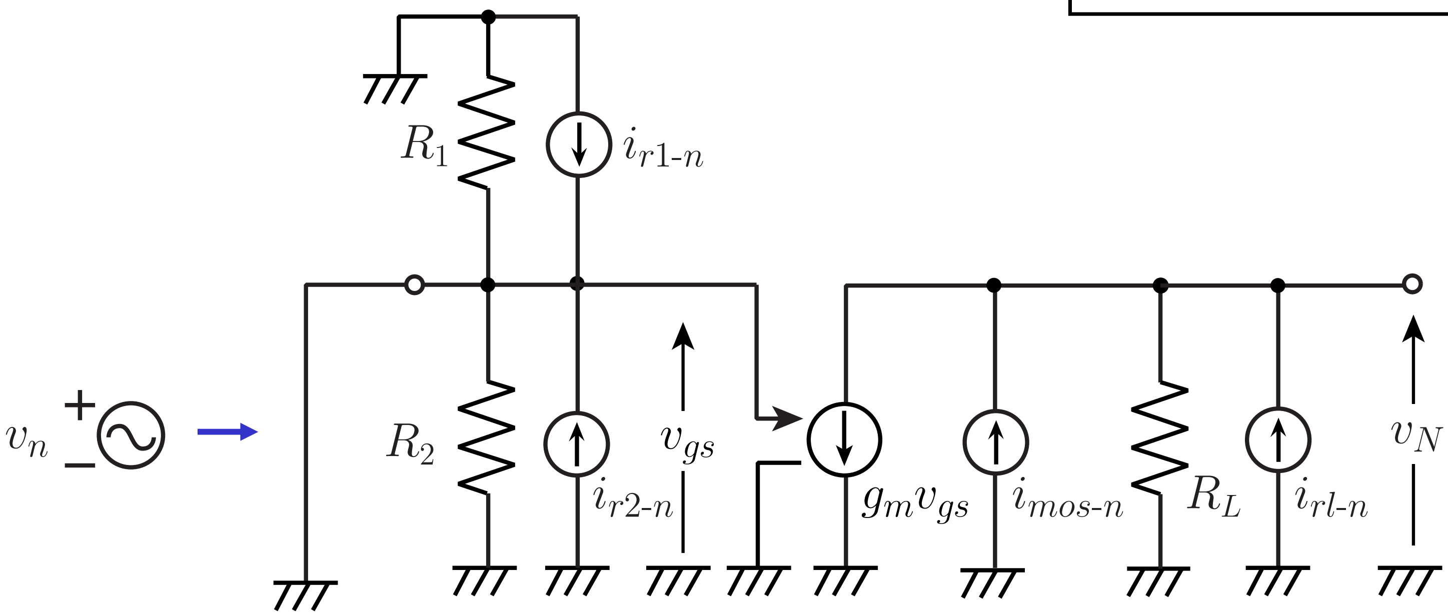
$$F=\frac{SNR_{in}}{SNR_{out}}$$

$$=1+\frac{1}{4kTR_S}\left\{\left|\frac{R_1R_2+R_2R_S+R_SR_1}{g_mR_1R_2}\right|^2(\overline{i_{rl\text{-}n}^2}+\overline{i_{mos\text{-}n}^2})\right.$$

$$\left.+R_S^2(\overline{i_{r1\text{-}n}^2}+\overline{i_{r2\text{-}n}^2})\right\}$$

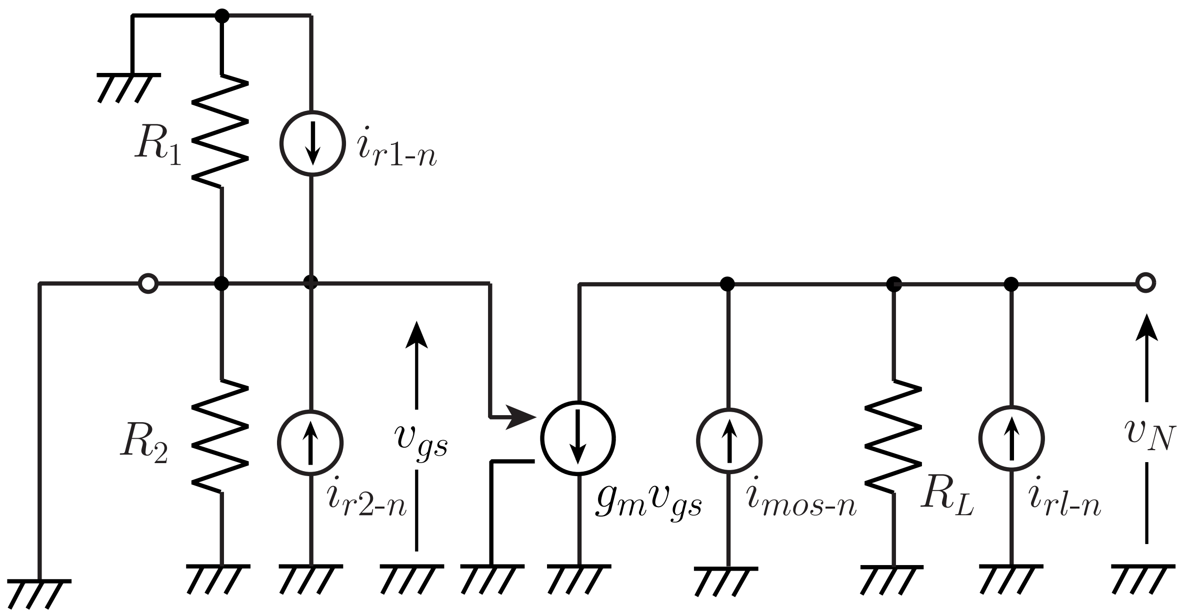
v_nの求め方





$$A=\frac{v_{out}}{v_n}=-g_m R_L$$

$$\overline{v_n^2}=\frac{\overline{v_N^2}}{A^2}$$

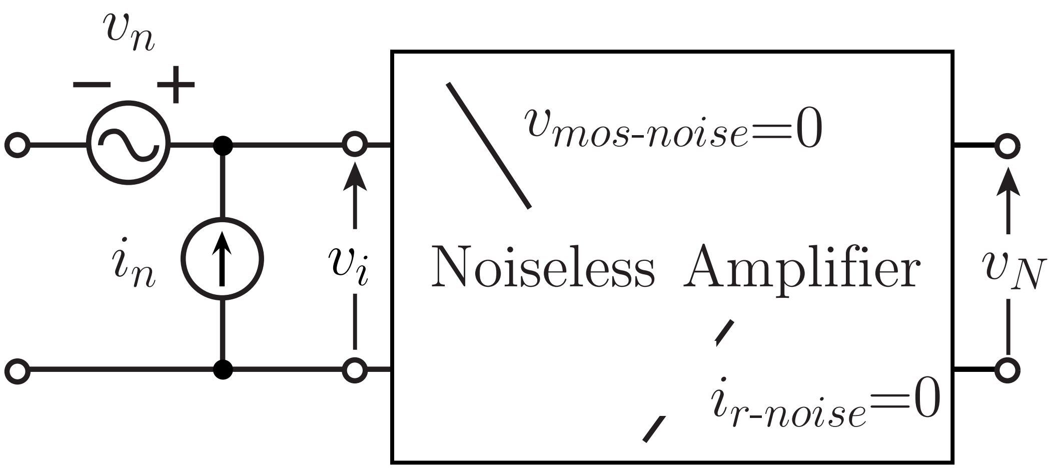


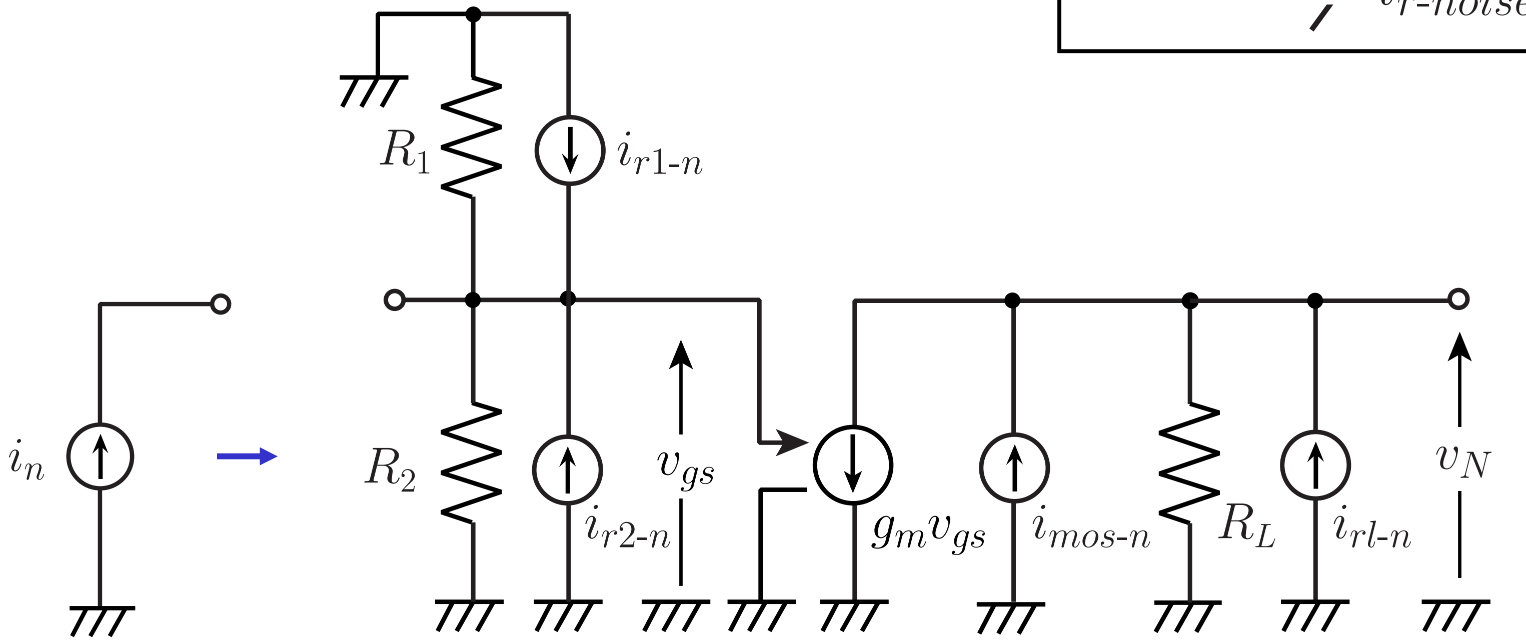
$$\overline{v_N^{\,2}} = R_L^{\,2}\left(\overline{i_{rl\text{-}n}^{\,2}} + \overline{i_{mos\text{-}n}^{\,2}}\right)$$

$$A = \frac{v_{out}}{v_n} = -g_m R_L$$

$$\overline{v_n^{\,2}} = \frac{1}{g_m^{\,2}}\left(\overline{i_{rl\text{-}n}^{\,2}} + \overline{i_{mos\text{-}n}^{\,2}}\right)$$

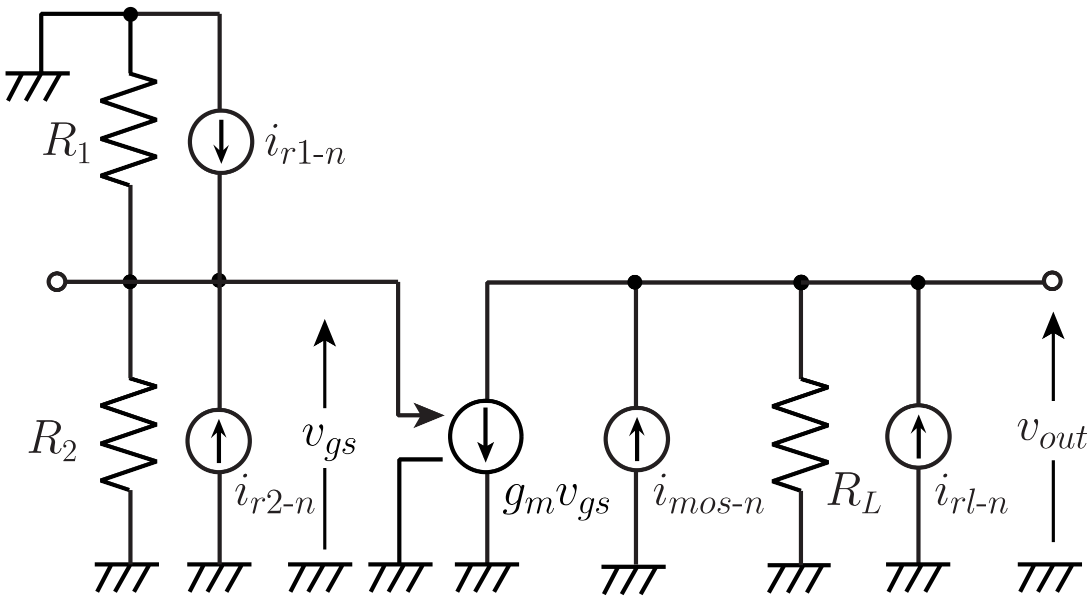
i_nの求め方





$$Z_T = -g_m R_L \frac{R_1 R_2}{R_2 + R_1}$$

$$\overline{i_n^{\,2}} = \frac{\overline{v_N^{\,2}}}{Z_T^{\,2}}$$

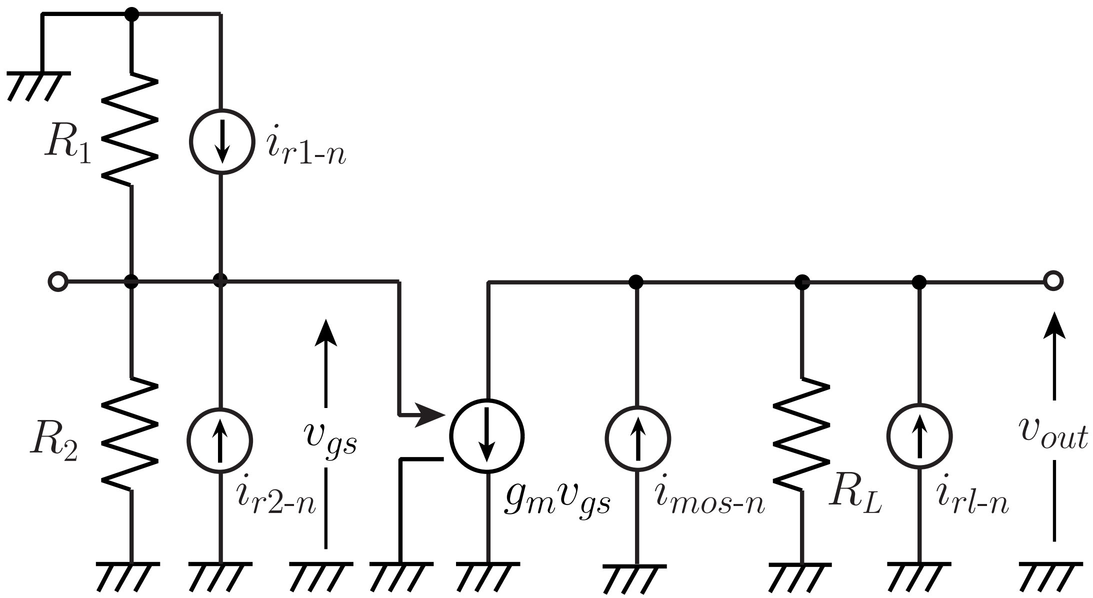


$$\overline{v_{o\text{-}n1}^2} = R_L^2 \left(\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2}\right)$$

$$\overline{v_{o\text{-}n2}^2} = \left| g_m R_L \frac{R_1 R_2}{R_2 + R_1} \right|^2 \left(\overline{i_{r1\text{-}n}^2} + \overline{i_{r2\text{-}n}^2}\right)$$

$$\overline{v_{out}^2} = \overline{v_{o\text{-}n1}^2} + \overline{v_{o\text{-}n2}^2}$$

$$\overline{v_N^2} = R_L^2(\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2}) + \left| g_m R_L \frac{R_1 R_2}{R_2 + R_1} \right|^2 (\overline{i_{r1\text{-}n}^2} + \overline{i_{r2\text{-}n}^2})$$



$$i_n = \frac{v_N}{Z_T}$$

$$Z_T = -g_m R_L \frac{R_1 R_2}{R_2 + R_1}$$

$$\overline{i_n^2} = \left| \frac{R_1 + R_2}{g_m R_2 R_1} \right|^2 (\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2}) + (\overline{i_{r1\text{-}n}^2} + \overline{i_{r2\text{-}n}^2})$$

$$\overline{v_n^2} = \frac{1}{g_m^2}\left(\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2}\right)$$

$$\overline{i_n^2} = \left|\frac{R_1 + R_2}{g_m R_2 R_1}\right|^2 \left(\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2}\right) + \left(\overline{i_{r1\text{-}n}^2} + \overline{i_{r2\text{-}n}^2}\right)$$

$$F = 1 + \frac{\overline{v_n^2} + R_S^2\,\overline{i_n^2}}{4kTR_S}$$ より

$$F = 1 + \frac{1}{4kTR_S}\left[\left\{\frac{1}{g_m^2} + \left|\frac{(R_1 + R_2)R_S}{g_m R_1 R_2}\right|^2\right\}\left(\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2}\right) + R_S^2\left(\overline{i_{r1\text{-}n}^2} + \overline{i_{r2\text{-}n}^2}\right)\right]$$

v_nとi_nの間の相関も考慮

$$F = 1 + \frac{1}{4kTR_S}\left\{\left|\frac{R_1R_2 + R_2R_S + R_SR_1}{g_mR_1R_2}\right|^2 (\overline{i_{rl\text{-}n}^{\,2}} + \overline{i_{mos\text{-}n}^{\,2}}) + R_S^{\,2}(\overline{i_{r1\text{-}n}^{\,2}} + \overline{i_{r2\text{-}n}^{\,2}})\right\}$$

v_nとi_nは無相関と仮定

$$F = 1 + \frac{1}{4kTR_S}\left[\left\{\frac{1}{g_m^{\,2}} + \left|\frac{(R_1 + R_2)R_S}{g_mR_1R_2}\right|^2\right\}(\overline{i_{rl\text{-}n}^{\,2}} + \overline{i_{mos\text{-}n}^{\,2}}) + R_S^{\,2}(\overline{i_{r1\text{-}n}^{\,2}} + \overline{i_{r2\text{-}n}^{\,2}})\right]$$

低雑音増幅回路の設計(1)

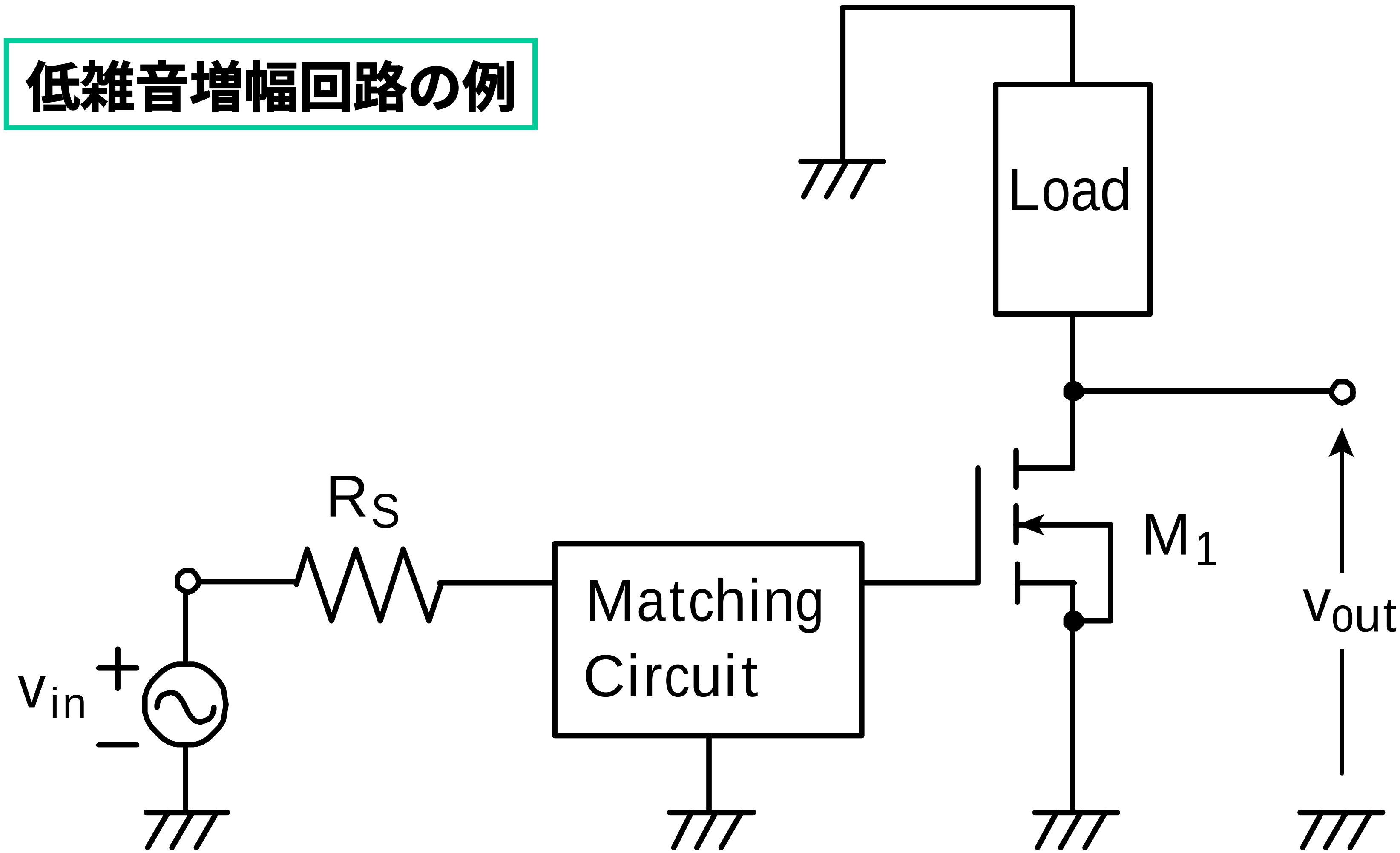
通信機器用低雑音増幅回路

高周波(数GHz)

狭帯域

LC共振特性の応用

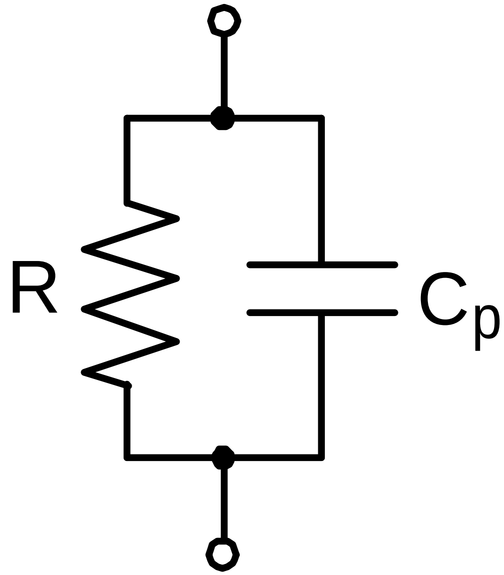
低雑音増幅回路の例



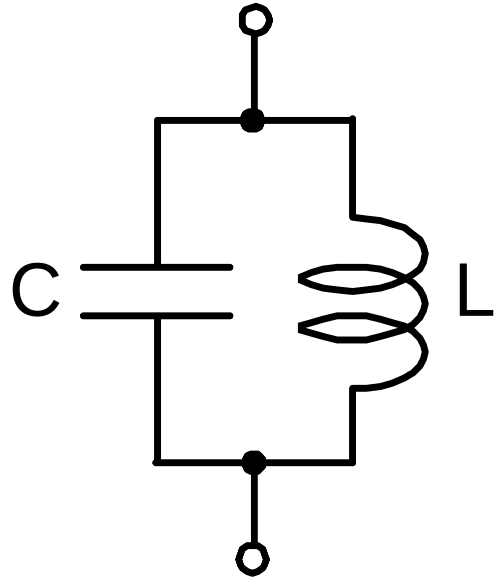
バイアス回路は省略

低雑音増幅回路における負荷の例

抵抗 (寄生容量付き)

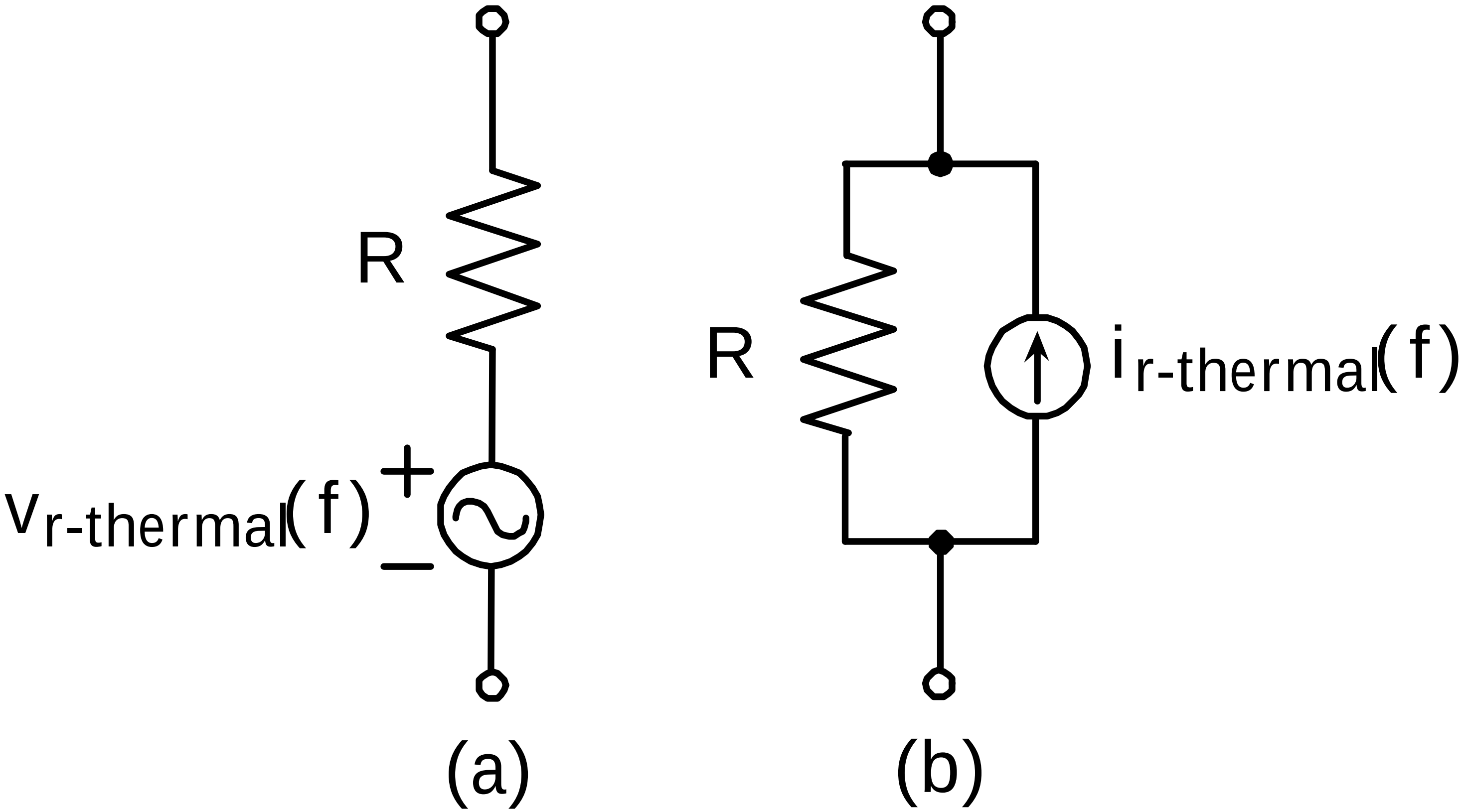


LC共振回路



LC共振回路は無雑音?

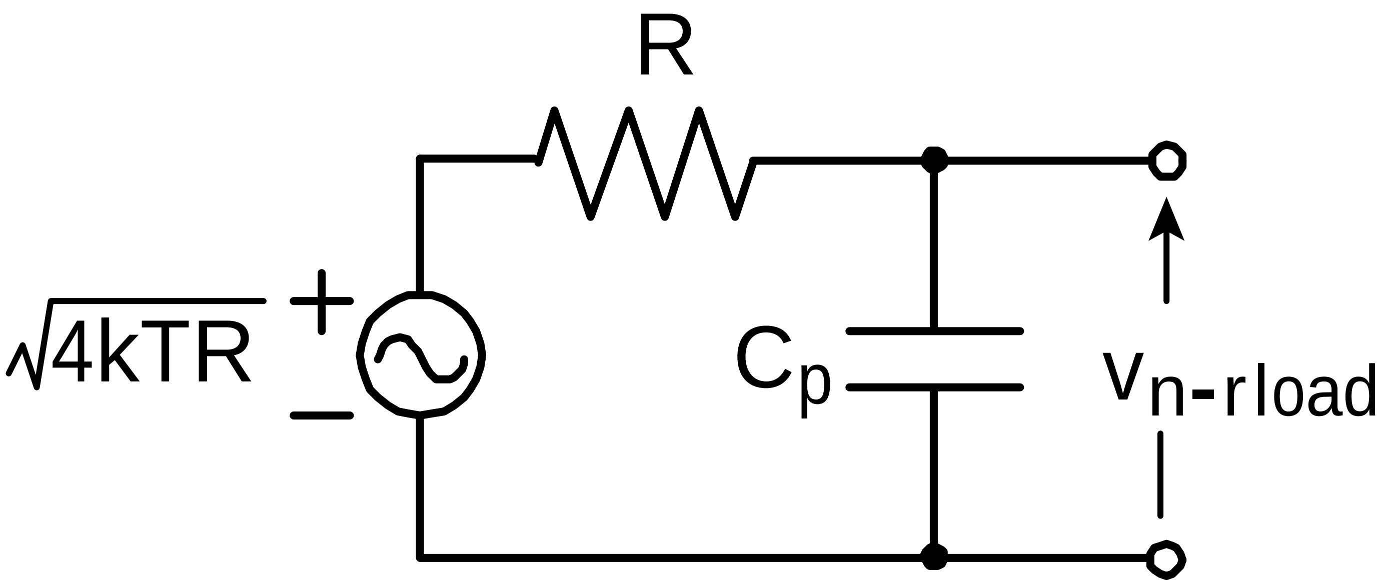
抵抗の熱雑音



$$\overline{v_{r-thermal}^{\,2}(f)} = 4kTR$$

$$\overline{i_{r-thermal}^{\,2}(f)} = 4kT\frac{1}{R}$$

負荷抵抗が発生する雑音



$$T_{rload}(s)=\frac{1}{1+sC_pR}\Rightarrow\left|T_{rload}(j\omega)\right|^2=\frac{1}{1+\omega^2C_p{}^2R^2}$$

$$T_{rload}(s)=\frac{1}{1+sC_pR}\Rightarrow\left|T_{rload}(j\omega)\right|^2=\frac{1}{1+\omega^2C_p{}^2R^2}$$

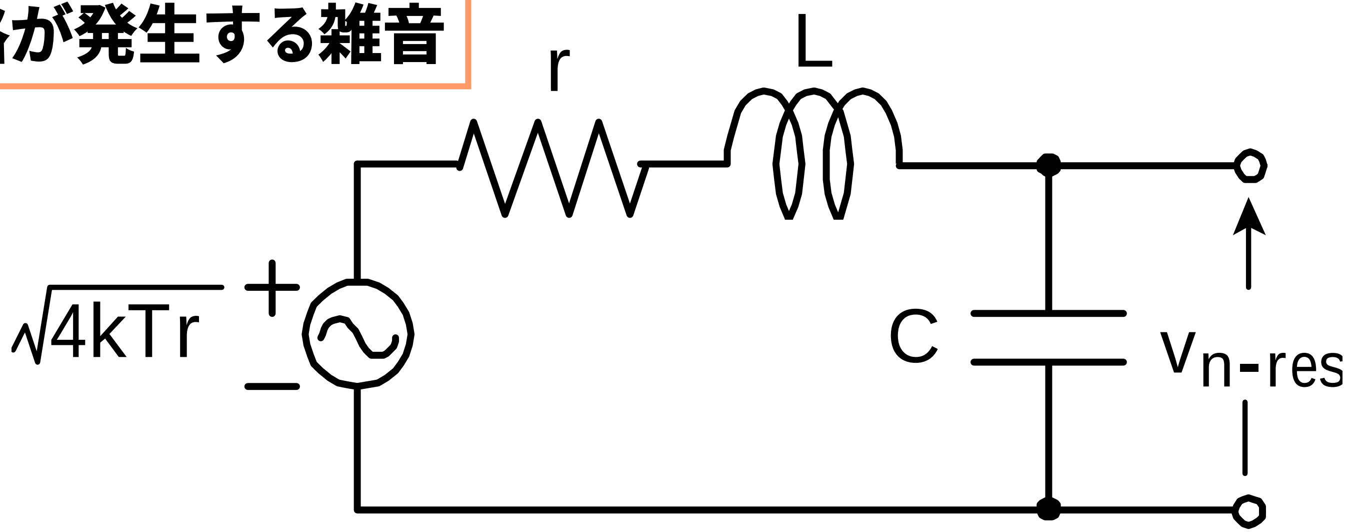
$$\overline{v_{n-rload}{}^2}=\int_0^\infty\left|T_{rload}(j\omega)\right|^2\frac{4kTR}{2\pi}d\omega$$

$$=\int_0^\infty\frac{1}{1+\omega^2C_p{}^2R^2}\frac{4kTR}{2\pi}d\omega$$

$$=\frac{2kT}{C_p{}^2R\pi}\int_0^\infty\frac{1}{\omega^2+\frac{1}{C_p{}^2R^2}}d\omega=\frac{2kT}{C_p{}^2R\pi}\left[C_pR\tan^{-1}(C_pR\omega)\right]_0^\infty$$

$$=\frac{2kT}{C_p{}^2R\pi}C_pR\left(\frac{\pi}{2}-0\right)=\frac{kT}{C_p}$$

LC共振回路が発生する雑音



$$T_{res}(s) = \frac{\frac{1}{LC}}{s^2 + s\frac{r}{L} + \frac{1}{LC}}$$

$$\Rightarrow \left|T_{res}(j\omega)\right|^2 = \frac{\frac{1}{L^2C^2}}{\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{r}{L}\right)^2 \omega^2}$$

$$T_{res}(s) = \frac{\frac{1}{LC}}{s^2 + s\frac{r}{L} + \frac{1}{LC}}$$

$$\Rightarrow \left|T_{res}(j\omega)\right|^2 = \frac{\frac{1}{L^2C^2}}{\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{r}{L}\right)^2 \omega^2}$$

$$\overline{v_{n-res}{}^2} = \int_0^\infty \left|T_{res}(j\omega)\right|^2 \frac{4kTr}{2\pi} d\omega$$

$$\overline{v_{n-res}{}^2} = \int_0^\infty \frac{\dfrac{1}{L^2C^2}}{\left(\omega^2 - \dfrac{1}{LC}\right)^2 + \left(\dfrac{r}{L}\right)^2 \omega^2} \cdot \frac{4kTr}{2\pi} d\omega$$

$$\int_0^\infty \frac{x^2}{\left(x^2 - a^2\right)^2 + b^2 x^2} dx = \frac{\pi}{2b}$$

ただし，$0 < \dfrac{b}{2} < a$でなければならない．

$$\omega = \frac{1}{x} \text{と置く} \Rightarrow d\omega = -\frac{1}{x^2} dx$$

$$\overline{v_{n-res}^{2}} = \frac{2kTr}{\pi L^2 C^2}\int_0^{\infty} \frac{1}{\left(\omega^2 - \frac{1}{LC}\right)^2 + \left(\frac{r}{L}\right)^2 \omega^2} d\omega$$

$$= \frac{2kTr}{\pi L^2 C^2}\int_{\infty}^{0} \frac{1}{\left(\frac{1}{x^2} - \frac{1}{LC}\right)^2 + \left(\frac{r}{L}\right)^2 \frac{1}{x^2}} \cdot \frac{-1}{x^2} dx$$

$$= \frac{2kTr}{\pi L^2 C^2}\int_0^{\infty} \frac{1}{\left(\frac{1}{x^2} - \frac{1}{LC}\right)^2 x^2 + \left(\frac{r}{L}\right)^2} dx$$

$$= \frac{2kTr}{\pi L^2 C^2}\int_0^{\infty} \frac{x^2}{\left(1 - \frac{1}{LC} x^2\right)^2 + \left(\frac{r}{L}\right)^2 x^2} dx$$

$$\overline{v_{n-res}^{2}}=\frac{2kTr}{\pi L^2 C^2}\int_0^{\infty}\frac{x^2}{\left(1-\frac{1}{LC}x^2\right)^2+\left(\frac{r}{L}\right)^2 x^2}dx$$

$$=\frac{2kTr}{\pi}\int_0^{\infty}\frac{x^2}{\left(x^2-LC\right)^2+(Cr)^2 x^2}dx$$

$$\int_0^{\infty}\frac{x^2}{\left(x^2-a^2\right)^2+b^2 x^2}dx=\frac{\pi}{2b}$$

ただし,$0<\frac{b}{2}<a$でなければならない.

$$\overline{v_{n-res}{}^2} = \frac{2kTr}{\pi}\int_0^{\infty}\frac{x^2}{\left(x^2 - LC\right)^2 + (Cr)^2 x^2}dx$$

$a = \sqrt{LC}$　$b = Cr$ より

$\sqrt{\frac{L}{C}} > \frac{r}{2}$ が成り立てばよい

$$\overline{v_{n-res}{}^2} = \frac{2kTr}{\pi}\cdot\frac{\pi}{2Cr} = \frac{kT}{C}$$

LC共振回路と抵抗負荷が発生する雑音の比較

- 雑音電力はいずれの場合も容量値のみに依存
- $C = C_p$であれば雑音電力は等しい

信号増幅周波数の上限に関する考察

条件(1): $C=C_p$

(負荷の雑音電力が等しい)

条件(2): LC共振回路の共振時の抵抗$R_{res}=R$

(増幅利得が等しい)

$$Z_{res}(s)=\frac{sL+r}{s^2LC+sCr+1}=R_{res}$$

$$\Rightarrow j\omega_{res}L+r=j\omega_{res}CR_{res}r+R_{res}(1-\omega_{res}{}^2LC)$$

$$j\omega_{res}L+r=j\omega_{res}CR_{res}r+R_{res}(1-\omega_{res}{}^{2}LC)$$

$$\Rightarrow L=CR_{res}r,\quad r=R_{res}(1-\omega_{res}{}^{2}LC)$$

$$\omega_{res}=\sqrt{\frac{1}{LC}\left(1-\frac{r}{R_{res}}\right)}=\sqrt{\frac{1}{C^{2}R_{res}r}\left(1-\frac{r}{R_{res}}\right)}=\frac{1}{CR_{res}}\sqrt{\frac{R_{res}}{r}-1}$$

$$Q_L=\frac{\omega_{res}L}{r}=\left(\frac{1}{CR_{res}}\sqrt{\frac{R_{res}}{r}-1}\right)CR_{res}=\sqrt{\frac{R_{res}}{r}-1}$$

$$\omega_{res}=\frac{1}{CR_{res}}Q_L$$

$$\omega_{res}=\frac{1}{CR_{res}}Q_L$$

抵抗負荷の場合の遮断角周波数：$\omega_C=\dfrac{1}{C_pR}$

$C=C_p$かつ$R_{res}=R$ならばLC共振回路負荷の場合の信号増幅周波数の上限はQ_L倍

増幅利得に関する考察

LC共振回路負荷の場合：$R_{res}=\dfrac{1}{C\omega_{res}}Q_L$

抵抗負荷の場合：$R=\dfrac{1}{C_p\omega_C}$

$C=C_p$かつ$\omega_{res}=\omega_C$ならば
LC共振回路負荷の場合の増幅利得は
抵抗負荷の場合のQ_L倍

雑音に関する考察

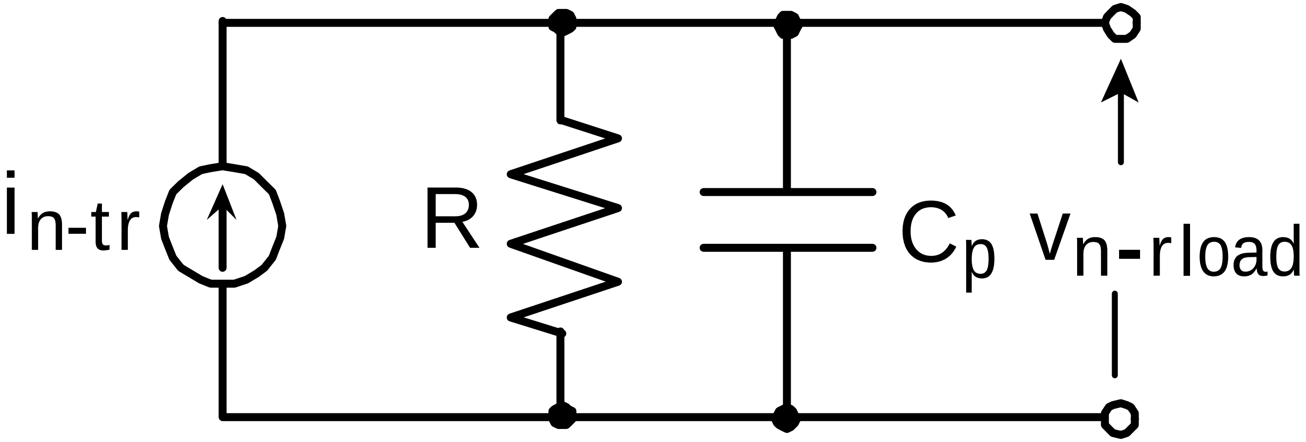
LC共振回路負荷の場合：$C=\dfrac{1}{R_{res}\omega_{res}}Q_L$

抵抗負荷の場合：$C_p=\dfrac{1}{R\omega_C}$

$\omega_{res}=\omega_C$かつ$R_{res}=R$ならば
LC共振回路負荷の場合の雑音は
抵抗負荷の場合の$\dfrac{1}{Q_L}$倍

トランジスタからの雑音の影響

抵抗負荷の場合



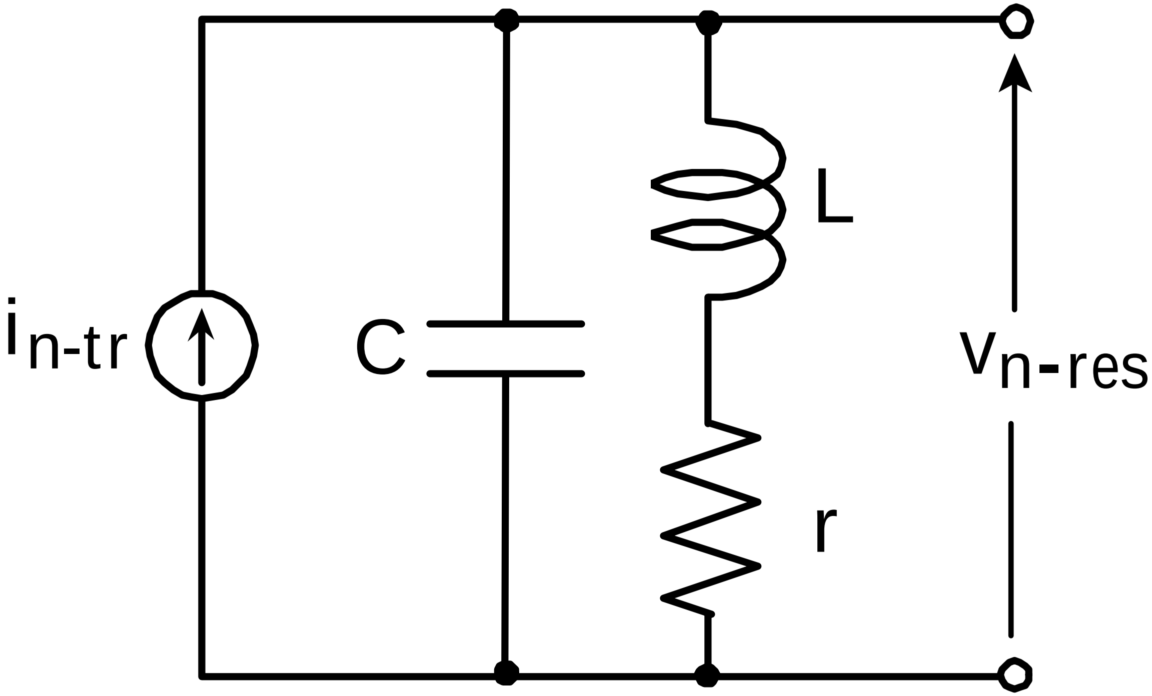
$$\overline{v_{n-rload}{}^2}=\int_0^{\infty}\frac{R^2}{1+\omega^2 C_p{}^2 R^2}\overline{i_{n-tr}{}^2}\,d\omega$$

$$\overline{v_{n-rload}^{2}} = \int_0^\infty \frac{R^2}{1+\omega^2 C_p^2 R^2} \overline{i_{n-tr}^{2}} d\omega$$

$$= \frac{\overline{i_{n-tr}^{2}}}{C_p^2} \int_0^\infty \frac{1}{\omega^2 + \frac{1}{C_p^2 R^2}} d\omega = \frac{\overline{i_{n-tr}^{2}}}{C_p^2} \left[C_p R \tan^{-1}(C_p R \omega) \right]_0^\infty$$

$$= \frac{\overline{i_{n-tr}^{2}}}{C_p^2} \cdot C_p R \frac{\pi}{2} = \frac{R\pi}{2C_p} \overline{i_{n-tr}^{2}}$$

LC共振回路負荷の場合



$$Z_{res}(s) = \frac{sL + r}{s^2LC + sCr + 1}$$

$$\overline{{v_{n-res}}^2} = \int_0^\infty \frac{\omega^2L^2 + r^2}{\left(1 - \omega^2LC\right)^2 + \omega^2C^2r^2}\overline{{i_{n-tr}}^2}d\omega$$

$$\overline{v_{n-res}^{2}} = \int_0^\infty \frac{\omega^2 L^2 + r^2}{\left(1-\omega^2 LC\right)^2 + \omega^2 C^2 r^2} \overline{i_{n-tr}^{2}} d\omega$$

$$= \int_0^\infty \frac{\omega^2 L^2 \overline{i_{n-tr}^{2}}}{\left(1-\omega^2 LC\right)^2 + \omega^2 C^2 r^2} d\omega + \int_0^\infty \frac{r^2 \overline{i_{n-tr}^{2}}}{\left(1-\omega^2 LC\right)^2 + \omega^2 C^2 r^2} d\omega$$

$$= \frac{1}{C^2} \int_0^\infty \frac{\omega^2 \overline{i_{n-tr}^{2}}}{\left(\omega^2 - \frac{1}{LC}\right)^2 + \frac{r^2}{L^2}\omega^2} d\omega + \int_0^\infty \frac{r^2 \overline{i_{n-tr}^{2}} x^2}{\left(x^2 - LC\right)^2 + C^2 r^2 x^2} dx$$

$$= \left(\frac{1}{C^2}\frac{L\pi}{2r} + r^2 \frac{\pi}{2Cr}\right) \overline{i_{n-tr}^{2}} = \left(\frac{L}{Cr} + r\right) \frac{\pi}{2C} \overline{i_{n-tr}^{2}}$$

$$\overline{v_{n-res}^{2}}=\left(\frac{L}{Cr}+r\right)\frac{\pi}{2C}\overline{i_{n-tr}^{2}}$$

$$\overline{v_{n-rload}^{2}}=R\frac{\pi}{2C_p}\overline{i_{n-tr}^{2}}$$

$$j\omega_{res}L+r=j\omega_{res}CR_{res}r+R_{res}(1-\omega_{res}^{2}LC)$$

$$\Rightarrow L=CR_{res}r,\quad r=R_{res}(1-\omega_{res}^{2}LC)$$

$$\overline{v_{n-res}^{2}}=(R_{res}+r)\frac{\pi}{2C}\overline{i_{n-tr}^{2}}$$

負荷の雑音レベルと増幅利得が等しい場合

$$Q_L=\sqrt{\frac{R_{res}}{r}-1}\text{より } r=\frac{R_{res}}{1+Q_L^{\ 2}}$$

$Q_L=3$のとき，　$R_{res}+r$はRの1.1倍=0.41dB

$Q_L=5$のとき，　$R_{res}+r$はRの1.04倍=0.16dB

$Q_L=10$のとき，　$R_{res}+r$はRの1.01倍=0.043dB

最大利得周波数：LC共振回路負荷=抵抗負荷×Q_L

増幅利得と最大利得周波数が等しい場合

$$C = \frac{1}{R_{res}\omega_{res}} Q_L \qquad C_p = \frac{1}{R\omega_C}$$

$$\overline{v_{n-res}^{\,2}} = \left(R_{res} + r\right)\frac{\pi}{2C}\overline{i_{n-tr}^{\,2}} = \frac{R_{res}\left(R_{res} + r\right)\omega_{res}}{2Q_L}\pi\overline{i_{n-tr}^{\,2}}$$

$$\overline{v_{n-rload}^{\,2}} = R\frac{\pi}{2C_p}\overline{i_{n-tr}^{\,2}} = \frac{R_{res}^{\,2}\omega_{res}}{2}\pi\overline{i_{n-tr}^{\,2}}$$

最大利得周波数と負荷の雑音レベルが等しい場合

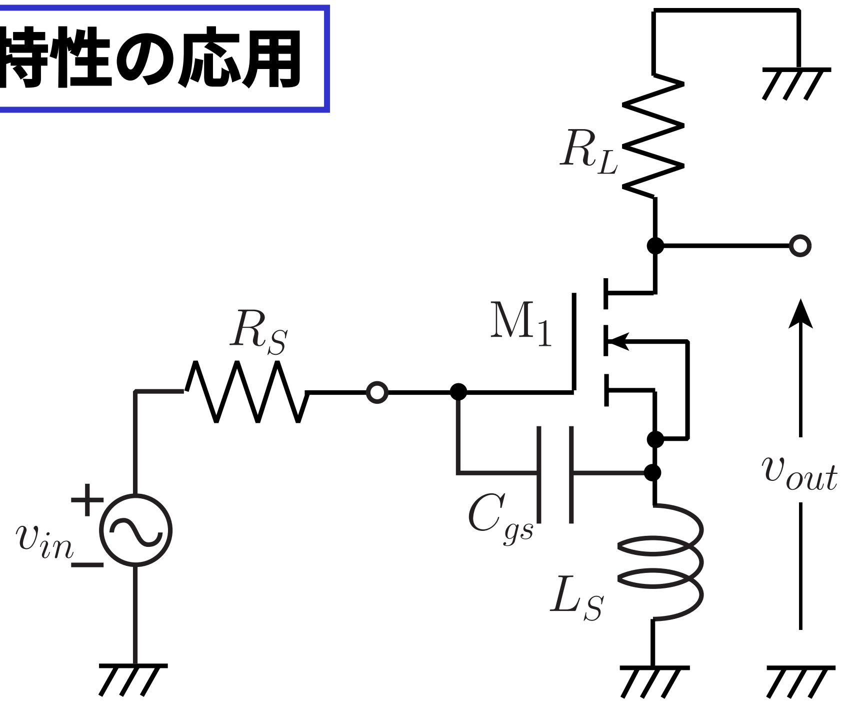
$$R_{res} = \frac{1}{\omega_{res} C} Q_L \qquad R = \frac{1}{\omega_C C_p}$$

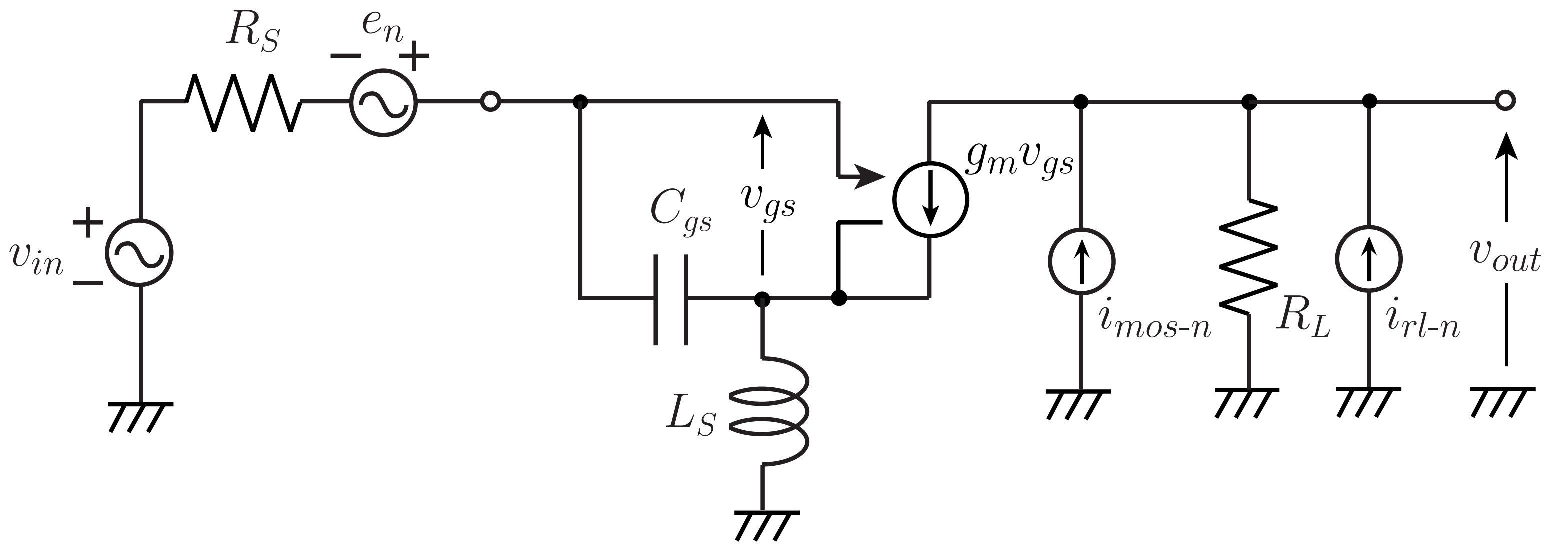
$$\overline{v_{n-res}{}^2} = (R_{res} + r)\frac{\pi}{2C}\overline{i_{n-tr}{}^2} = \left(\frac{Q_L}{\omega_{res} C} + r\right)\frac{\pi}{2C}\overline{i_{n-tr}{}^2}$$

$$\overline{v_{n-rload}{}^2} = R\frac{\pi}{2C_p}\overline{i_{n-tr}{}^2} = \frac{1}{\omega_{res} C}\frac{\pi}{2C}\overline{i_{n-tr}{}^2}$$

低雑音増幅回路の設計(2)

LC共振特性の応用





$$\overline{v_N{}^2} = R_L{}^2(\overline{i_{rl\text{-}n}{}^2} + \overline{i_{mos\text{-}n}{}^2})$$

$$A = \frac{-g_m R_L}{1 + sL_S(g_m + sC_{gs})}$$

$$Z_T = \frac{g_m}{sC_{gs}} R_L$$

$$\overline{v_N^2} = R_L^2 (\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2})$$

$$A = \frac{-g_m R_L}{1 + sL_S(g_m + sC_{gs})} \qquad Z_T = \frac{g_m}{sC_{gs}} R_L$$

$$\overline{v_n^2} = \left| \frac{1 + sL_S(g_m + sC_{gs})}{g_m} \right|^2 (\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2})$$

$$\overline{i_n^2} = \left| \frac{sC_{gs}}{g_m} \right|^2 (\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2})$$

$$\overline{v_n^2} = \left| \frac{1 + sL_S(g_m + sC_{gs})}{g_m} \right|^2 (\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2})$$

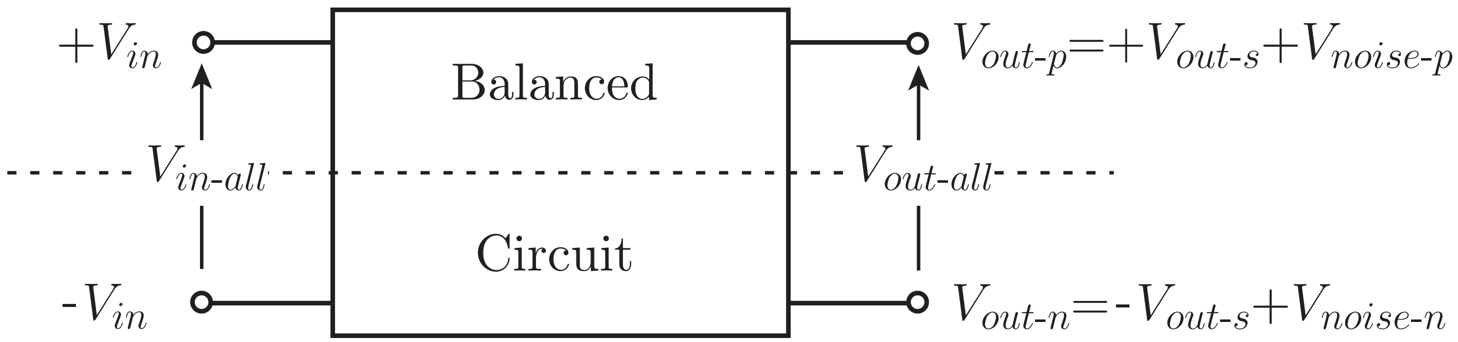
$$\overline{i_n^2} = \left| \frac{sC_{gs}}{g_m} \right|^2 (\overline{i_{rl\text{-}n}^2} + \overline{i_{mos\text{-}n}^2})$$

$$\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} = \left| \frac{1 + sL_S(g_m + sC_{gs})}{sC_{gs}} \right|$$

$$\left| \frac{1 + sL_S(g_m + sC_{gs})}{sC_{gs}} \right| = R_S$$

$f = \frac{1}{2\pi}\sqrt{\frac{1}{L_S C_{gs}}}$ のとき $\frac{L_S g_m}{C_{gs}}$ の純抵抗

平衡型構成の利用



Symmetry Structure

$$V_{noise\text{-}d}=\frac{V_{noise\text{-}p}-V_{noise\text{-}n}}{2}$$

$$V_{noise\text{-}p}=+V_{noise\text{-}d}+V_{noise\text{-}c}$$

$$V_{noise\text{-}c}=\frac{V_{noise\text{-}p}+V_{noise\text{-}n}}{2}$$

$$V_{noise\text{-}n}=-V_{noise\text{-}d}+V_{noise\text{-}c}$$

$$V_{out\text{-}all}=V_{out\text{-}p}-V_{out\text{-}n}=2V_{out\text{-}s}+2V_{noise\text{-}d}$$

不平衡型回路のSNR

$$V_{out} = V_{out\text{-}s} + V_{noise}$$

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T V_{out\text{-}s}{}^2 dt = \overline{V_{out\text{-}s}}^2 \qquad \lim_{T\to\infty}\frac{1}{T}\int_0^T V_{noise}{}^2 dt = \overline{V_{noise}}^2$$

$$SNR_{imbal} = \frac{\overline{V_{out\text{-}s}}^2}{\overline{V_{noise}}^2}$$

平衡型回路のSNR

$$V_{out\text{-}all}=V_{out\text{-}p}-V_{out\text{-}n}=2(V_{out\text{-}s}+V_{noise\text{-}d})$$

$$\lim_{T\to\infty}\frac{1}{T}\int_0^T V_{out\text{-}s}{}^2 dt=\overline{V_{out\text{-}s}}^2$$

$$\overline{V_{noise\text{-}d}}^2=\lim_{T\to\infty}\frac{1}{T}\int_0^T V_{noise\text{-}d}{}^2 dt=\lim_{T\to\infty}\frac{1}{T}\int_0^T V_{noise\text{-}c}{}^2 dt=\frac{1}{2}\overline{V_{noise}}^2$$

SNRの改善： $SNR_{bal}=\dfrac{\overline{V_{out\text{-}s}}^2}{\overline{V_{noise\text{-}d}}^2}=2\dfrac{\overline{V_{out\text{-}s}}^2}{\overline{V_{noise}}^2}$

ただし，回路規模は2倍