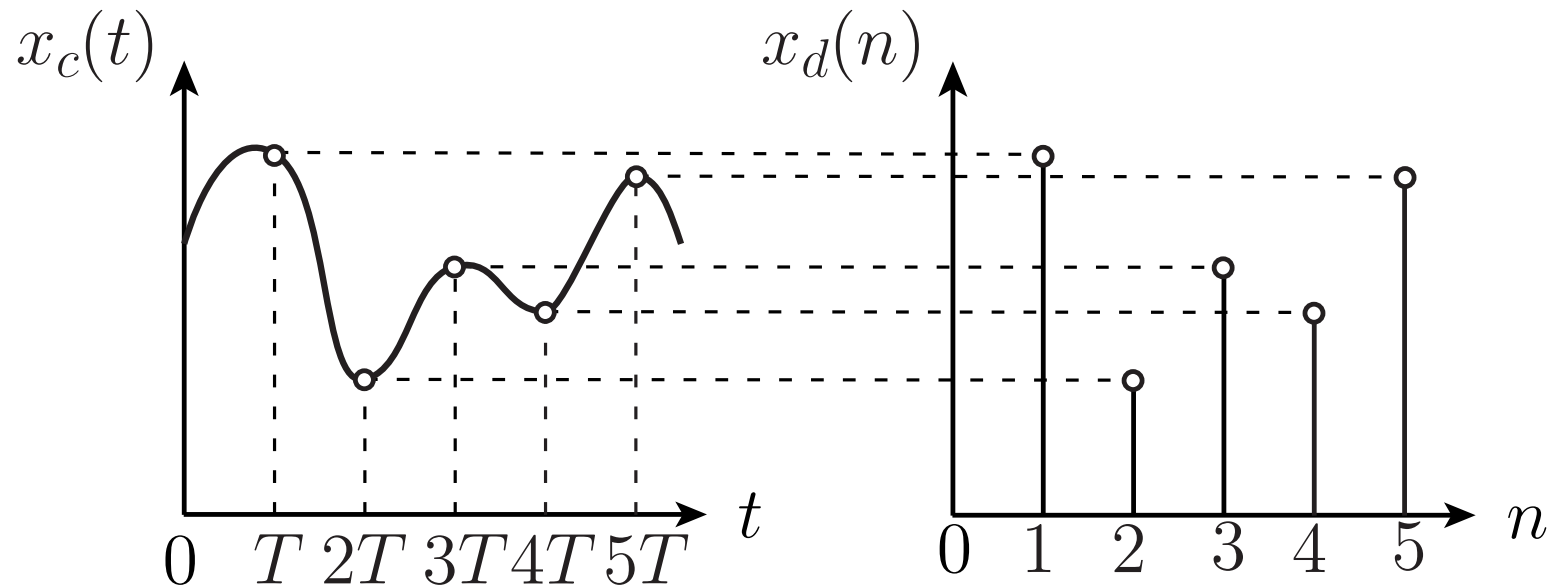


離散時間系フィルタの構成

スイッチトキャパシタフィルタ

連続時間系回路と離散時間系回路 (ラプラス変換から z 変換へ)



$x_c(t)$: 連続時間信号
 $x_d(n)$: 離散時間信号

理想サンプリング

$\delta(t)$: デルタ関数

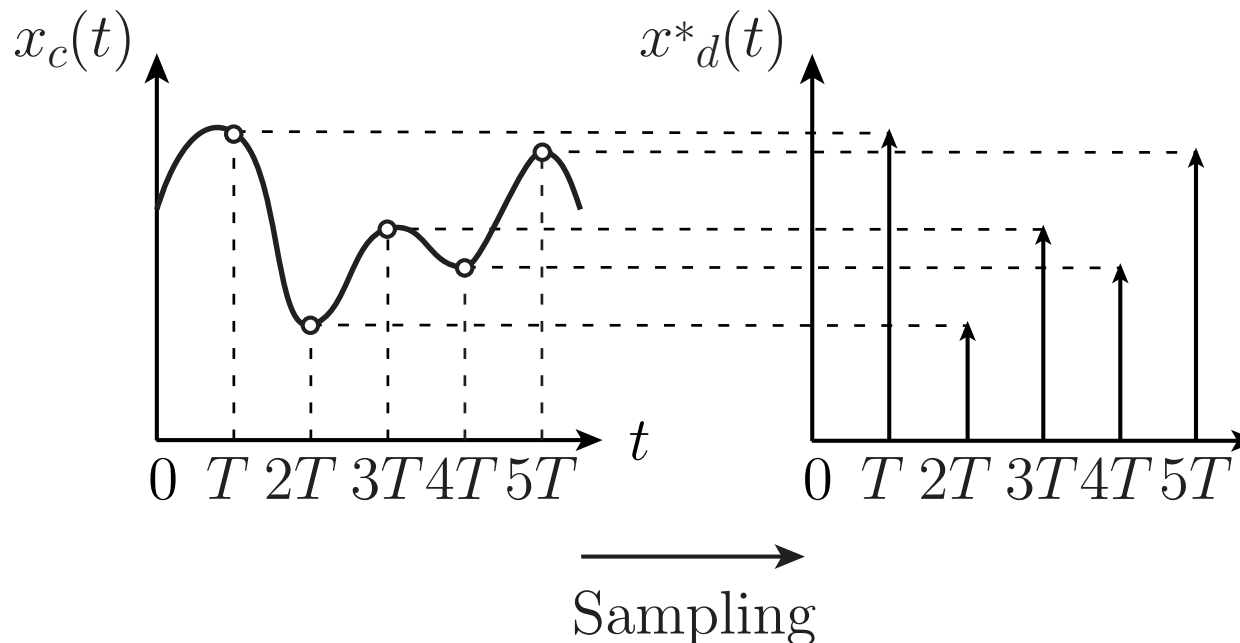
定義： $\int_{-\infty}^{\infty} \delta(t)\varphi(t)dt = \varphi(0)$

ただし、 $\varphi(t)$ はテスト関数と呼ばれ、
ある t_0 について $|t| > t_0$ のときは $\varphi(t) = 0$ となり、
何回でも微分可能な関数。

$$x_d^*(t) = \sum_{n=-\infty}^{\infty} x_c(t) \delta(t - nT) = x_c(t) \delta_T(t)$$

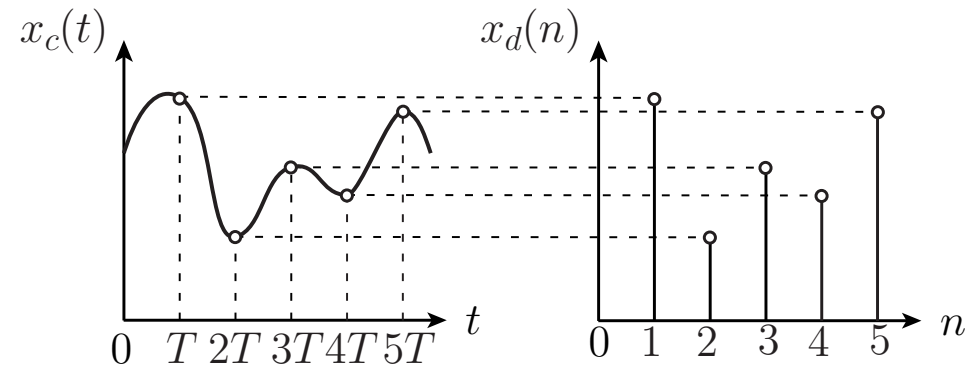
$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \quad \int_{-\infty}^{\infty} \delta(t) \phi(t) dt = \phi(0)$$

離散時間信号の連続時間**仮**表現



離散時間信号の連続時間**仮**表現と離散時間表現

$$x_d^*(t) = \sum_{n=-\infty}^{\infty} x_c(t) \delta(t-nT) = x_c(t) \delta_T(t) \quad \delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$



連続時間**仮**表現:

$$x_d^*(t) = x_c(t) \delta_T(t)$$

離散時間表現:

$$x_d(n) = \int_{-\infty}^{\infty} x_c(t) \delta_T(t) dt$$

時間応答：畳み込み積分と畳み込み和

$x_c(t)$: 入力 $g_c(t)$: 出力 $h_c(t)$: インパルス応答

$x_d(n)$: 入力 $g_d(n)$: 出力 $h_d(n)$: インパルス応答

$$\text{畳み込み積分} : g_c(t) = \int_{-\infty}^{\infty} h_c(t-\tau) x_c(\tau) d\tau$$

$$\text{畳み込み和} : g_d(n-k) = \sum_{k=-\infty}^{\infty} h_d(n-k) x_d(k)$$

離散時間信号の連続時間^仮表現による畳み込み積分

$$\begin{aligned} \int_{-\infty}^{\infty} h_d^*(t-\tau) x_d^*(\tau) d\tau &= \int_{-\infty}^{\infty} h_c(t-\tau) \delta_T(t-\tau) x_c(\tau) \delta_T(\tau) d\tau \\ &= \int_{-\infty}^{\infty} h_c(t-\tau) x_c(\tau) \delta_T(t) d\tau = \int_{-\infty}^{\infty} h_c(t-\tau) x_c(\tau) d\tau \delta_T(t) \\ &= \sum_{n=-\infty}^{\infty} g_c(t) \delta(t-nT) = g_d^*(t) \end{aligned}$$

畳み込み積分による離散時間信号の連続時間表現

$$g_c(t) = \int_{-\infty}^{\infty} h_c(t-\tau)x_c(\tau)d\tau \text{なので } g_d(n) \text{は}$$

$$g_d(n) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_c(t-\tau)x_c(\tau)d\tau\delta(t-nT)dt$$

$$= \int_{-\infty}^{\infty} h_c(t-\tau)\delta(t-nT)dt x_c(\tau)d\tau = \int_{-\infty}^{\infty} h_c(nT-\tau)x_c(\tau)d\tau$$

畳み込み和による離散時間信号の連続時間表現

一方, $h_d(n) = \int_{-\infty}^{\infty} h_c(t) \delta(t - nT) dt$, $x_d(n) = \int_{-\infty}^{\infty} x_c(t) \delta(t - nT) dt$

であるから, $h_d(n)$ と $x_d(n)$ の畳み込み和は

$$g_d(n) = \sum_{k=-\infty}^{\infty} h_d(n-k) x_d(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} h_c(t) \delta(t - (n-k)T) d\tau \int_{-\infty}^{\infty} x_c(\tau) \delta(\tau - kT) d\tau$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_c(t) x_c(\tau) \sum_{k=-\infty}^{\infty} \delta(t - (n-k)T) \delta(\tau - kT) dt d\tau$$

$$= \int_{-\infty}^{\infty} x_c(\tau) \int_{-\infty}^{\infty} h_c(t) \delta(t - nT + \tau) dt d\tau$$

$$= \int_{-\infty}^{\infty} x_c(\tau) h_c(nT - \tau) d\tau$$

一致!

離散時間信号の連続時間**仮**表現と離散時間表現による周波数特性

$x_c(t)$ のフーリエ変換 $\rightarrow X_c(j\omega)$

$x_d^*(t)$ のフーリエ変換 $\rightarrow X_d^*(j\omega)$

$\delta_T(t)$ は周期関数 $\rightarrow$ フーリエ級数展開による表現

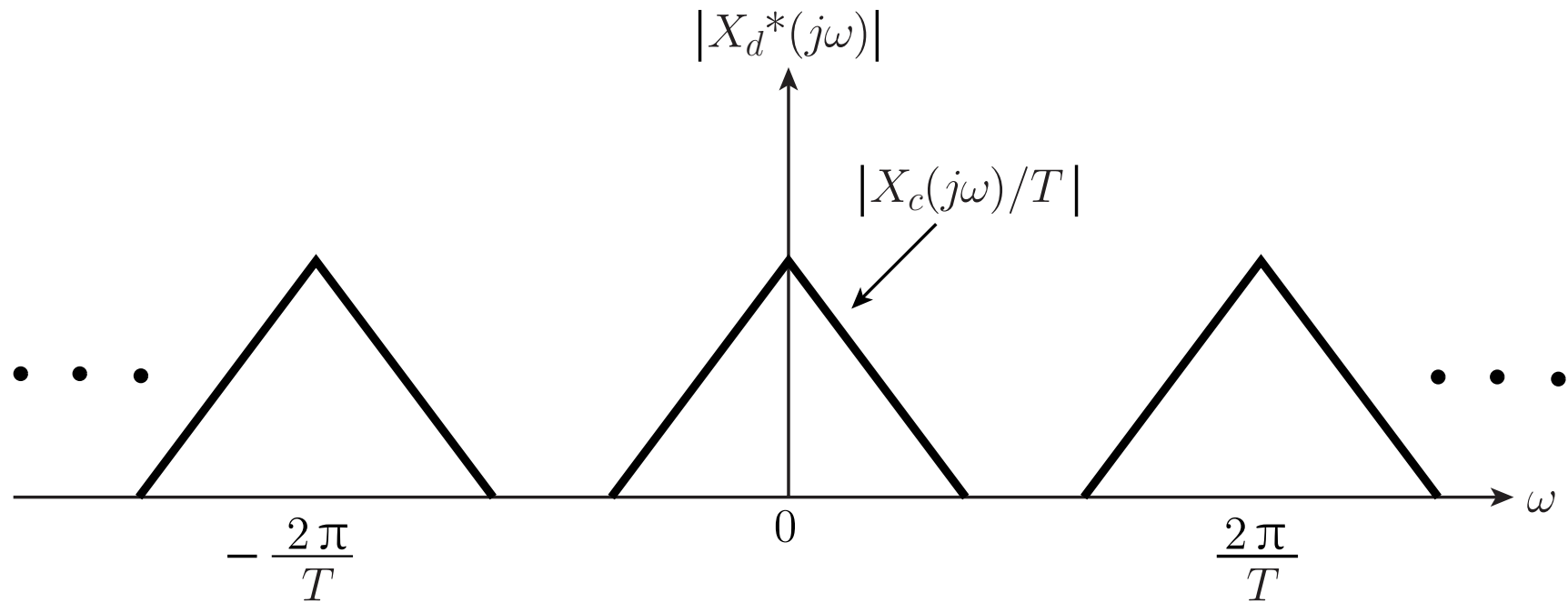
$$\delta_T(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\frac{2\pi}{T}t}$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} \delta_T(t) e^{-jk\frac{2\pi}{T}t} dt = \frac{1}{T} \longrightarrow \delta_T(t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} e^{jk\frac{2\pi}{T}t}$$

$$X_d^*(j\omega) = \int_{-\infty}^{\infty} x_c(t) \frac{1}{T} \sum_{k=-\infty}^{\infty} e^{jk\frac{2\pi}{T}t} e^{-j\omega t} dt = \frac{1}{T} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} x_c(t) e^{j(k\frac{2\pi}{T} - \omega)t} dt$$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j\omega - jk\frac{2\pi}{T})$$

周期的



(両側)z変換

$$\text{定義： } X_d(z) = \sum_{n=-\infty}^{\infty} x_d(n) z^{-n}$$

ただし、 z は複素変数

$$x_d(n) = \int_{-\infty}^{\infty} x_c(t) \delta(t - nT) dt$$

z を e^{sT} で置き換えると

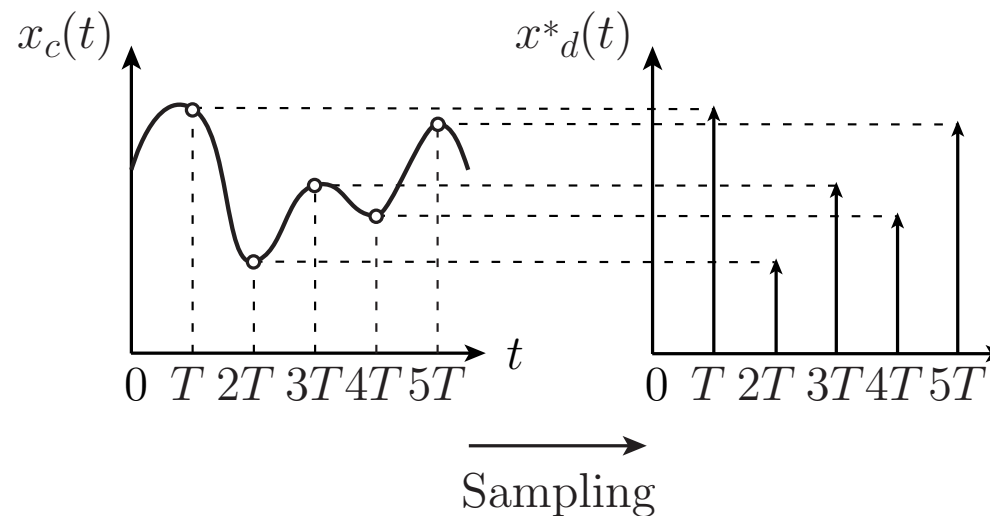
$$X_d(z) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} x_c(t) \delta(t - nT) dt e^{-snT} = \sum_{n=-\infty}^{\infty} x_c(nT) e^{-snT}$$

一方, $X_d^*(s)$ は

$$X_d^*(s) = \int_{-\infty}^{\infty} x_d^*(t) e^{-st} dt = \sum_{n=-\infty}^{\infty} x_c(nT) e^{-snT} = X_d(z)$$

(両側ラプラス変換)

$X_d(z) = X_d^*(s)$ が成り立つ



$$x_d^*(t) = \sum_{n=-\infty}^{\infty} x_c(t) \delta(t - nT) = x_c(t) \delta_T(t)$$

z変換

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n}$$

z変換の性質(1)

$$\sum_{n=-\infty}^{\infty} h(n-1)z^{-n} = \sum_{l=-\infty}^{\infty} h(l)z^{-(l+1)} = \left(\sum_{l=-\infty}^{\infty} h(l)z^{-l} \right) z^{-1} = H(z)z^{-1}$$

z変換の性質(2)

$H(z)$ の周波数特性

$z=e^{j\omega T}$ を代入

z変換の性質(3)

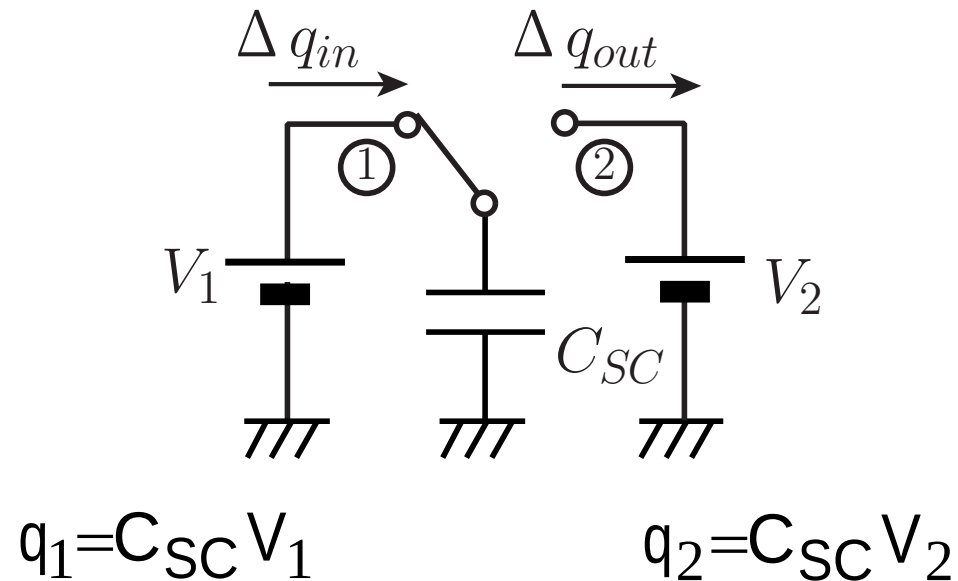
$s_i=\sigma_i+j\omega_i$ と置いたとき

$|z_i|=|e^{s_i T}|$ に $s_i=\sigma_i+j\omega_i$ を代入すると

$$|z_i|=|e^{(\sigma_i+j\omega_i)T}|=|e^{\sigma_i T}| \cdot |e^{j\omega_i T}|=|e^{\sigma_i T}| < 1$$

$\sigma_i < 0$ なら安定

スイッチトキャパシタ回路の動作原理



$$q_2 + (q_1 - q_2) = q_1 \quad \longrightarrow \quad \Delta q_{in} = q_1 - q_2 = C_{SC} (V_1 - V_2)$$

$$q_1 - (q_1 - q_2) = q_2 \quad \longrightarrow \quad \Delta q_{out} = q_1 - q_2 = C_{SC} (V_1 - V_2)$$

$$\Delta q = \Delta q_{out} = \Delta q_{in}$$

近似モデル

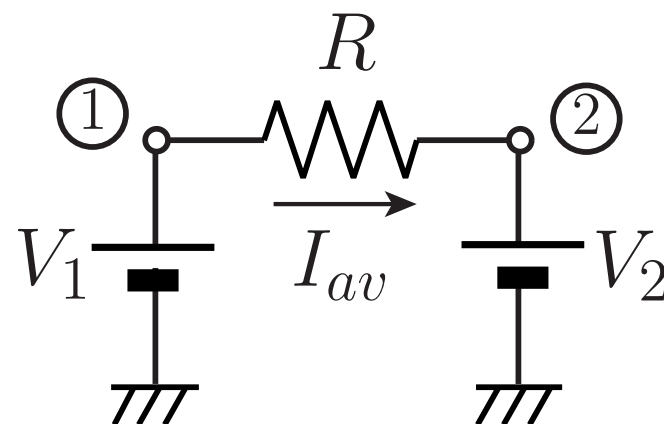
$$\Delta q = \Delta q_{\text{out}} = \Delta q_{\text{in}}$$

スイッチング周期: T

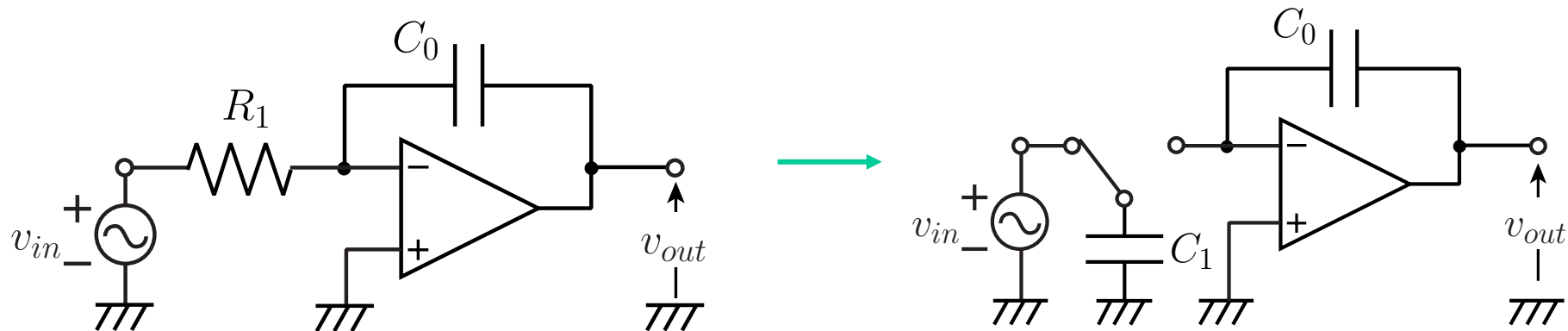
T 秒間に Δq の電荷が移動

平均電流:
$$I_{\text{av}} = \frac{\Delta q}{T} = \frac{C_{\text{SC}} (V_1 - V_2)}{T}$$

等価抵抗:
$$R = \frac{T}{C_{\text{SC}}}$$



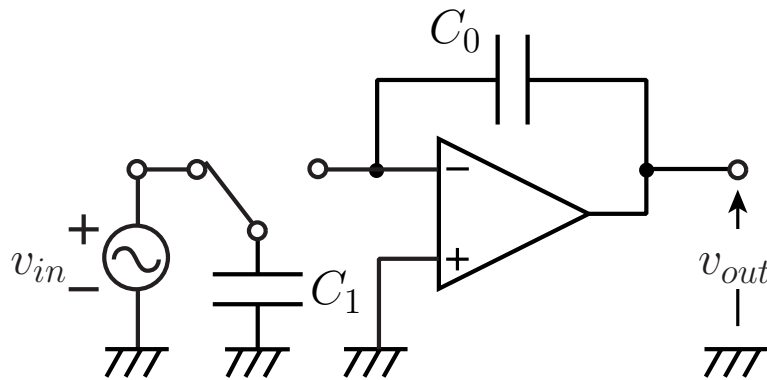
基本スイッチトキャパシタ積分回路



$$R_1 = \frac{T}{C_1}$$

$$T(s) = \frac{v_{out}}{v_{in}} = \frac{-1}{sC_0R_1}$$

$$T(s) = \frac{v_{out}}{v_{in}} = \frac{-C_1}{sC_0T}$$



$$T(s) = \frac{V_{out}}{V_{in}} = \frac{-C_1}{sC_0T}$$

特性： 容量比 & スイッチング周期

$$C_0=1\text{nF}, C_1=1\text{nF}$$

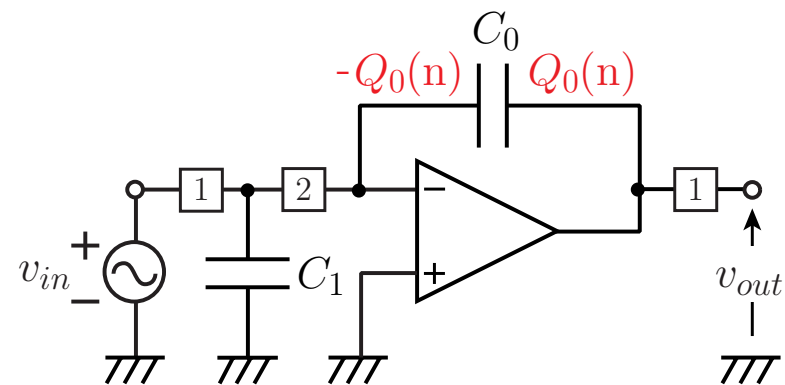
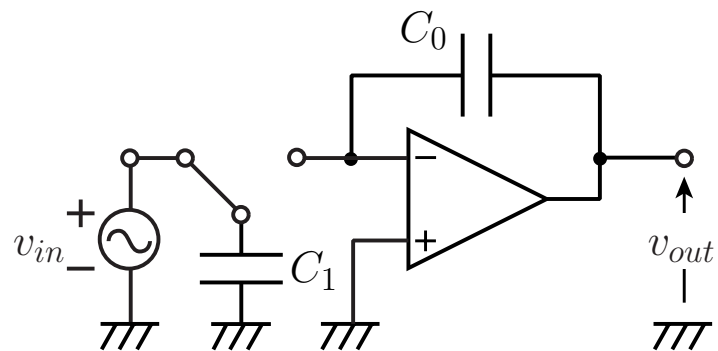
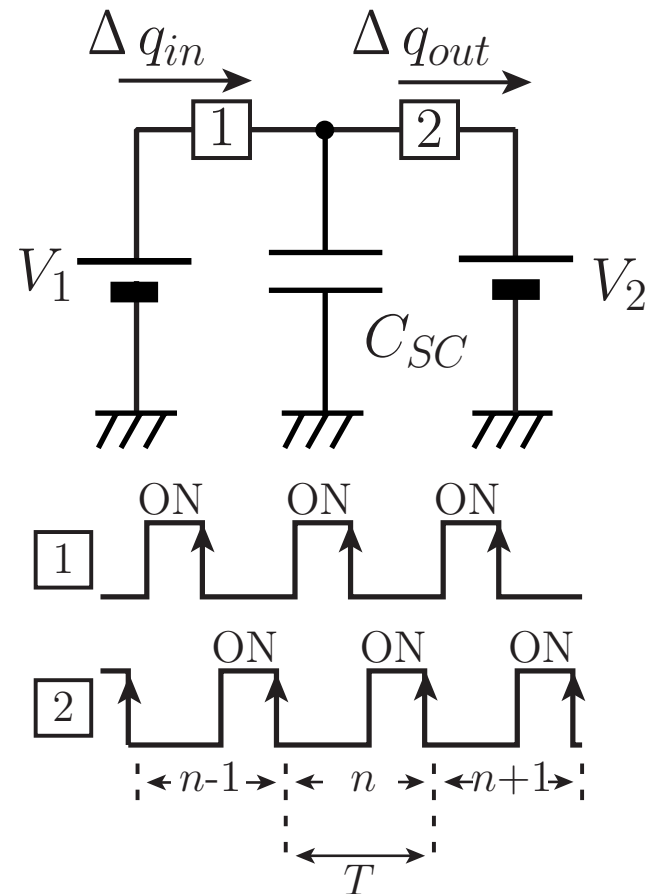
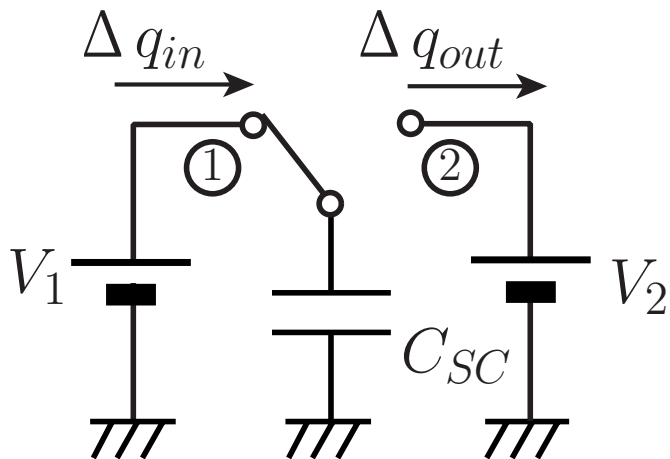
=

$$C_0=1\text{pF}, C_1=1\text{pF}$$

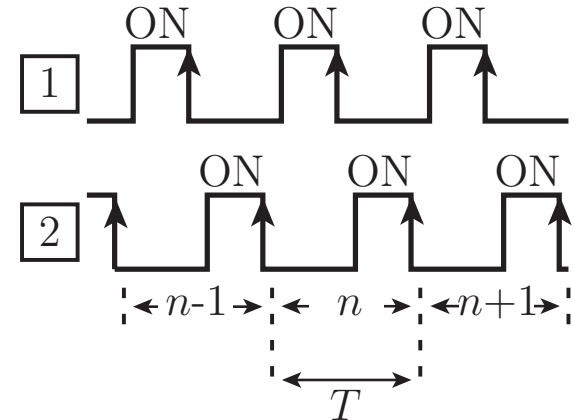
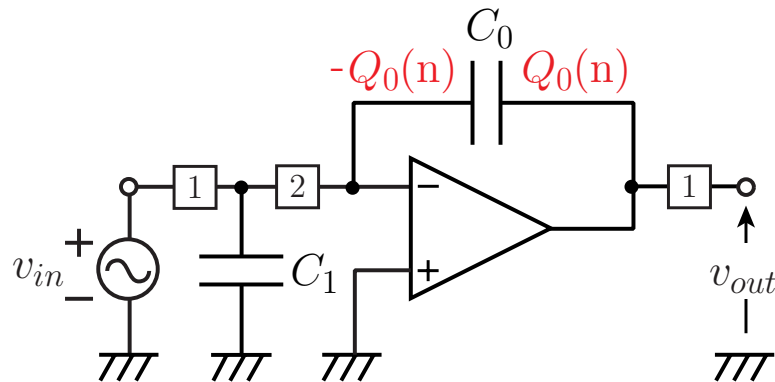
T: 水晶発振回路により正確

容量, 演算増幅器, スイッチ: 実現容易

スイッチトキャパシタ回路の解析

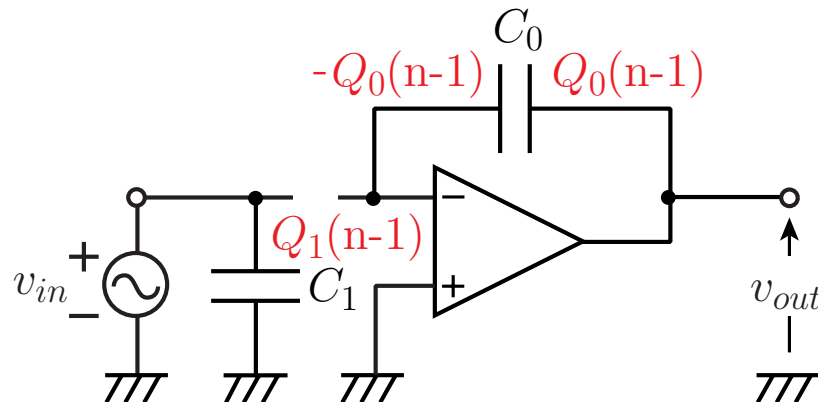


基本スイッチトキャパシタ積分回路の解析

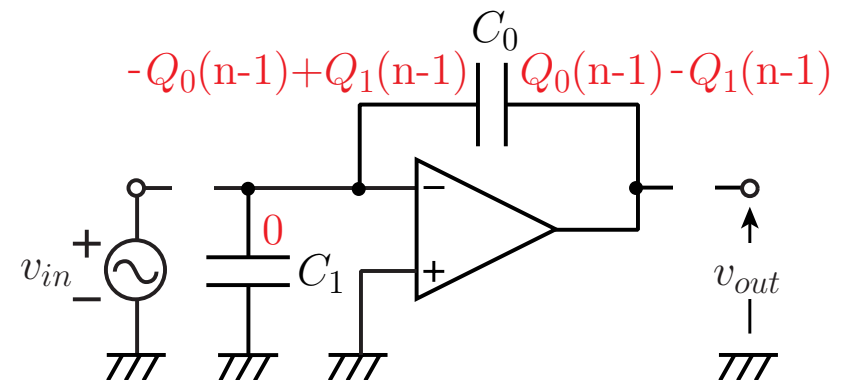


時刻 $n-1$

スイッチ1が短絡
スイッチ2が開放

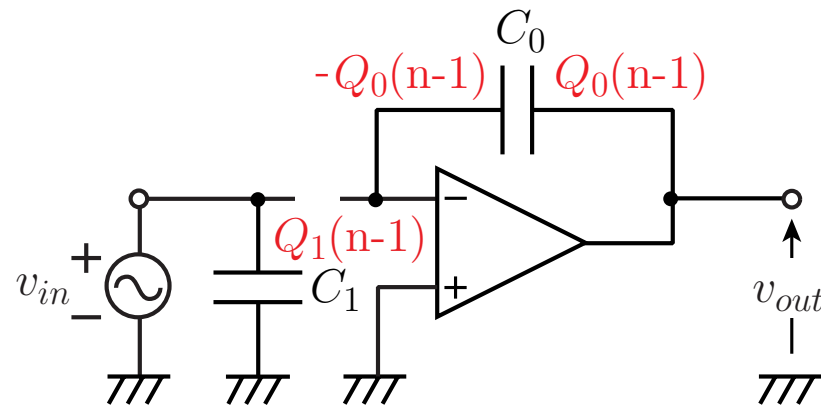


スイッチ1が開放
スイッチ2が短絡

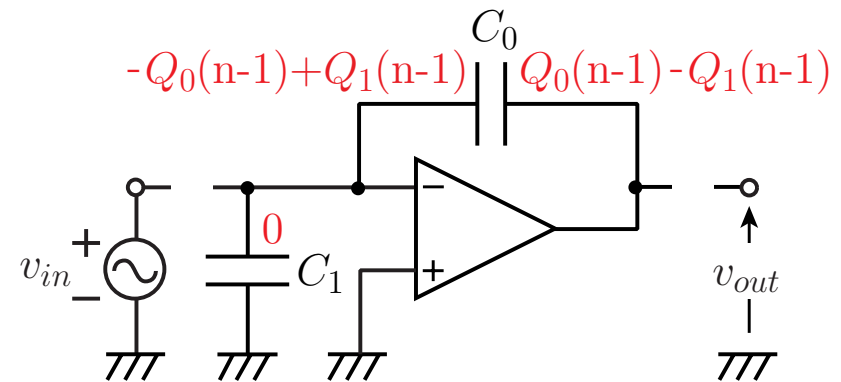


時刻 $n-1$

スイッチ1が短絡
スイッチ2が開放



スイッチ1が開放
スイッチ2が短絡



$$-Q_0(n) = -Q_0(n-1) + Q_1(n-1)$$

$$Q_1(n-1) = C_1 v_{in}(n-1)$$

$$Q_0(n-1) = C_0 v_{out}(n-1)$$

$$Q_0(n) = C_0 v_{out}(n)$$

$$-Q_0(n) = -Q_0(n-1) + Q_1(n-1)$$

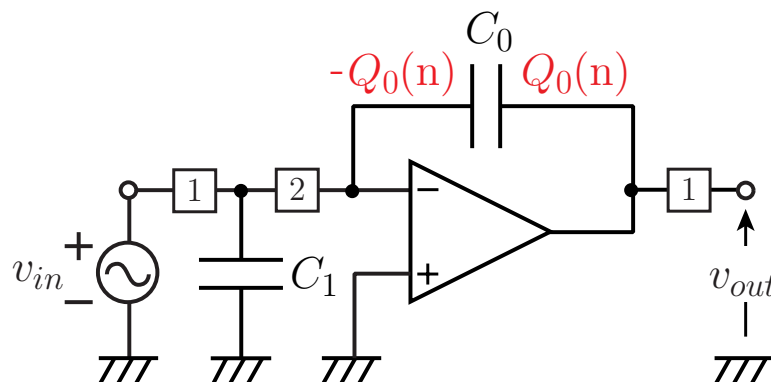


$$Q_0(n) = Q_0(n-1) - Q_1(n-1)$$

$$Q_0(n) = C_0 v_{out}(n) \quad Q_0(n-1) = C_0 v_{out}(n-1) \quad Q_1(n-1) = C_1 v_{in}(n-1)$$

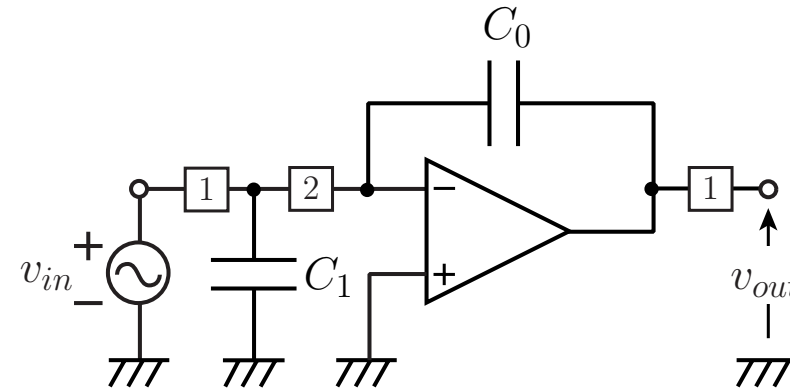
$$C_0 v_{out}(n) = C_0 v_{out}(n-1) - C_1 v_{in}(n-1)$$

$$C_0 V_{out}(z) = C_0 V_{out}(z) z^{-1} - C_1 V_{in}(z) z^{-1}$$



$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1 z^{-1}}{C_0(1 - z^{-1})}$$

$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1 z^{-1}}{C_0(1-z^{-1})}$$



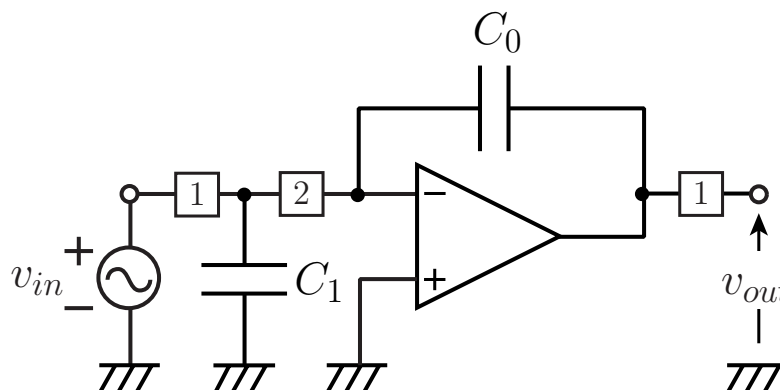
$z=e^{j\omega T}$ を代入

$$\begin{aligned} H(j\omega) &= \frac{-C_1 e^{-j\omega T}}{C_0(1-e^{-j\omega T})} = \frac{-C_1 e^{-j\omega T/2}}{C_0(e^{j\omega T/2}-e^{-j\omega T/2})} \\ &= \frac{-C_1 e^{-j\omega T/2}}{C_0\{\cos(\omega T/2)+j\sin(\omega T/2)-\cos(\omega T/2)+j\sin(\omega T/2)\}} \\ &= \frac{-C_1 e^{-j\omega T/2}}{j2C_0\sin(\omega T/2)} \approx \frac{-C_1}{j\omega C_0 T} \quad (\because x \ll 1 \text{ のとき } \sin x \approx x) \end{aligned}$$

スイッチトキャパシタ積分回路の構成

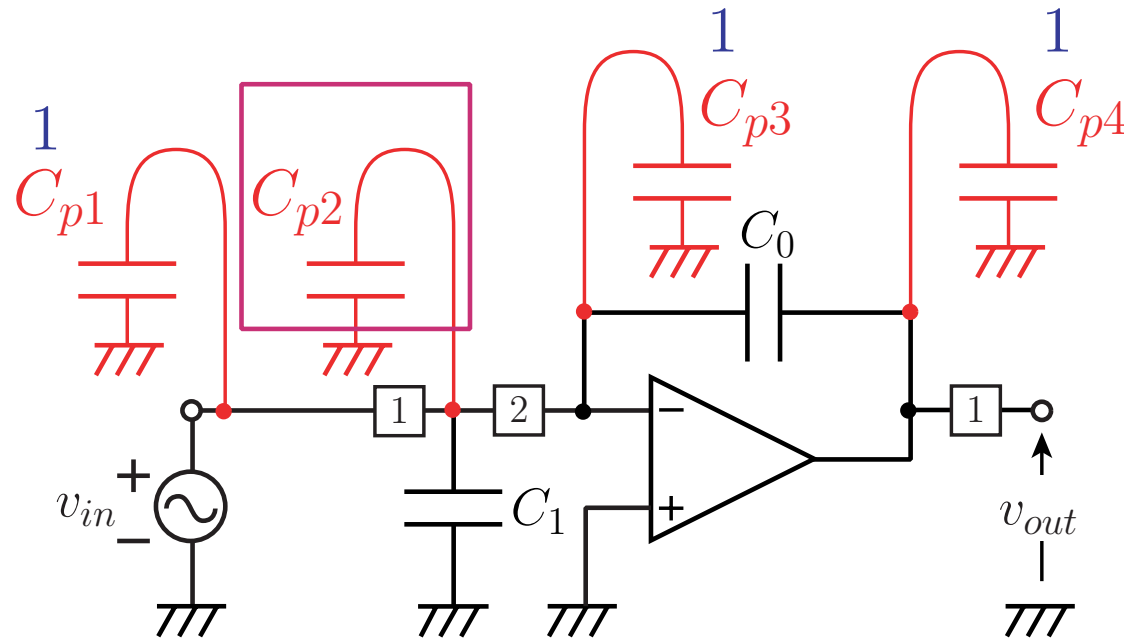
基本スイッチトキャパシタ積分回路の問題点

寄生容量



特性に影響を与えない寄生容量

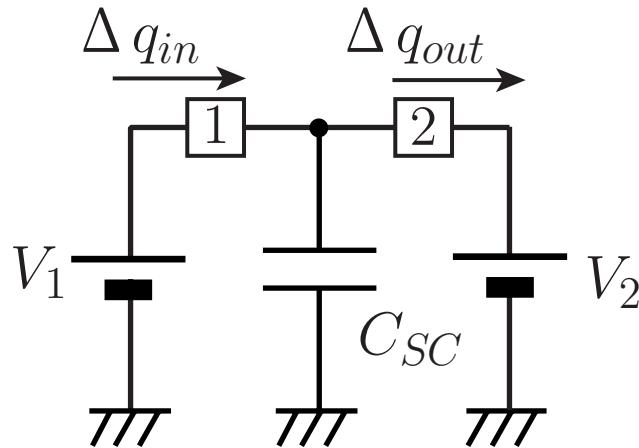
1. 常に電圧源または接地端子，仮想接地端子のいずれかに接続されている寄生容量
2. 電圧源と接地端子に交互に接続される寄生容量
3. 仮想接地と接地端子に交互に接続される寄生容量



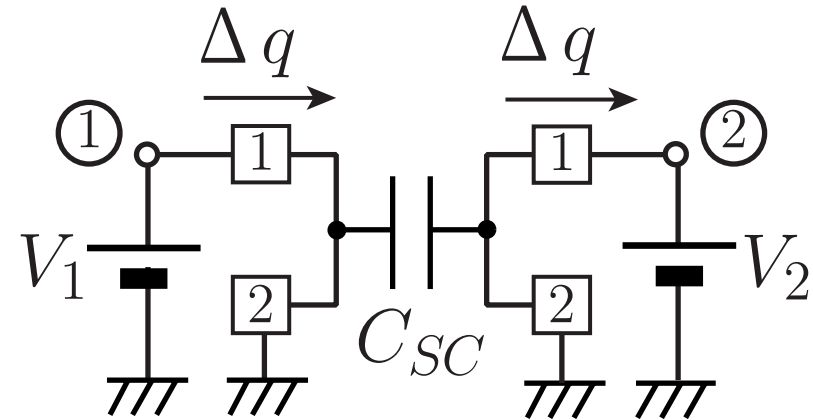
寄生容量に有感なスイッチトキャパシタ回路

1. 常に電圧源または接地端子，仮想接地端子のいずれかに接続されている寄生容量
2. 電圧源と接地端子に交互に接続される寄生容量
3. 仮想接地と接地端子に交互に接続される寄生容量

寄生容量への対策



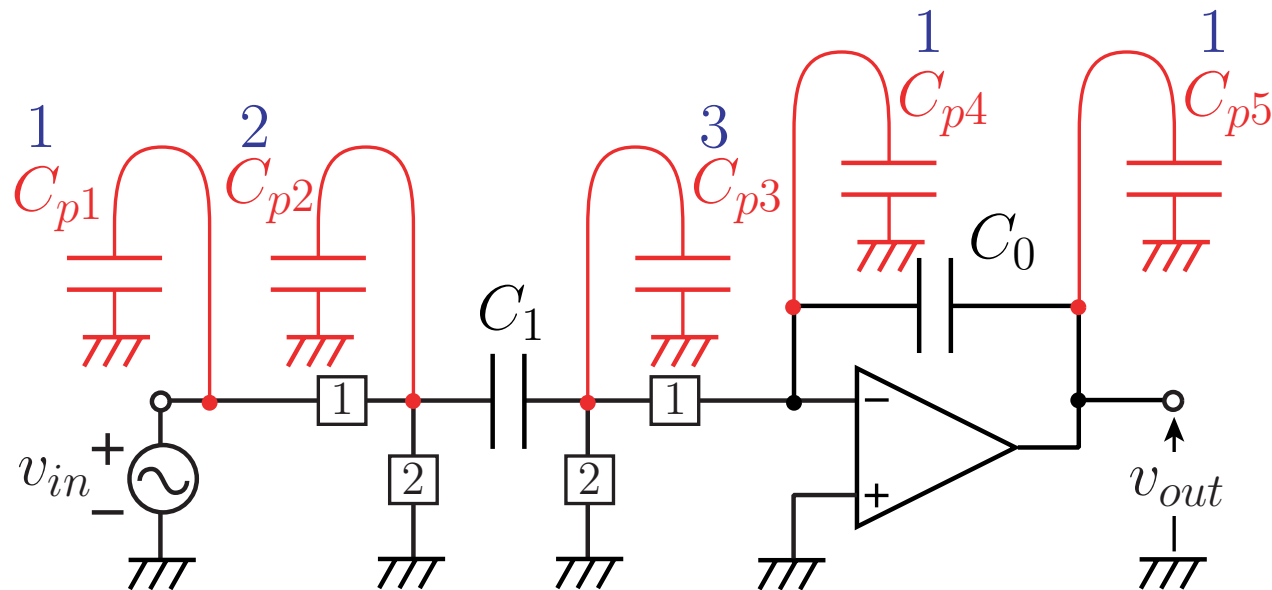
$$\Delta q_{out} = \Delta q_{in} = C_{SC} (V_1 - V_2)$$



$$\Delta q = C_{SC} (V_1 - V_2)$$

電荷の移動量: 同じ

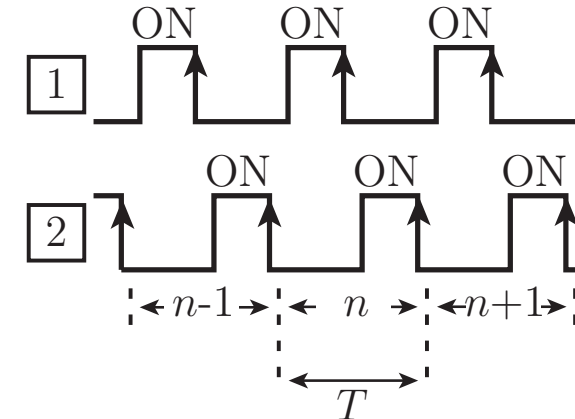
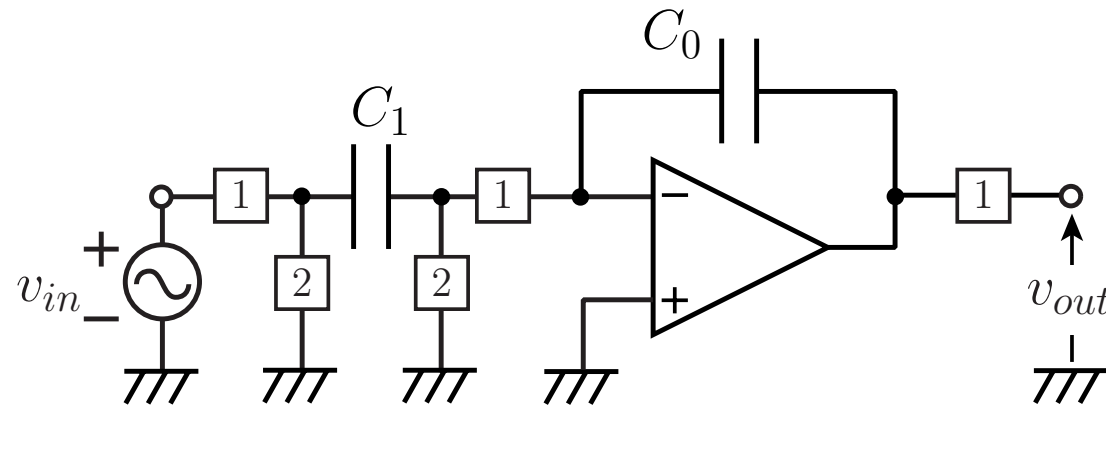
電荷の移動のタイミング: 異なる



寄生容量に不感なスイッチトキャパシタ回路

1. 常に電圧源または接地端子，仮想接地端子のいずれかに接続されている寄生容量
2. 電圧源と接地端子に交互に接続される寄生容量
3. 仮想接地と接地端子に交互に接続される寄生容量

寄生容量に不感なスイッチトキャパシタ積分回路 (1)

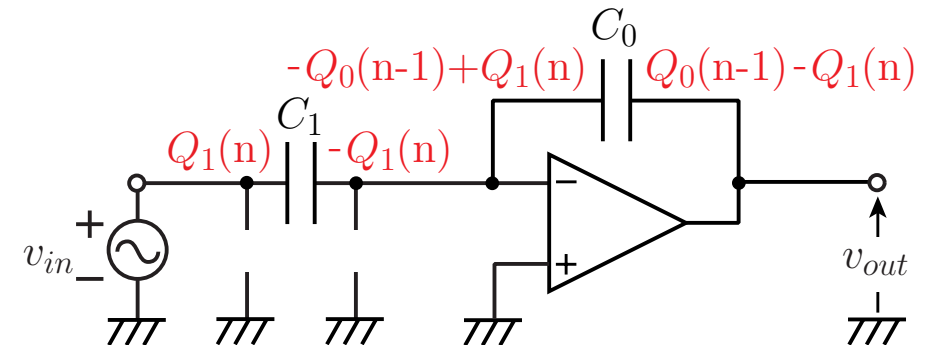
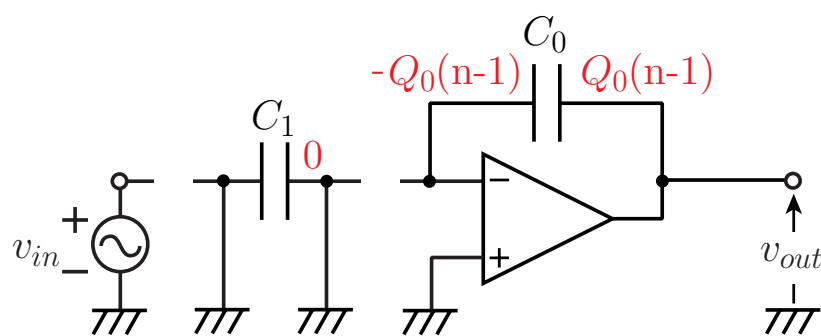


時刻 $n-1$

スイッチ1が開放
スイッチ2が短絡

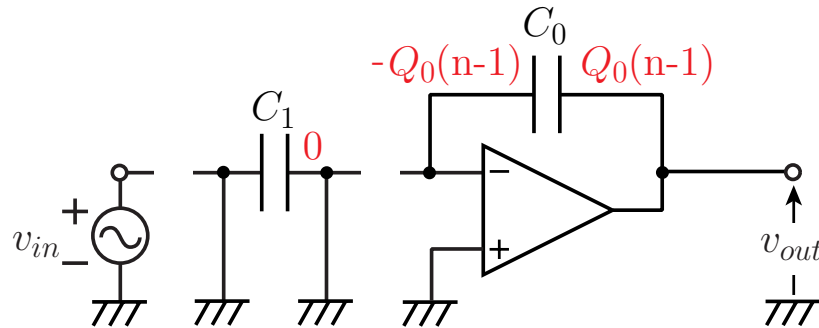
時刻 n

スイッチ1が短絡
スイッチ2が開放



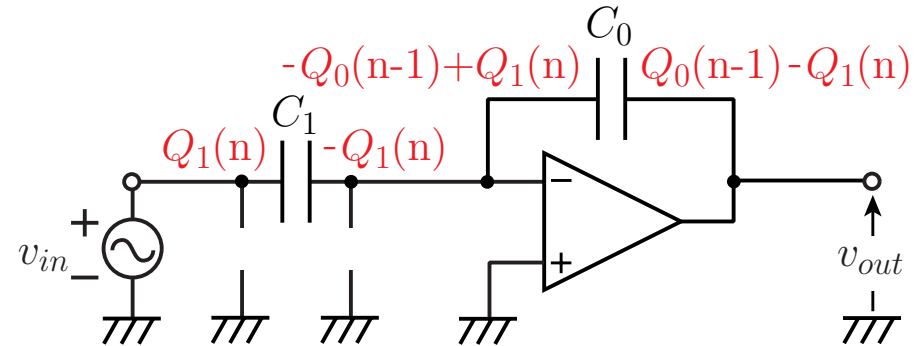
時刻 $n-1$

スイッチ1が開放
スイッチ2が短絡



時刻 n

スイッチ1が短絡
スイッチ2が開放



$$-Q_0(n) = -Q_0(n-1) + Q_1(n)$$

$$Q_1(n) = C_1 v_{in}(n)$$

$$Q_0(n-1) = C_0 v_{out}(n-1)$$

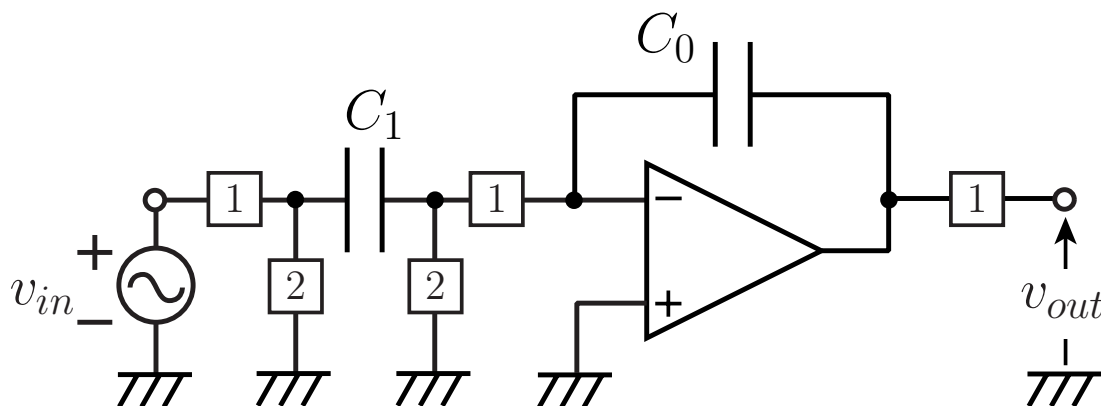
$$Q_0(n) = C_0 v_{out}(n)$$

$$-Q_0(n) = -Q_0(n-1) + Q_1(n) \quad \longrightarrow \quad Q_0(n) = Q_0(n-1) - Q_1(n)$$

$$Q_0(n) = C_0 v_{out}(n) \quad Q_0(n-1) = C_0 v_{out}(n-1) \quad Q_1(n) = C_1 v_{in}(n)$$

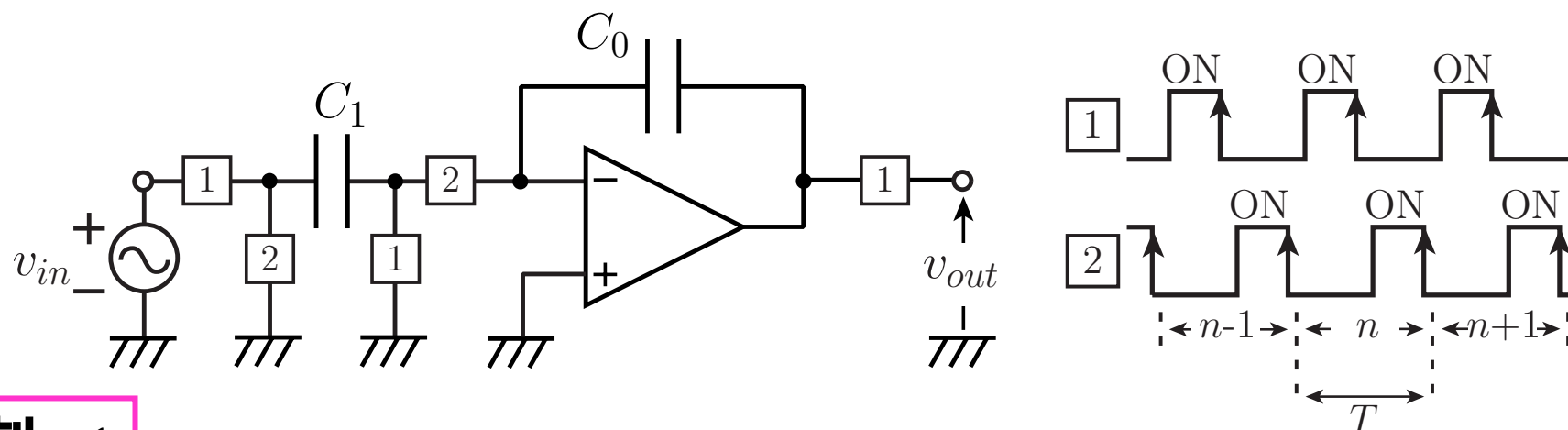
$$C_0 v_{out}(n) = C_0 v_{out}(n-1) - C_1 v_{in}(n)$$

$$C_0 V_{out}(z) = C_0 V_{out}(z) z^{-1} - C_1 V_{in}(z)$$



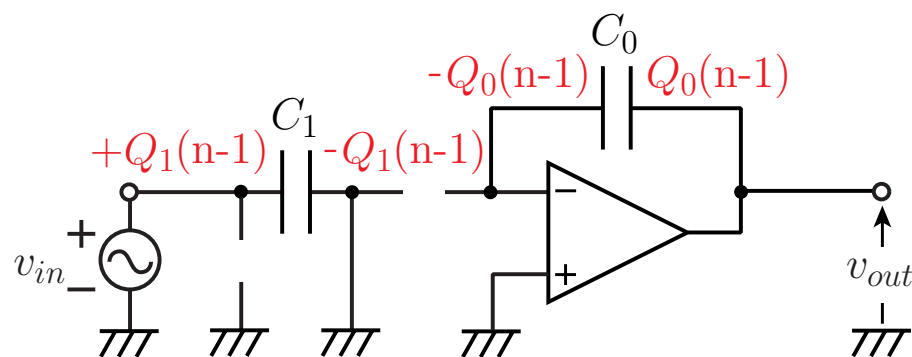
$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1}{C_0(1-z^{-1})}$$

寄生容量に不感なスイッチトキャパシタ積分回路 (2)

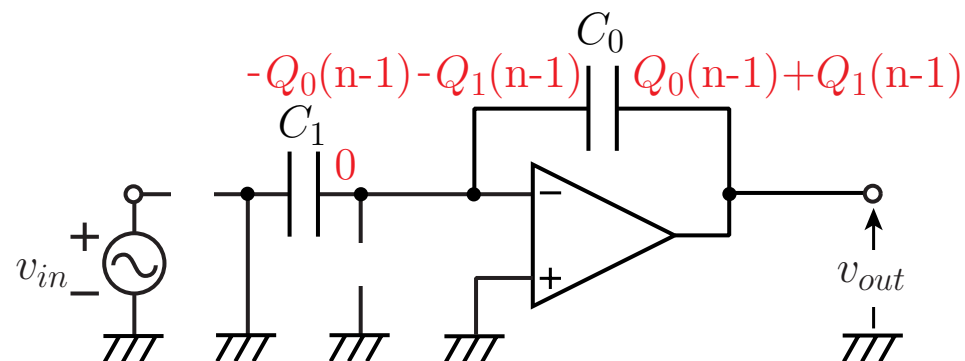


時刻 $n-1$

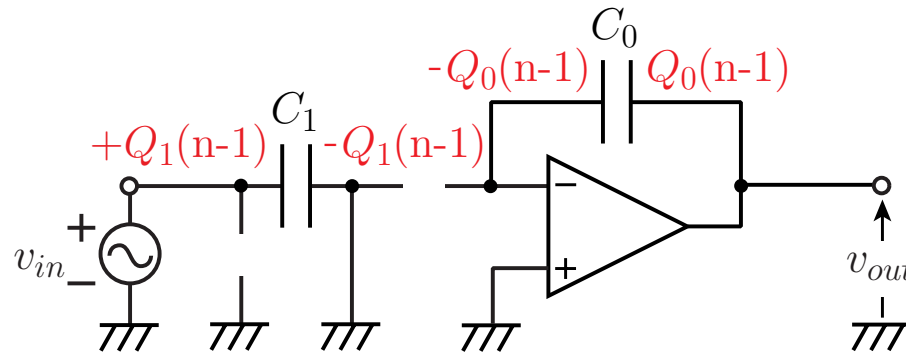
スイッチ1が短絡
スイッチ2が開放



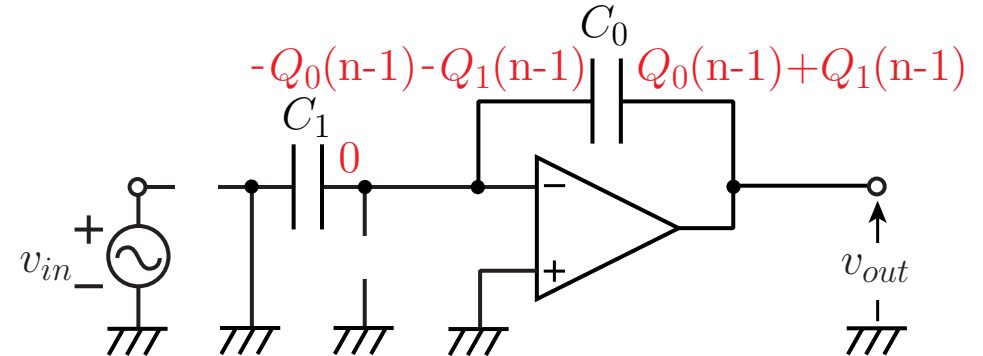
スイッチ1が開放
スイッチ2が短絡



スイッチ1が短絡
スイッチ2が開放



スイッチ1が開放
スイッチ2が短絡



$$-Q_0(n) = -Q_0(n-1) - Q_1(n-1)$$



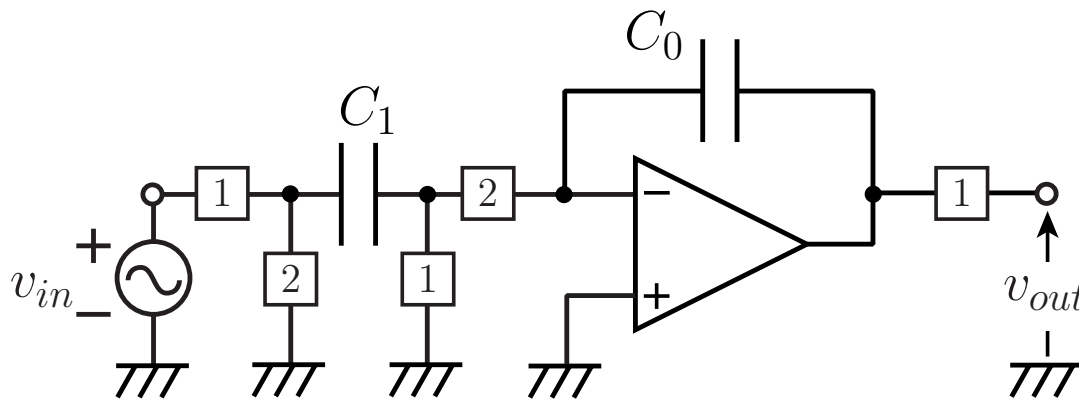
$$Q_0(n) = Q_0(n-1) + Q_1(n-1)$$

$$Q_0(n) = C_0 v_{out}(n) \quad Q_0(n-1) = C_0 v_{out}(n-1) \quad Q_1(n-1) = C_1 v_{in}(n-1)$$

$$C_0 v_{out}(n) = C_0 v_{out}(n-1) + C_1 v_{in}(n-1)$$

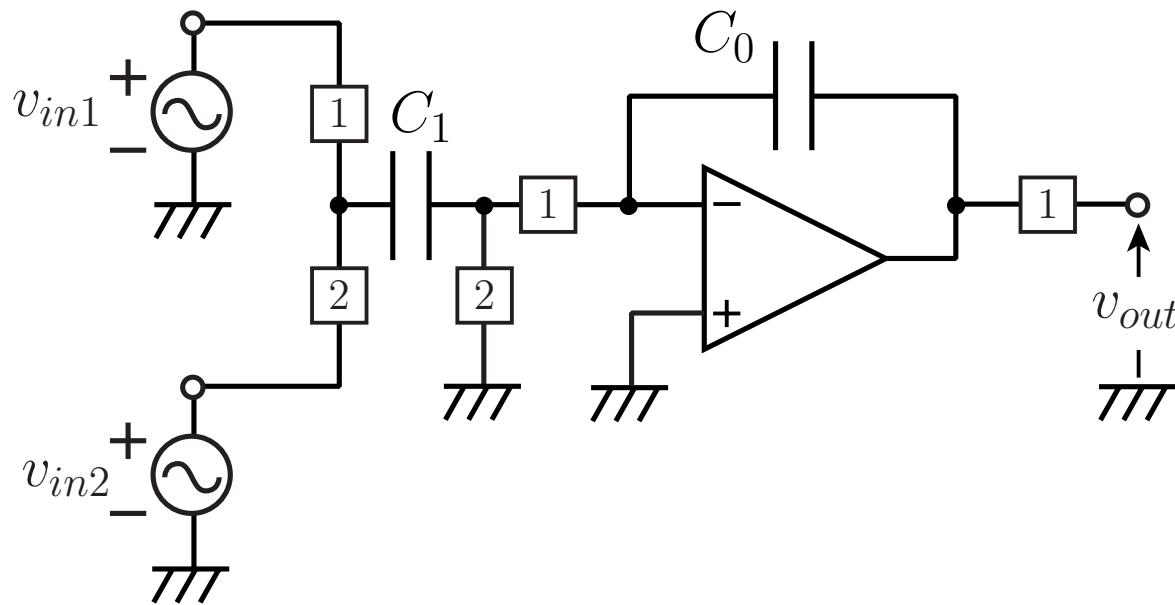
$$C_0 v_{\text{out}}(n) = C_0 v_{\text{out}}(n-1) + C_1 v_{\text{in}}(n-1)$$

$$C_0 V_{\text{out}}(z) = C_0 V_{\text{out}}(z) z^{-1} + C_1 V_{\text{in}}(z) z^{-1}$$



$$\frac{V_{\text{out}}(z)}{V_{\text{in}}(z)} = \frac{C_1 z^{-1}}{C_0 (1 - z^{-1})}$$

寄生容量に不感なスイッチトキャパシタ積分回路 (3)



$$C_0 v_{out}(n) = C_0 v_{out}(n-1) - C_1 v_{in1}(n) + C_1 v_{in2}(n-1)$$

$$C_0 V_{out}(z) = C_0 V_{out}(z) z^{-1} - C_1 V_{in1}(z) + C_1 V_{in2}(z) z^{-1}$$

$$V_{out}(z) = \frac{C_1 \{ z^{-1} V_{in2}(z) - V_{in1}(z) \}}{C_0 (1 - z^{-1})}$$

スイッチトキャパシタフィルタの構成

双一次 s - z 変換

$$s \rightarrow \frac{2(1-z^{-1})}{T(1+z^{-1})}$$

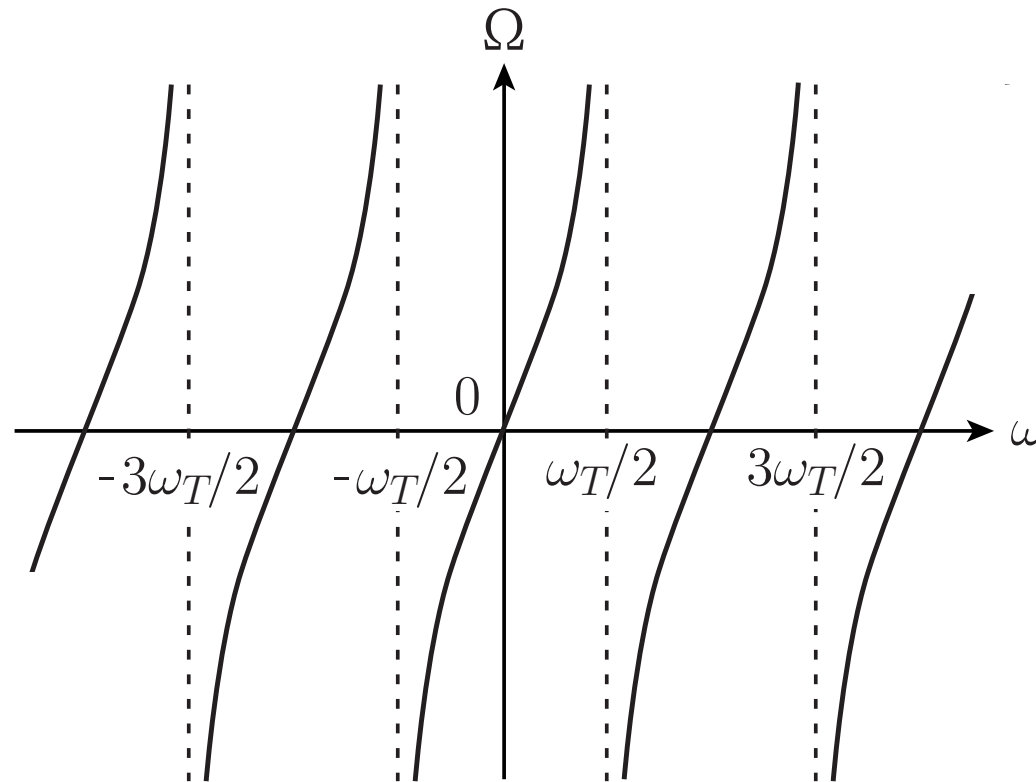
$s=j\Omega$, $z=e^{j\omega T}$ を代入

$$\frac{2(1-z^{-1})}{T(1+z^{-1})} = \frac{2(1-e^{-j\omega T})}{T(1+e^{-j\omega T})} = \frac{2(e^{j\omega T/2}-e^{-j\omega T/2})}{T(e^{j\omega T/2}+e^{-j\omega T/2})}$$

$$= \frac{2\{\cos(\omega T/2)+j\sin(\omega T/2)-\cos(\omega T/2)+j\sin(\omega T/2)\}}{T\{\cos(\omega T/2)+j\sin(\omega T/2)+\cos(\omega T/2)-j\sin(\omega T/2)\}}$$

$$= \frac{4j\sin(\omega T/2)}{2T\cos(\omega T/2)} = j\frac{2}{T}\tan\frac{\omega T}{2}$$

$$\boxed{s \rightarrow \frac{2(1-z^{-1})}{T(1+z^{-1})}} \quad \longrightarrow \quad \boxed{\Omega \rightarrow \frac{2}{T} \tan\left(\frac{\omega T}{2}\right)}$$



$$\omega_T = \frac{2\pi}{T}$$

$$\omega T \ll 1 \text{ のとき } \Omega = \frac{2}{T} \tan\left(\frac{\omega T}{2}\right) \approx \frac{2}{T} \times \frac{\omega T}{2} = \omega$$

LDI変換 (Lossless Discrete Integrator変換)

$$s \rightarrow \frac{z^{1/2} - z^{-1/2}}{T}$$

$s = j\Omega$, $z = e^{j\omega T}$ を代入

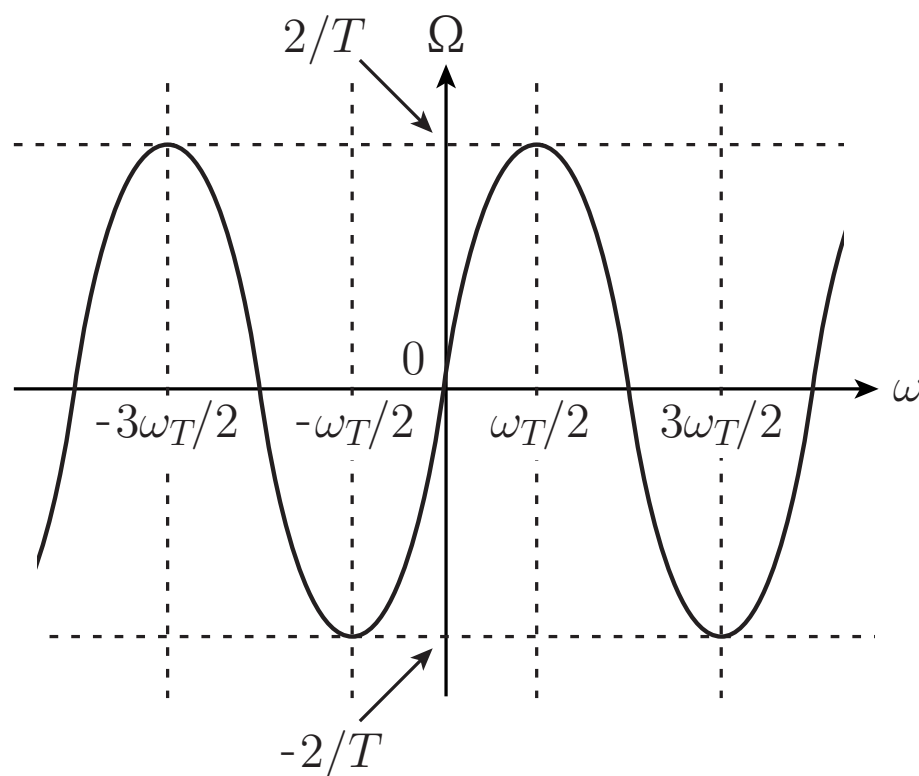
$$\begin{aligned} \frac{z^{1/2} + z^{-1/2}}{T} &= \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{T} \\ &= \frac{\{\cos(\omega T/2) + j\sin(\omega T/2)\} - \{\cos(\omega T/2) - j\sin(\omega T/2)\}}{T} \end{aligned}$$

$$= j \frac{2}{T} \sin(\omega T/2)$$

$$\Omega \rightarrow \frac{2}{T} \sin\left(\frac{\omega T}{2}\right)$$

$$\Omega \rightarrow \frac{2}{T} \sin\left(\frac{\omega T}{2}\right) \approx \omega$$

($\because x \ll 1$ のとき $\sin x \approx x$)



$$\omega_T = \frac{4\pi}{T}$$

Ωの一部がωへ写像

スイッチトキャパシタ2次区間回路

$$T_{\text{second}}(s) = \frac{N(s)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \quad + \quad \text{双一次}_{s-z} \text{変換}$$

$$\begin{aligned} T_{\text{SC2}}(z) &= T_{\text{second}}\left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}\right) = \frac{N\left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}\right)}{\left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}\right)^2 + \frac{\omega_0}{Q}\left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}\right) + \omega_0^2} \\ &= \frac{N\left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}\right)(1+z^{-1})^2}{\frac{4}{T^2}(1-z^{-1})^2 + \frac{\omega_0}{Q}(1-z^{-2}) + \omega_0^2(1+z^{-1})^2} \end{aligned}$$

$$T_{SC2}(z) = \frac{N \left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \right) (1+z^{-1})^2}{\frac{4}{T^2} (1-z^{-1})^2 + \frac{\omega_0}{Q} (1-z^{-2}) + \omega_0^2 (1+z^{-1})^2}$$

低域通過型

$$N(s) = K \omega_0^2$$

$$N_{SC2}(z) \propto (1+z^{-1})^2$$

帶域通過型

$$N(s) = K \frac{\omega_0}{Q} s \Rightarrow K \frac{\omega_0}{Q} \cdot \frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}$$

$$N_{SC2}(z) \propto (1-z^{-2})$$

高域通過型

$$N(s) = K s^2 \Rightarrow K \left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \right)^2$$

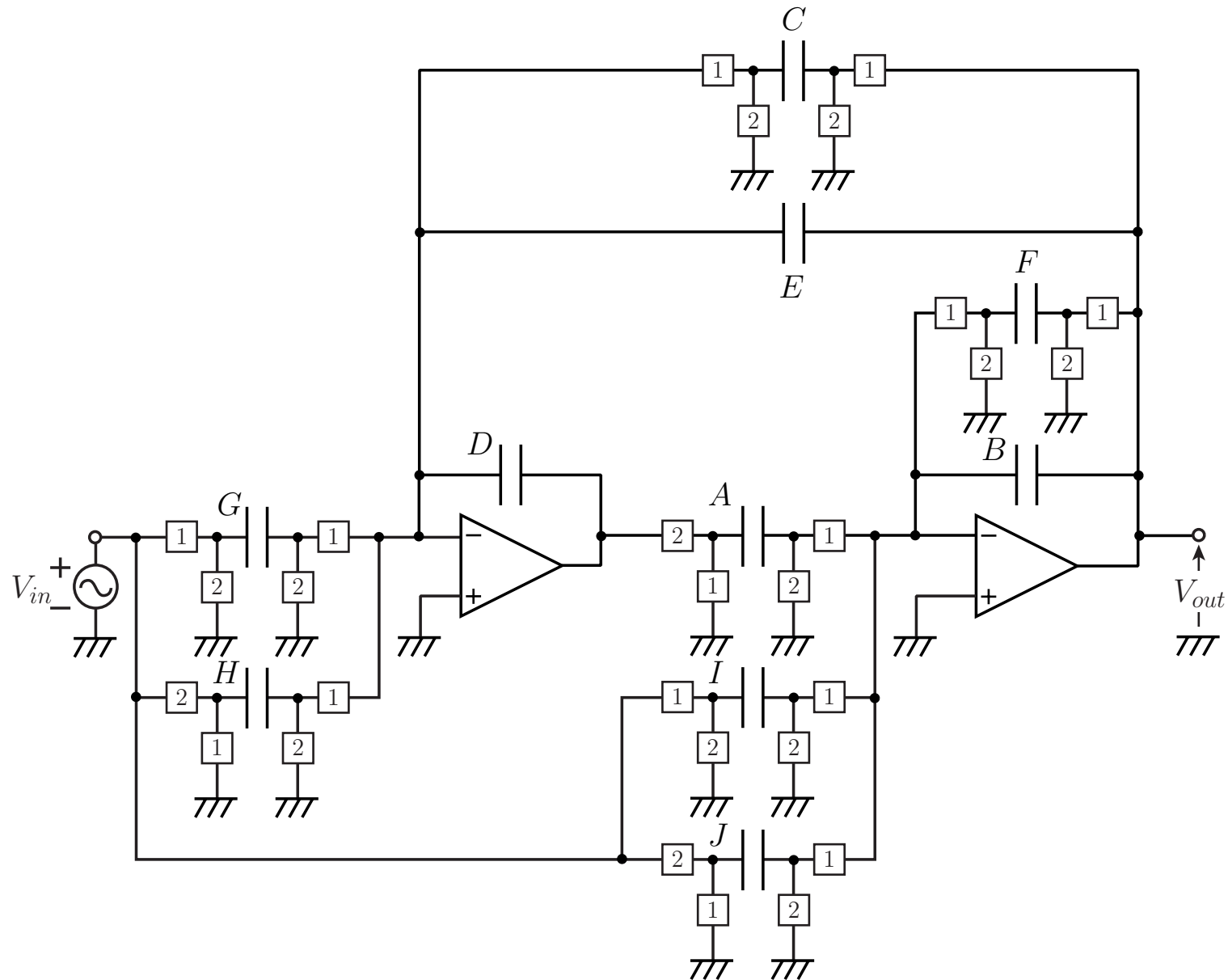
$$N_{SC2}(z) \propto (1-z^{-1})^2$$

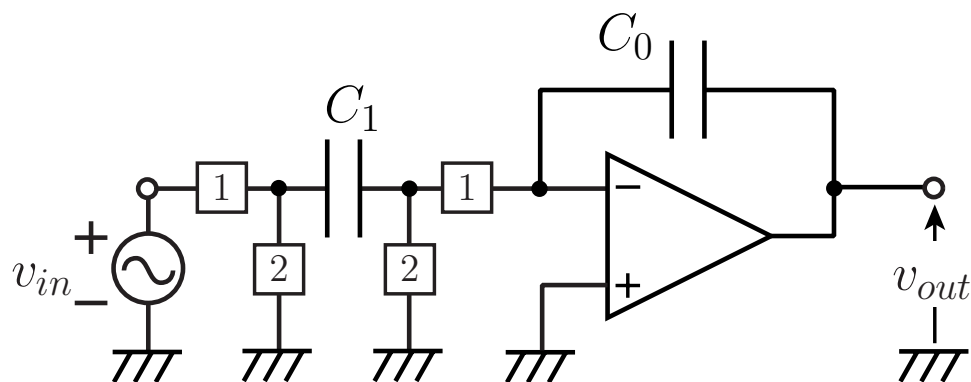
例：低域通過型伝達関数

$$T_{SC2}(z) = \frac{K \omega_0^2}{\left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \right)^2 + \frac{\omega_0}{Q} \left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \right) + \omega_0^2}$$

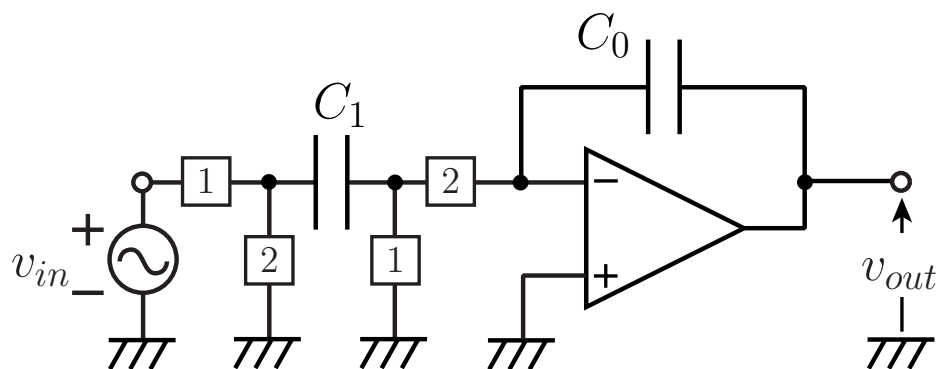
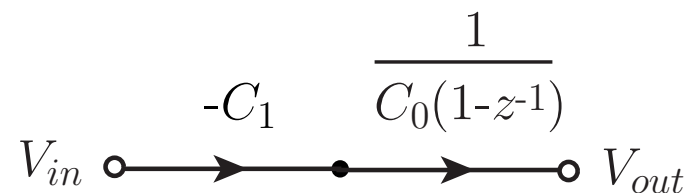
$$= \frac{\frac{K \omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} (1+z^{-1})^2}{1 - 2 \frac{\frac{4}{T^2} - \omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} z^{-1} + \frac{\frac{4}{T^2} - \frac{2\omega_0}{TQ} + \omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} z^{-2}}$$

FleischerとLakerの回路

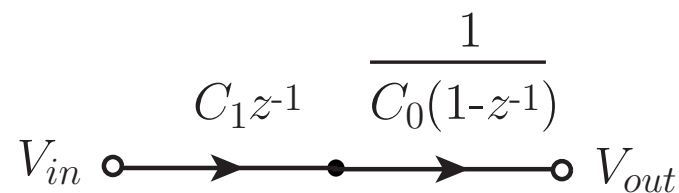


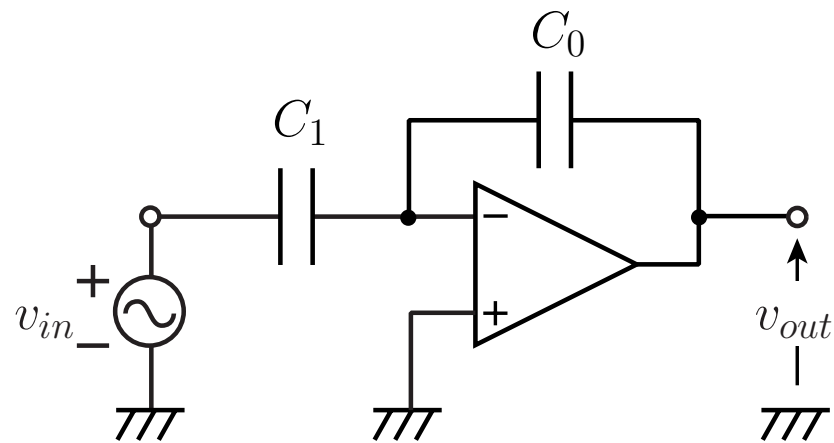


$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1}{C_0(1-z^{-1})} = -C_1 \times \frac{1}{C_0(1-z^{-1})}$$

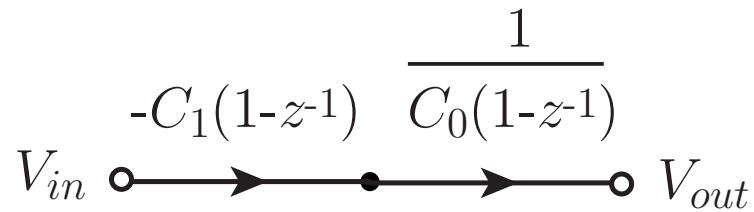


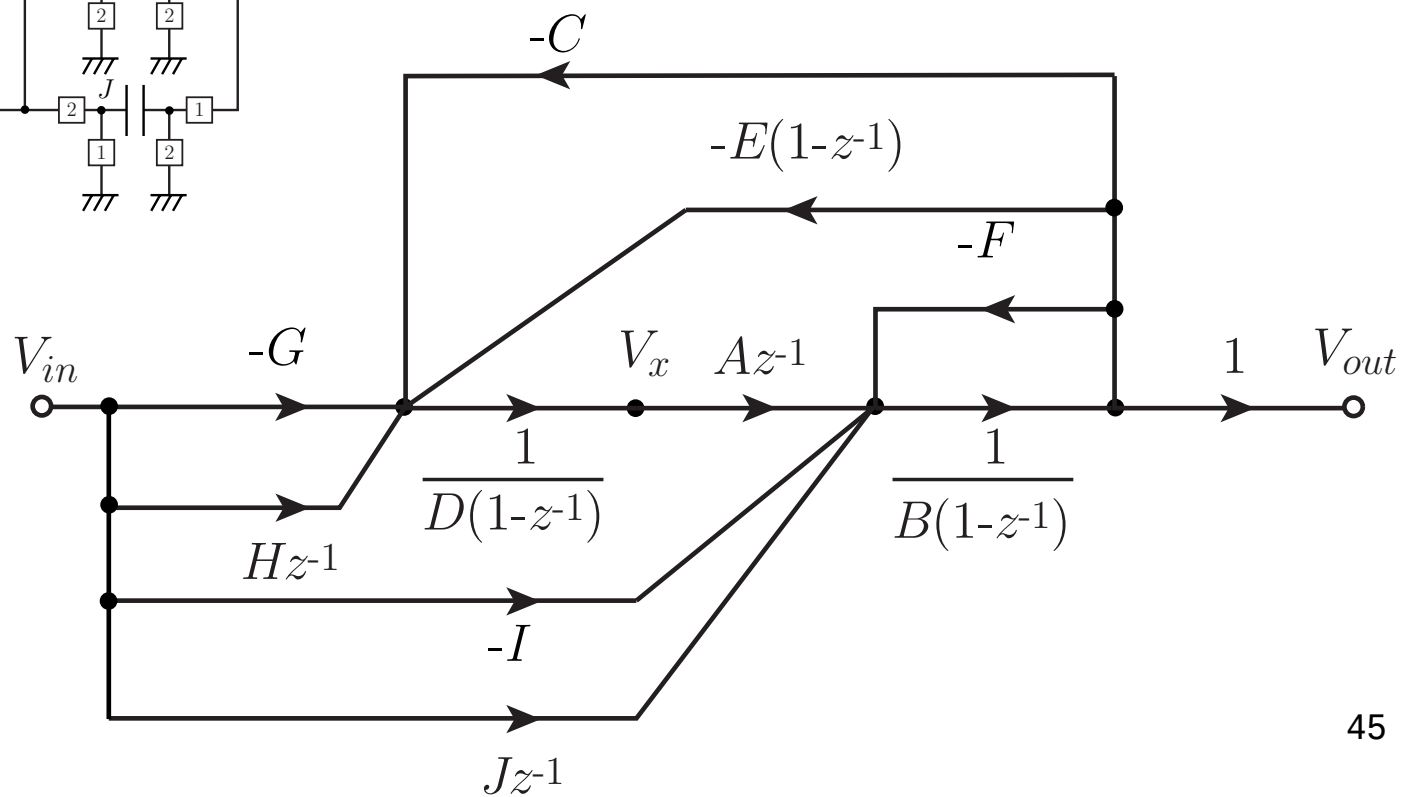
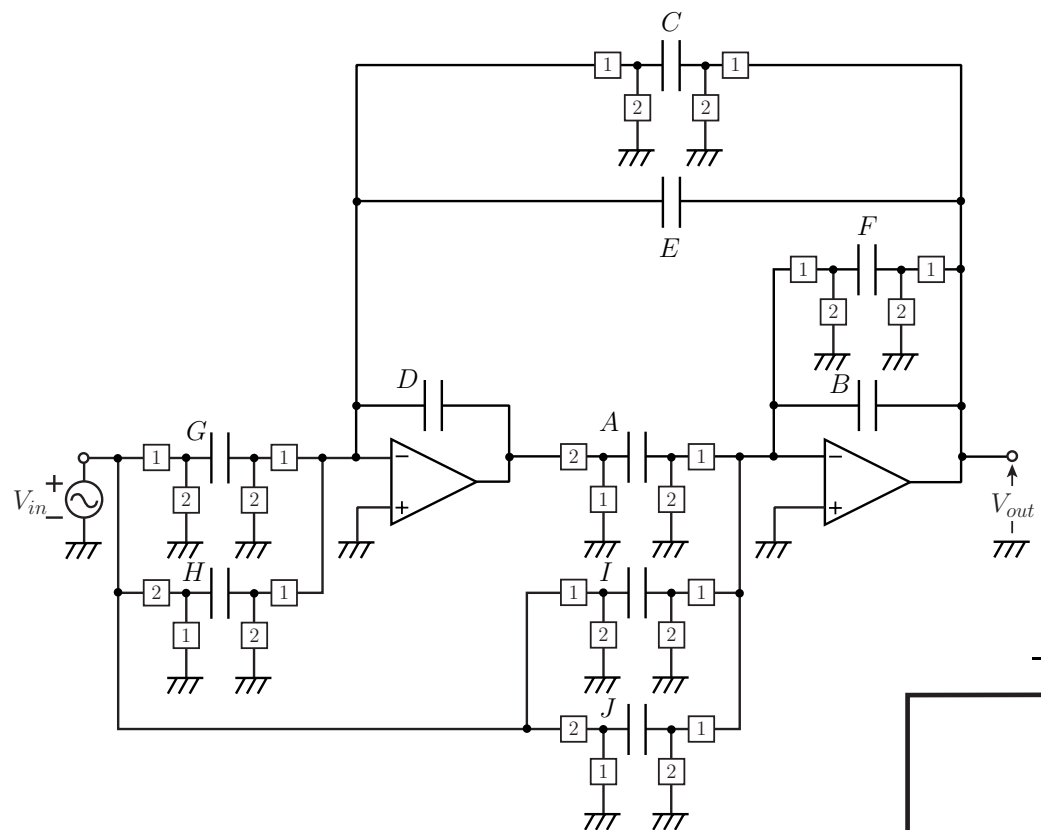
$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{C_1 z^{-1}}{C_0(1-z^{-1})} = C_1 z^{-1} \times \frac{1}{C_0(1-z^{-1})}$$

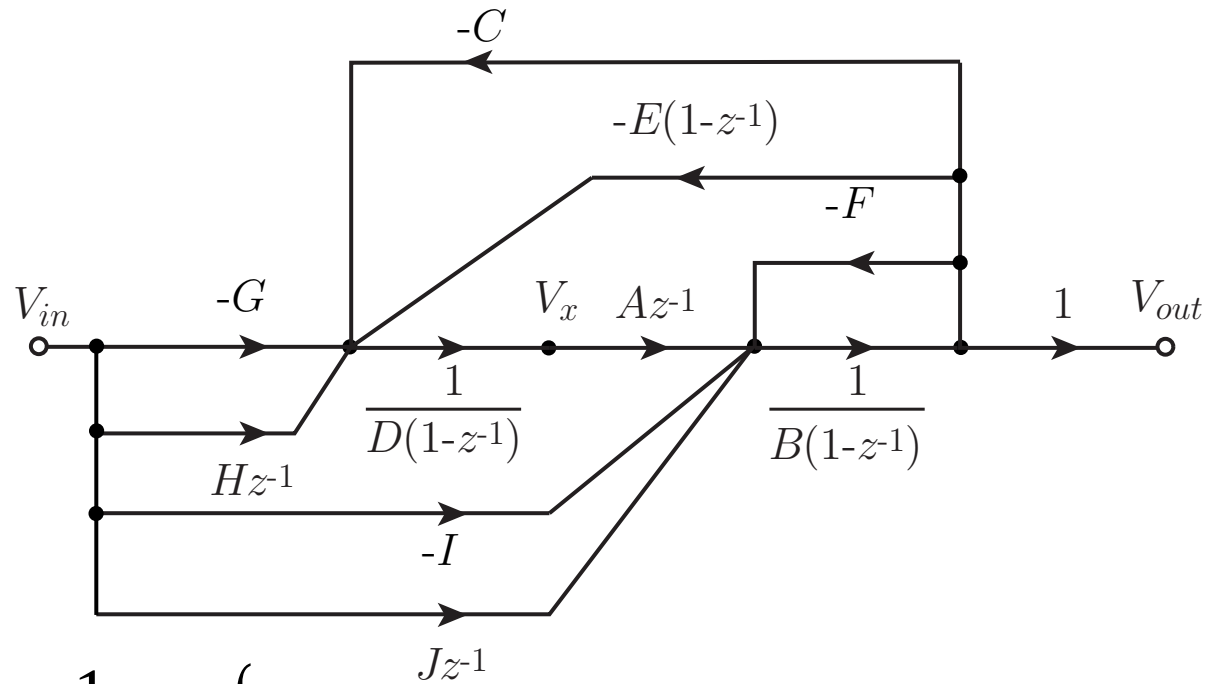




$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1}{C_0} = -C_1(1-z^{-1}) \times \frac{1}{C_0(1-z^{-1})}$$





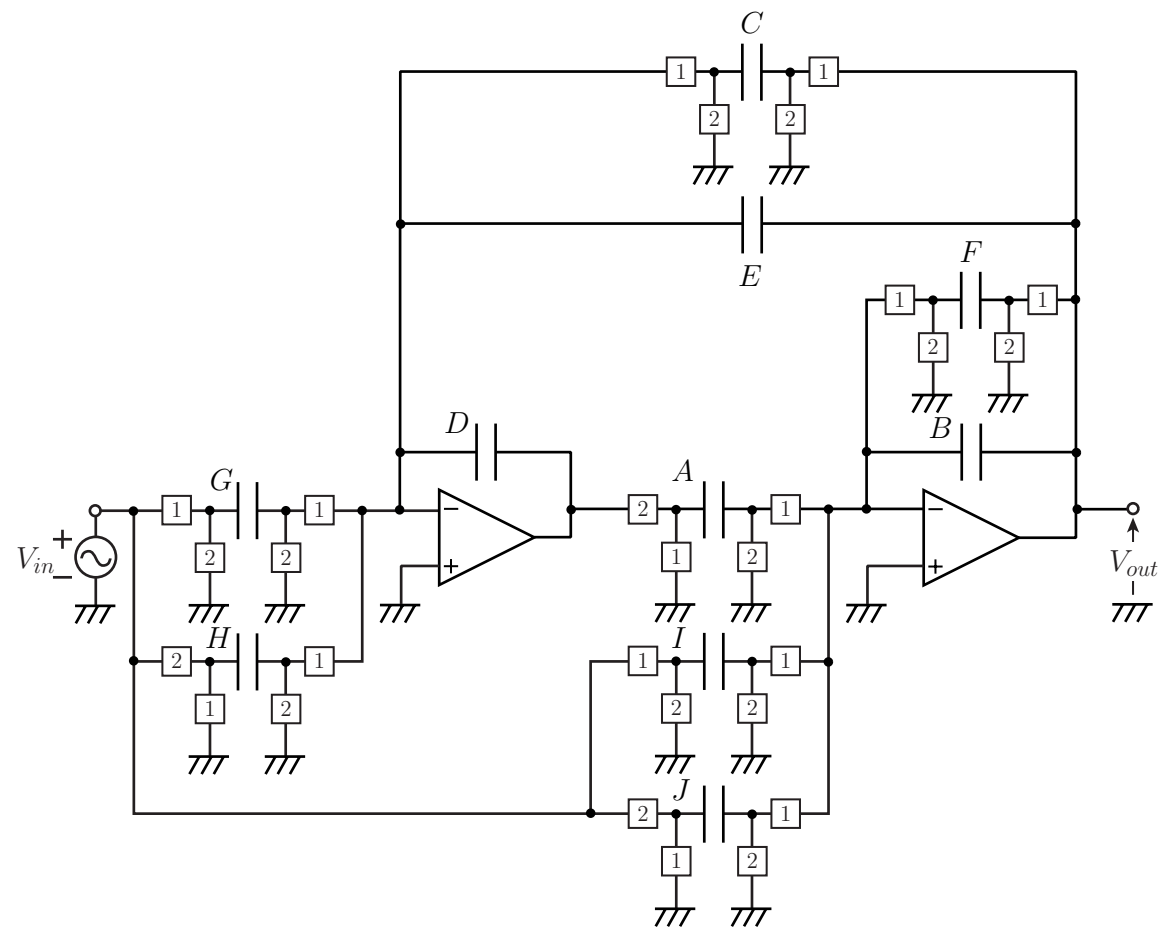


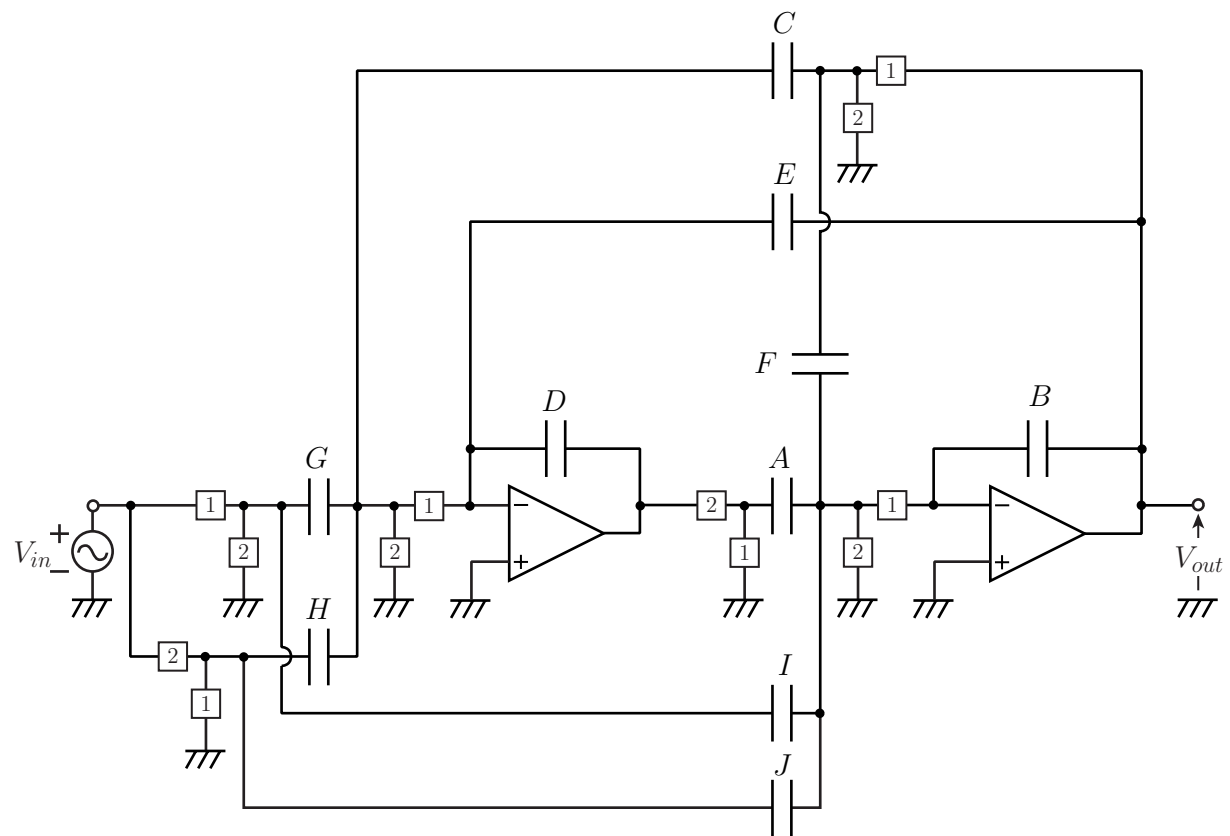
$$V_x = \frac{1}{D(1-z^{-1})} \left\{ -GV_{in} + Hz^{-1}V_{in} - CV_{out} - E(1-z^{-1})V_{out} \right\}$$

$$V_{out} = \frac{1}{B(1-z^{-1})} \left\{ Az^{-1}V_x - IV_{in} + Jz^{-1}V_{in} - FV_{out} \right\}$$

$$T_{FL}(z) = \frac{DI + (AG - DI - DJ)z^{-1} + (DJ - AH)z^{-2}}{D(B + F) + (AC + AE - DF - 2BD)z^{-1} + (BD - AE)z^{-2}}$$

スイッチの共有





$$T_{FL}(z) = \frac{DI + (AG - DI - DJ)z^{-1} + (DJ - AH)z^{-2}}{D(B + F) + (AC + AE - DF - 2BD)z^{-1} + (BD - AE)z^{-2}}$$

特性：容量比で決定



規格化



$$A = B = D = 1$$

$$T_{FL}(z) = \frac{I + (G - I - J)z^{-1} + (J - H)z^{-2}}{1 + F + (C + E - F - 2)z^{-1} + (1 - E)z^{-2}}$$

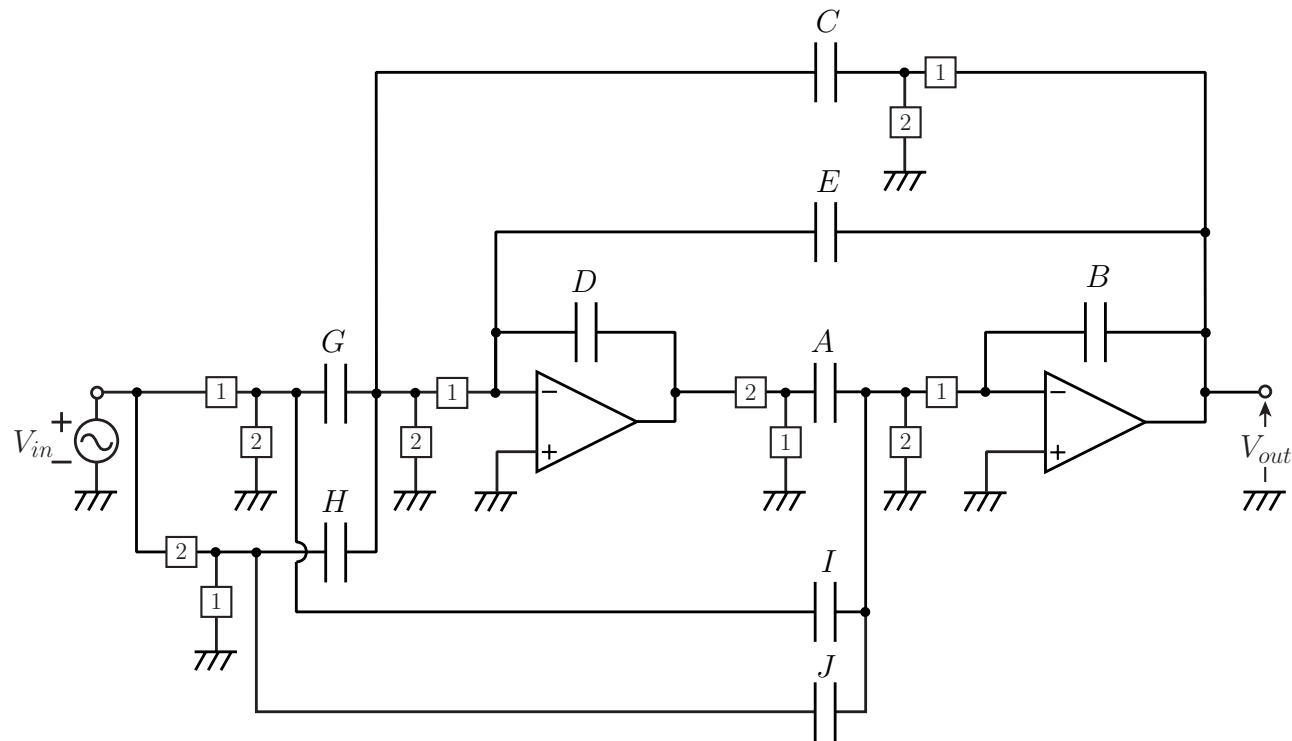
安定性：分母多項式の根は単位円内

$$1 + F > 1 - E$$

$$F = 0 \text{ or } E = 0$$

例: $F = 0$

$$T_{FL}(z) = \frac{1 + (G - I - J)z^{-1} + (J - H)z^{-2}}{1 + (C + E - 2)z^{-1} + (1 - E)z^{-2}}$$



FleischerとLakerのE回路

$$T_{SC2}(z) = \frac{\frac{K\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} (1+z^{-1})^2}{1 - 2 \frac{\frac{4}{T^2} - \omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} z^{-1} + \frac{\frac{4}{T^2} - \frac{2\omega_0}{TQ} + \omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} z^{-2}}$$

$$T_{FL}(z) = \frac{1 + (G - I - J)z^{-1} + (J - H)z^{-2}}{1 + (C + E - 2)z^{-1} + (1 - E)z^{-2}}$$

$$E = \frac{\frac{4\omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2}$$

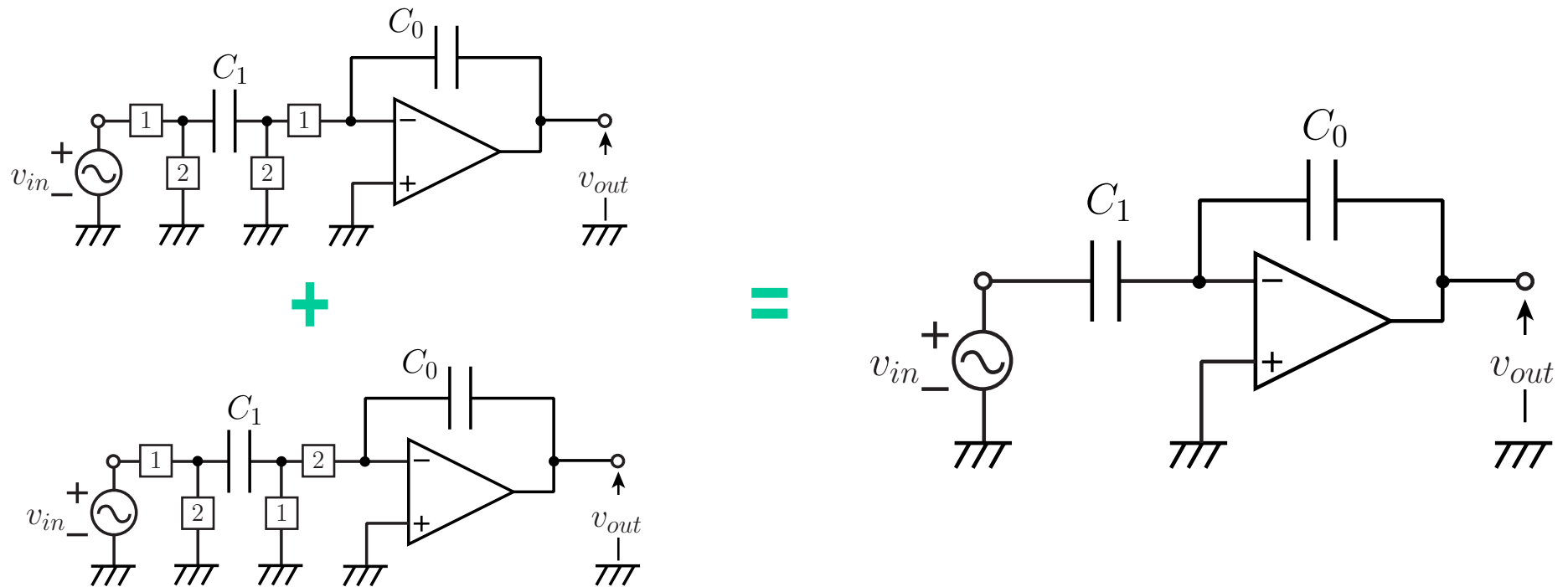
$$C = \frac{4\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2}$$

$$T_{FL}(z) = \frac{I + (G - I - J)z^{-1} + (J - H)z^{-2}}{1 + (C + E - 2)z^{-1} + (1 - E)z^{-2}}$$

$$T_{SC2}(z) = \frac{\frac{K\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} (1 + 2z^{-1} + z^{-2})}{1 - 2 \frac{\frac{4}{T^2} - \omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} z^{-1} + \frac{\frac{4}{T^2} - \frac{2\omega_0}{TQ} + \omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} z^{-2}}$$

$$I = \frac{K\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \quad J = \frac{K\omega_0^2}{\frac{4}{T} + \frac{2\omega_0}{TQ} + \omega_0^2} \quad H = 0 \quad G = \frac{4K\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2}$$

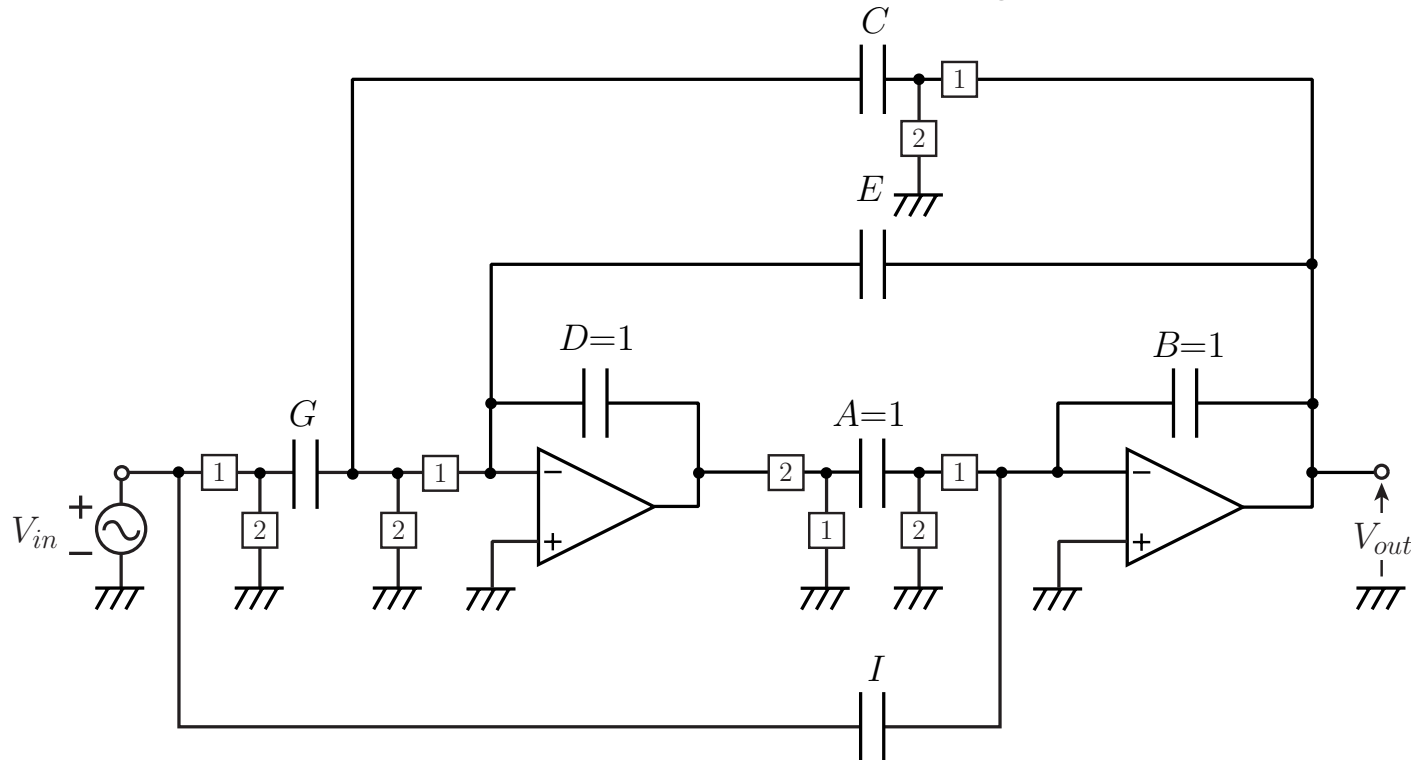
$$\begin{aligned}
A = B = D = 1 \quad C &= \frac{4\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \quad E = \frac{\frac{4\omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \\
I = J &= \frac{K\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \quad G = \frac{4K\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \quad H = 0
\end{aligned}$$



$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{-C_1}{C_0(1-z^{-1})} + \frac{C_1 z^{-1}}{C_0(1-z^{-1})} = \frac{-C_1(1-z^{-1})}{C_0(1-z^{-1})} = \frac{-C_1}{C_0}$$

$$A = B = D = 1 \quad C = \frac{4\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \quad E = \frac{\frac{4\omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2}$$

$$I = J = \frac{K\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \quad G = \frac{4K\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \quad H = 0$$



帯域通過型関数:

$$\begin{aligned}
 T_{SC2}(z) &= T_{\text{second}} \left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \right) = \frac{N \left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \right)}{\left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \right)^2 + \frac{\omega_0}{Q} \left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \right) + \omega_0^2} \\
 &= \frac{\frac{2K\omega_0}{TQ} (1-z^{-2})}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \\
 &= \frac{1 - 2 \frac{\frac{4}{T^2} - \omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} z^{-1} + \frac{\frac{4}{T^2} - \frac{2\omega_0}{TQ} + \omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} z^{-2}}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2}
 \end{aligned}$$

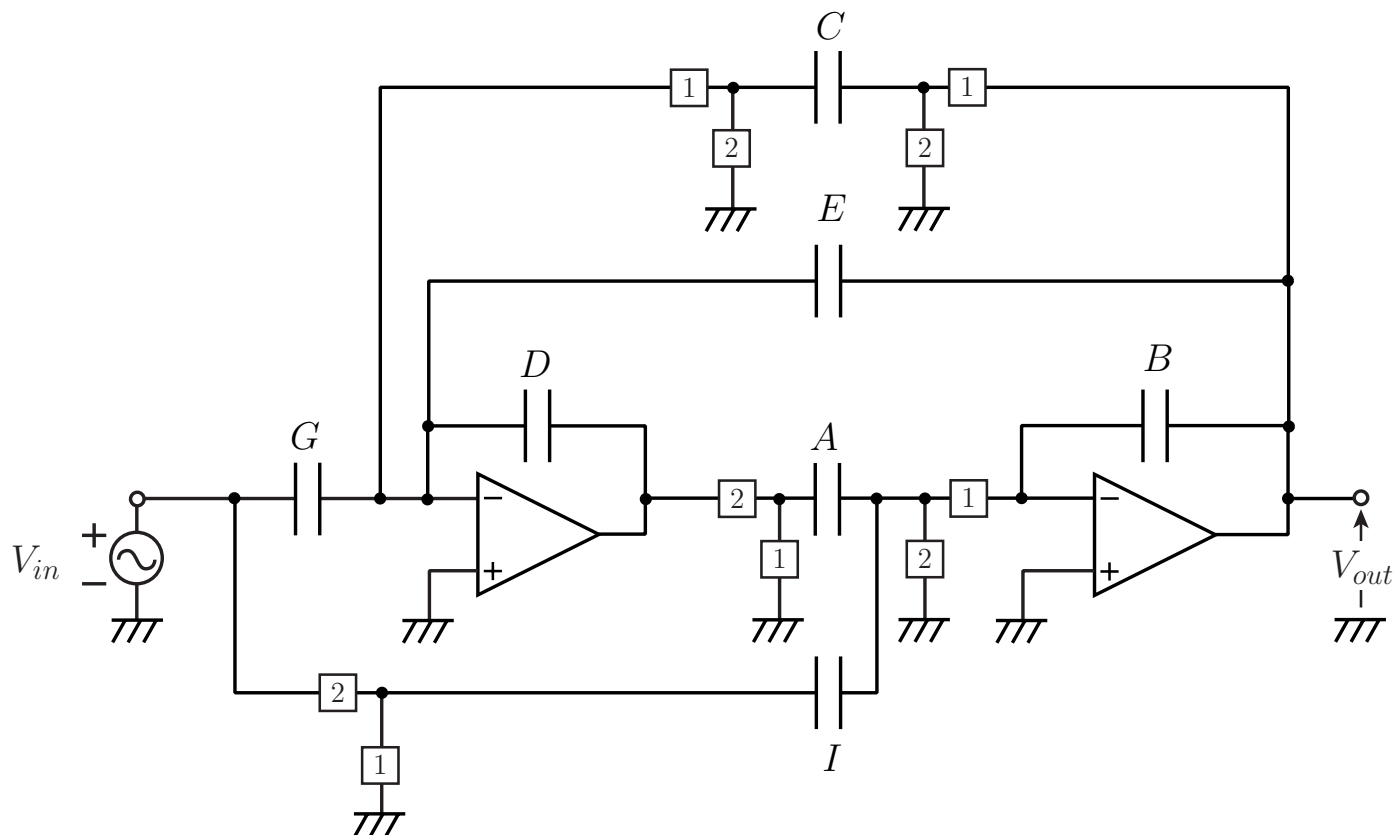
$$T_{SC2}(z) = \frac{\frac{\frac{2K\omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} (1-z^{-2})}{1 - 2 \frac{\frac{4}{T^2} - \omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} z^{-1} + \frac{\frac{4}{T^2} - \frac{2\omega_0}{TQ} + \omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} z^{-2}}$$

A=B=D=1とすると

$$T_{FL}(z) = \frac{1 + (G - I - J)z^{-1} + (J - H)z^{-2}}{1 + (C + E - 2)z^{-1} + (1 - E)z^{-2}}$$

$$C = \frac{4\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \quad E = \frac{\frac{4\omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \quad I = G = H = \frac{\frac{2K\omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} \quad J = 0$$

2次帯域通過フィルタ



例題

中心周波数1kHz, $Q=5$, $T=1/20000\text{s}$, $K=1$

$$\Omega \rightarrow \frac{2}{T} \tan\left(\frac{\omega T}{2}\right)$$

$$\omega_0 = 40000 \tan\left(\frac{2\pi \times 1000}{2} \times \frac{1}{20000}\right) = 2\pi \times 1008$$

$$T(z^{-1}) = \frac{\frac{2K\omega_0}{TQ}(1-z^{-2})}{\left(\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2\right) - 2\left(\frac{4}{T^2} - \omega_0^2\right)z^{-1} + \left(\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2\right)z^{-2}}$$

$$= \frac{0.0300(1-z^{-2})}{1-1.85z^{-1}+0.940z^{-2}}$$

$$\omega_0 = 2\pi \times 1000, \quad Q = 5, \quad T = 1/20000\text{s}, \quad K = 1$$

$$A = B = D = 1$$

$$\rightarrow 33.3\text{pF}$$

$$C = \frac{4\omega_0^2}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} = 0.0950$$

$$\rightarrow 3.17\text{pF}$$

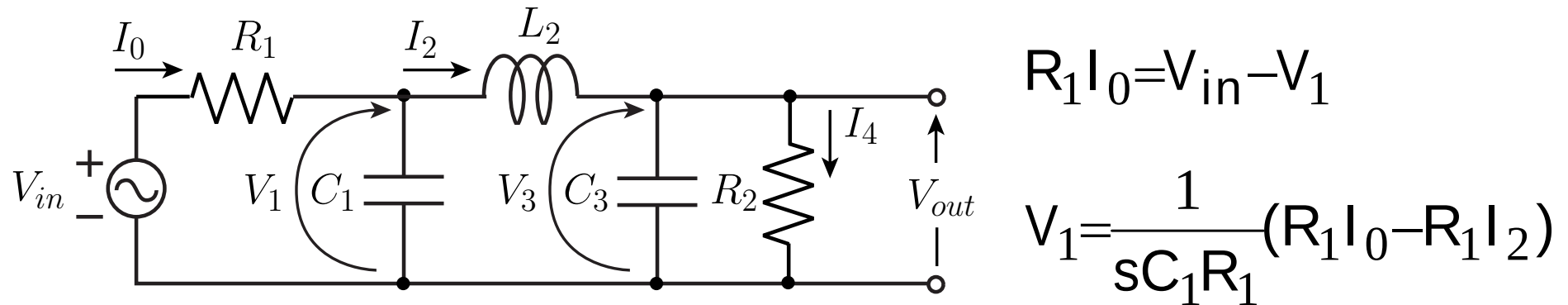
$$E = \frac{\frac{4\omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} = 0.0600$$

$$\rightarrow 2.00\text{pF}$$

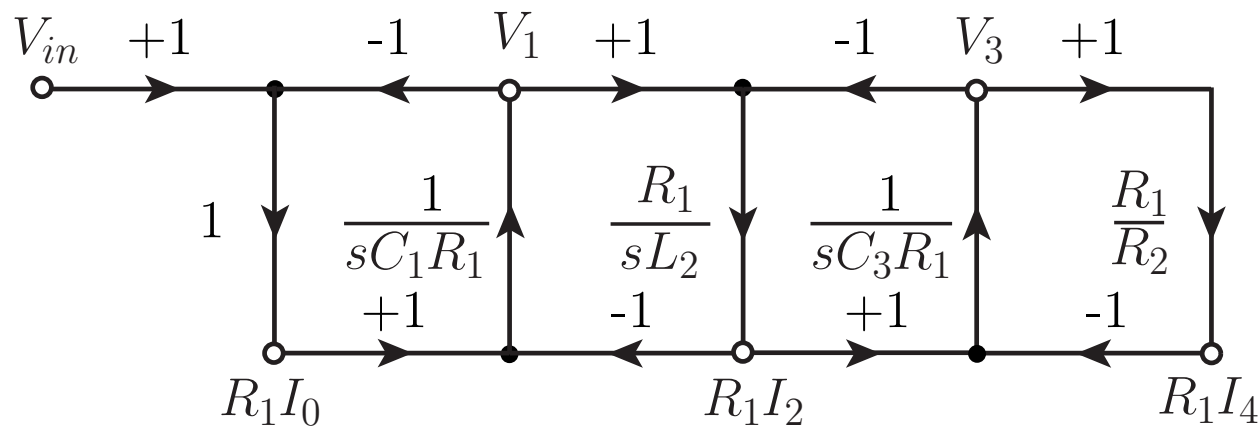
$$I = G = H = \frac{K \frac{2\omega_0}{TQ}}{\frac{4}{T^2} + \frac{2\omega_0}{TQ} + \omega_0^2} = 0.0300$$

$$\rightarrow 1.00\text{pF}$$

スイッチトキャパシタ回路によるリープフログ・シミュレーション

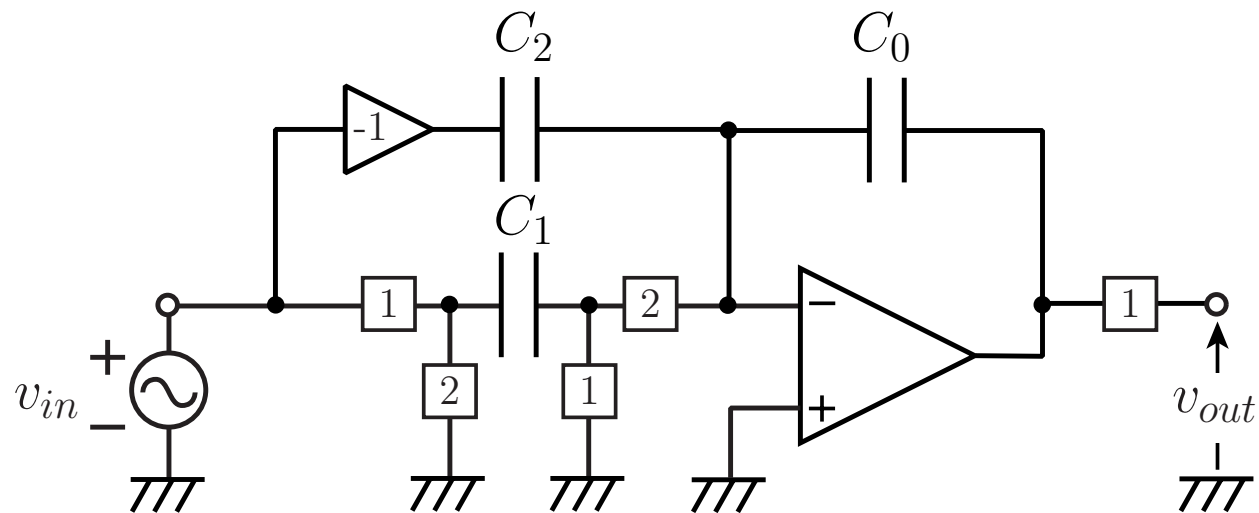


$$R_1 I_2 = \frac{R_1}{sL_2} (V_1 - V_3) \quad V_3 = \frac{1}{sC_3 R_1} (R_1 I_2 - R_1 I_4) \quad R_1 I_4 = \frac{R_1}{R_2} V_3$$



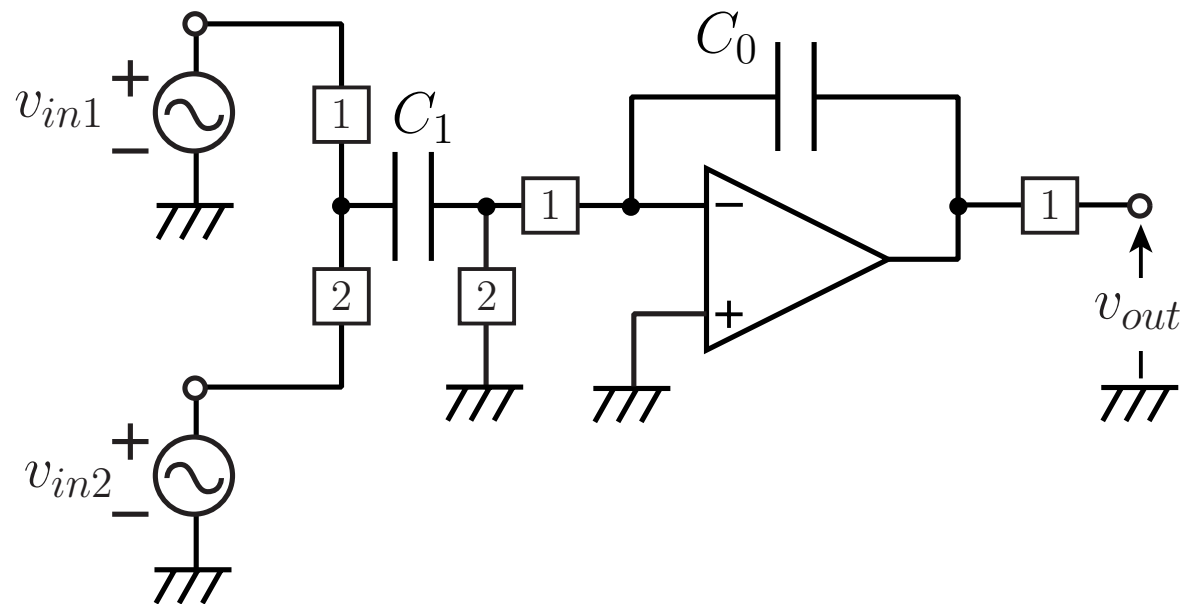
(a)

積分回路の選択

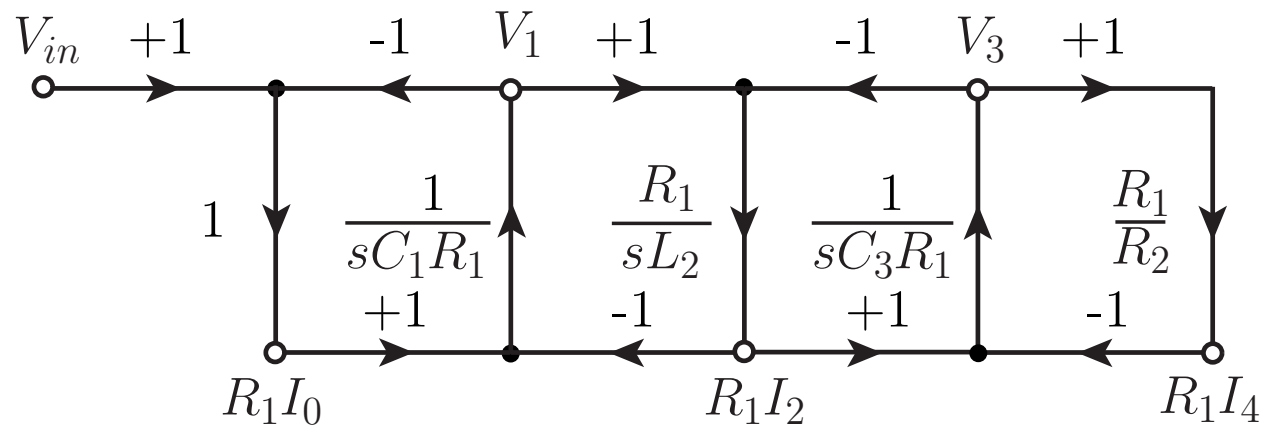


$C_1=2C_2$ のとき

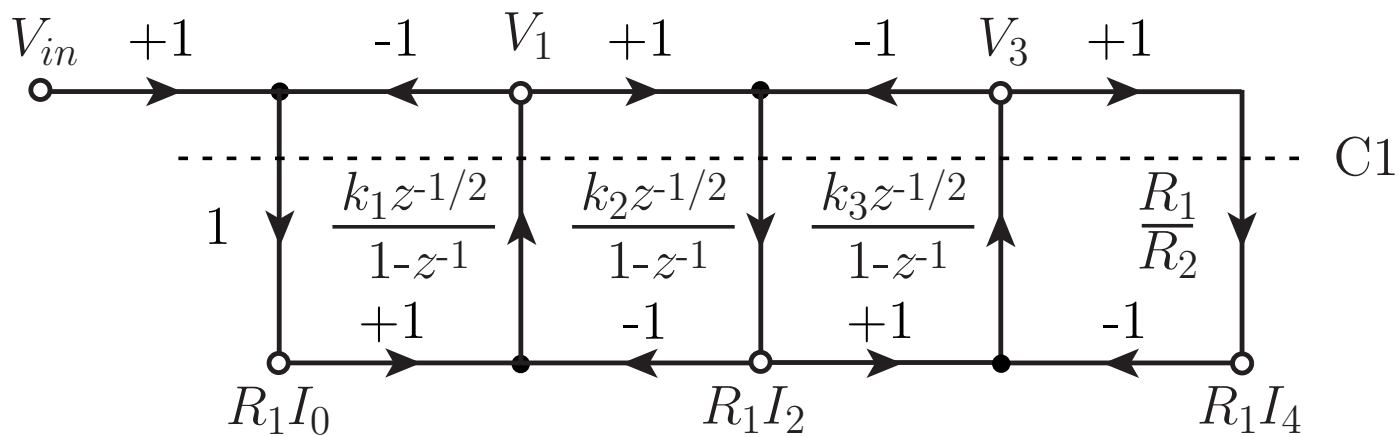
双一次積分回路
複雑



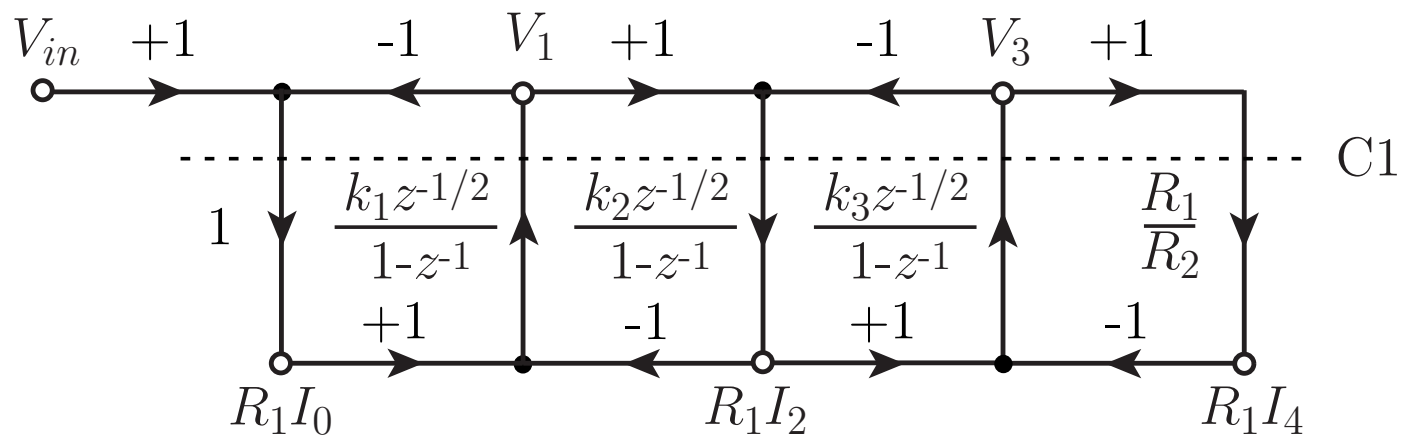
LDI積分回路
簡単



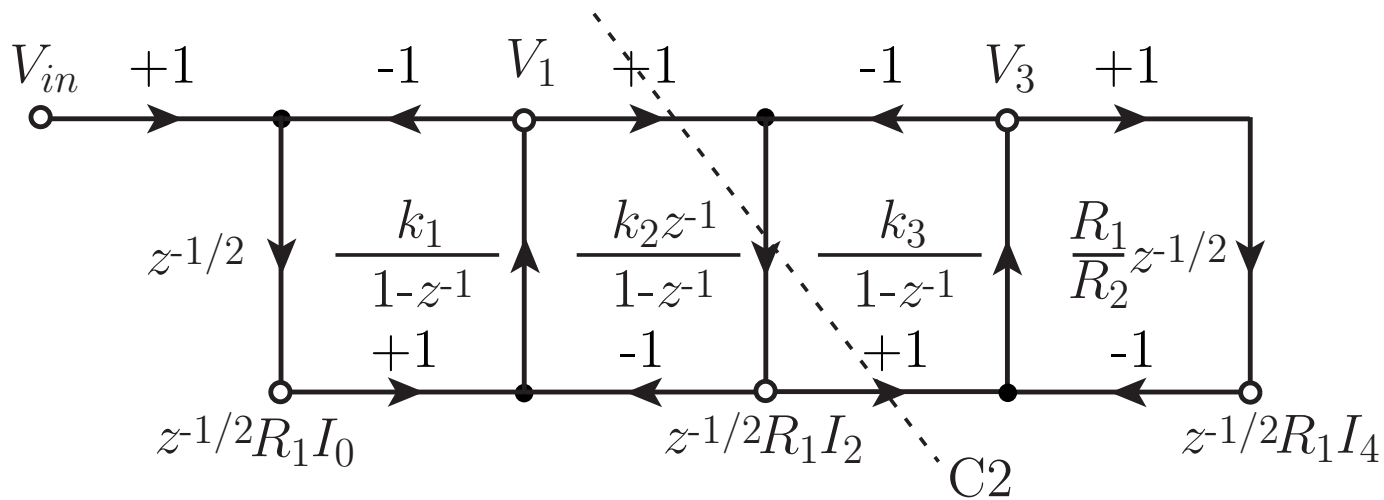
(a)



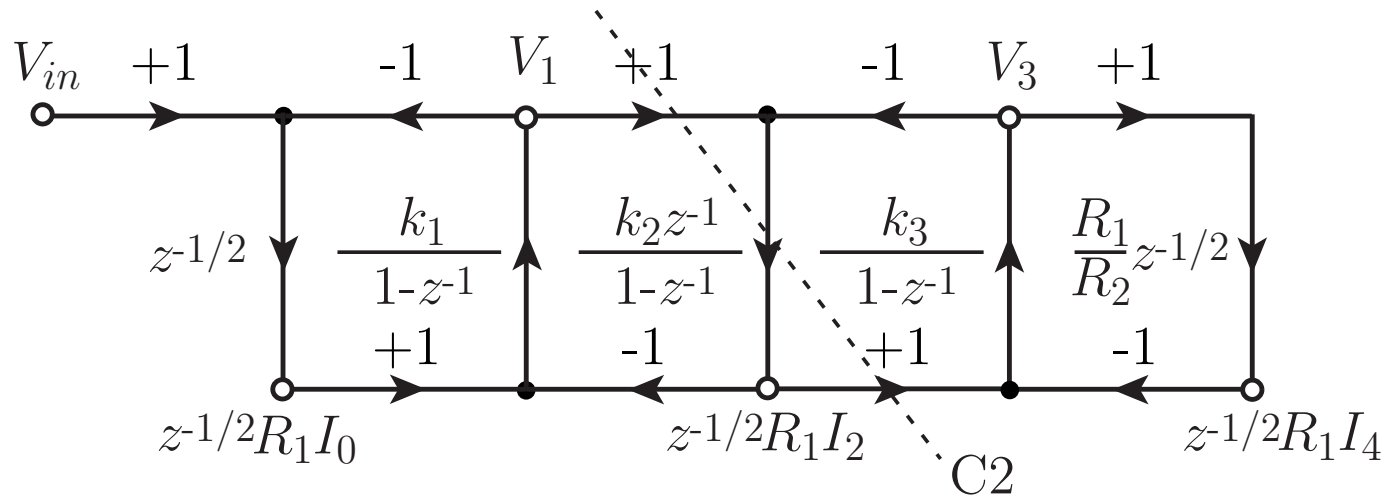
(b)



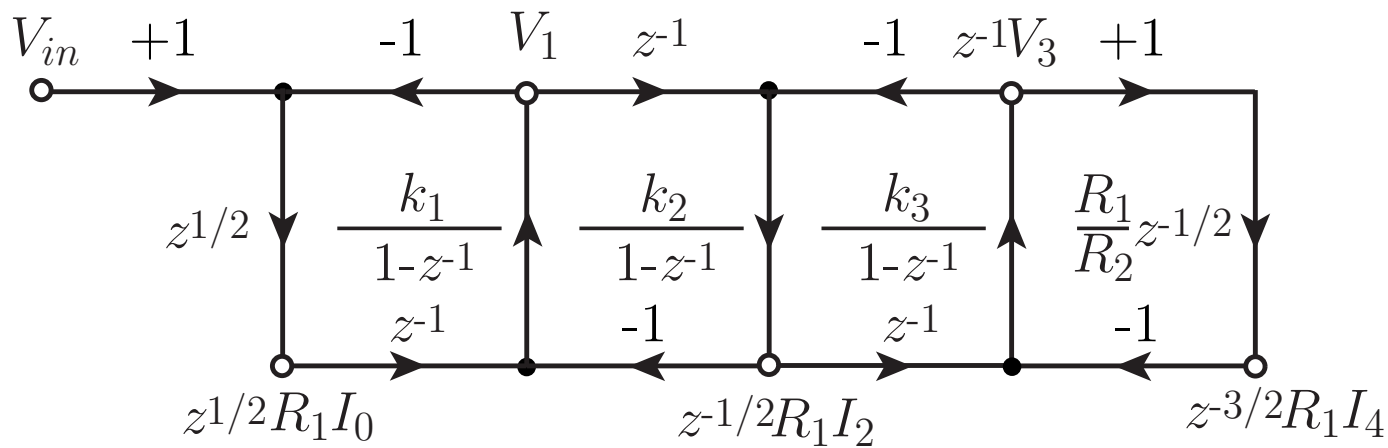
(b)



(c)

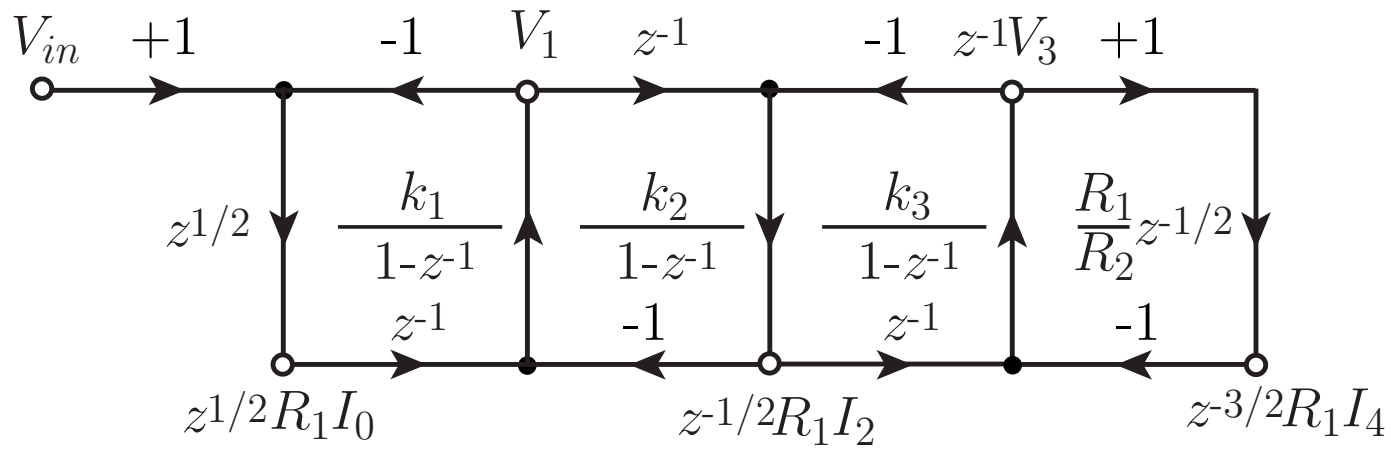


(c)



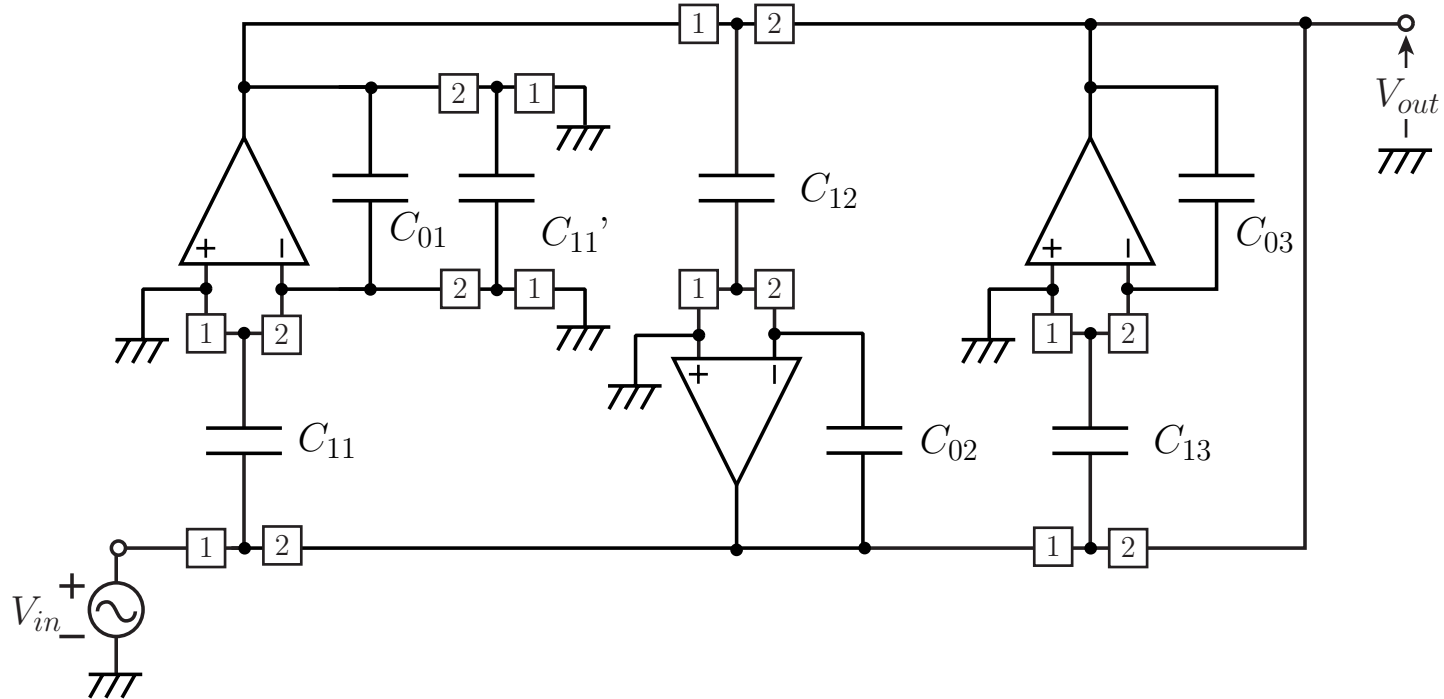
(d)

$$k_1 = \frac{T}{C_1 R_1} \quad k_2 = \frac{R_1 T}{L_2} \quad k_3 = \frac{T}{C_3 R_1}$$



(d)

$$k_1 = \frac{T}{C_1 R_1} \quad k_2 = \frac{R_1 T}{L_2} \quad k_3 = \frac{T}{C_3 R_1}$$



LDI変換の問題点

伝達関数 $T(s)$ の極:

$$s_i = \sigma_i + j \omega_i$$

$$s_i = \frac{z^{1/2} - z^{-1/2}}{T}$$



$$s_i^2 = \frac{z - 2 + z^{-1}}{T^2}$$

解:

$$z_i$$



$$z_i^{-1}$$

$$|z_i| < 1$$



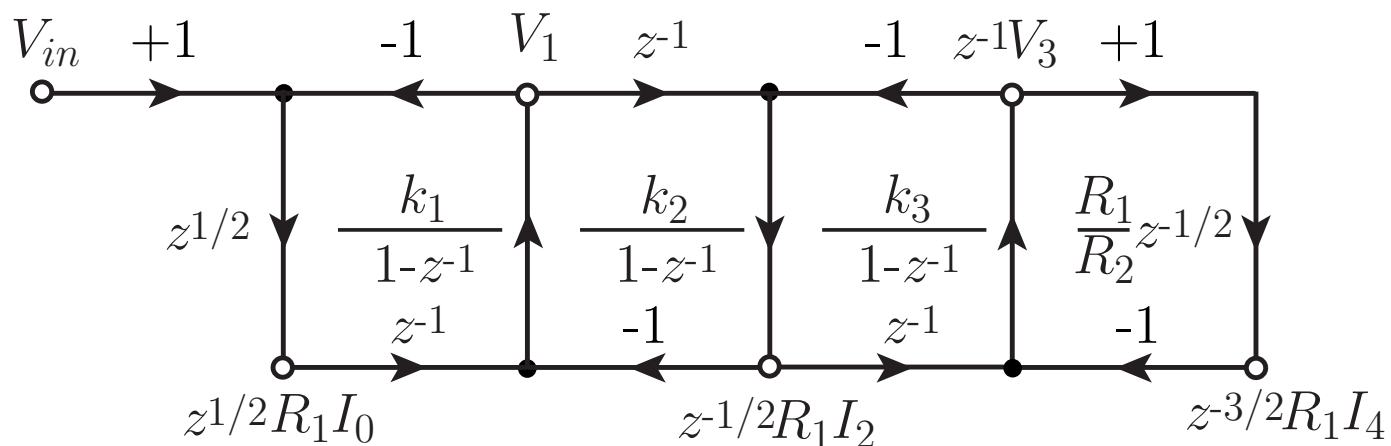
安定



$$|z_i^{-1}| > 1$$



不安定



(d)

$$k_1 = \frac{T}{C_1 R_1} \quad k_2 = \frac{R_1 T}{L_2} \quad k_3 = \frac{T}{C_3 R_1}$$

誤差要因

不安定性を除去

例題

遮断周波数1kHzの3次低域通過フィルタ

$$\Omega \rightarrow \frac{2}{T} \sin\left(\frac{\omega T}{2}\right)$$

$$T = 1/20000\text{s}$$

$$\omega_0 = 40000 \sin\left(\frac{2\pi \times 1000}{2} \times \frac{1}{20000}\right) = 2\pi \times 996$$

$$R_1 = R_2 = 1\text{k}\Omega$$

$$C_1 = C_3 = 0.159\mu\text{F}$$

$$L_2 = 0.318\text{H}$$



$$R_1 = R_2 = 1\text{k}\Omega$$

$$C_1 = C_3 = 0.160\mu\text{F}$$

$$L_2 = 0.319\text{H}$$

$$k_1 = \frac{T}{C_1 R_1} = \frac{C_{11}}{C_{01}} = \frac{C'_{11}}{C_{01}} = 0.319$$

$$k_2 = \frac{TR_1}{L_2} = \frac{C_{12}}{C_{02}} = 0.160$$

$$k_3 = \frac{T}{C_3 R_1} = \frac{C_{13}}{C_{03}} = 0.319$$

$C_{01}=C_{02}=C_{03}=1$ とすれば

$$C_{11}=C'_{11}=C_{13}=0.319, \quad C_{12}=0.160$$

$$C_{01}=C_{02}=C_{03}=6.25\text{pF}, \quad C_{11}=C'_{11}=C_{13}=1.99\text{pF}, \quad C_{12}=1.00\text{pF}$$

容量の総和 :25.7pF

$$k_1 = \frac{T}{C_1 R_1} = \frac{C_{11}}{C_{01}} = \frac{C'_{11}}{C_{01}} = 0.319$$

$$k_2 = \frac{TR_1}{L_2} = \frac{C_{12}}{C_{02}} = 0.160$$

$$k_3 = \frac{T}{C_3 R_1} = \frac{C_{13}}{C_{03}} = 0.319$$

$C_{01}=C_{03}=1, C_{02}=2$ とすれば

$$C_{11}=C'_{11}=C_{12}=C_{13}=0.319$$

$$C_{01}=C_{03}=3.13\text{pF}, C_{02}=6.25\text{pF}, C_{11}=C'_{11}=C_{12}=C_{13}=1.00\text{pF}$$

容量の総和 :13.4pF