Strategic Decision Making with Coarse Frames

Norimasa Kobayashi

Department of Value and Decision Science
Tokyo Institute of Technology

Techniques for Rational Analysis Lecture 13

Strategic Use of Coarse Models

Coarse Evaluation

- Strategic decisions are often made without specifying the details.
  i.e.) rough strategic plan $\rightarrow$ tactical details
  Q. Is it legitimate to make decisions such a way?
  Ans. Optimal substructure of set-valued solution provides a sufficient condition for the legitimacy.
- Set-valued solution characterizes hierarchical diversity in the society – finer coordination is possible within the expected range in the smaller communities

Local Dominance

Definition (Local Dominance)

- $a \in A$ locally dominates $a' \in A$ at the range of uncontrollable states $Y \subset X$ if
$$\forall x \in Y[u(a,x) \geq u(a',x)]$$
- $B \subset A$ is locally dominant at the range $Y \subset X$ if
$$\forall a \in B \forall a' \in A \setminus B[a \text{ locally dominates } a']$$

- Notice that the definition is applied to a subset of actions rather than a single best action.
- Local dominance describes the situation in which, if you know that the state of the world is within a certain range, then you can make choices from a certain range of alternatives.

天気と休日の外出の意思決定

$a \backslash \omega$	晴れ	曇り	雨
ピクニック	5	2	-5
ショッピング	2	3	-1
家にいる	1	1	1

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Application to Interactive Decision Situations

Definition

$B \subset A$ is closed under rational behavior (curb) [1] or persistent[2] if $\forall i \in N[B_i$ is locally dominant at $B_{-i}]$.

Example

1 \2	a	b	c	d
α	1, 1	2, 3	0, 0	0, 0
β	3, 2	1, 1	0, 0	0, 0
γ	0, 0	0, 0	1, 1	2, 3
δ	0, 0	0, 0	3, 2	1, 1

- $\{\alpha, \beta\} \times \{a, b\}$ is locally dominant.
- If player 1 knows that player 2 chooses within $\{a, b\}$, then she is better off choosing either α or β than choosing any other action.

Optimal Substructure

Theorem ([3])

Let $B \subset A$ be a curb subspace. For $\forall C \subset B$, if C is curb in the game restricted to B, then C is also curb in the original game.

Example

1 \2	a	b
α	1, 1	2, 3
β	3, 2	1, 1

$\Rightarrow$

1 \2	a	b	c	d
α	1, 1	2, 3	0, 0	0, 0
β	3, 2	1, 1	0, 0	0, 0
γ	0, 0	0, 0	1, 1	2, 3
δ	0, 0	0, 0	3, 2	1, 1

Partitioned Games

Definition (Partition game)

Let P^i be a frame on action space A_i for each player i, and $P := \times_{i \in N} P^i$. A partition game $\Gamma(P) = (N, P, \hat{u})$ of game Γ is a normal form game that satisfies

$$\forall i \in N, \forall B \in P : \min_{a \in B} u_i(a) \leq \hat{u}_i(B) \leq \max_{a \in B} u_i(a)$$

Theorem

Take a game Γ and a partition game $\Gamma(P)$. Assume that an action profile subspace $B^* \in P$. Then,
B^* is locally dominant in game $\Gamma \Rightarrow B^*$ is a Nash equilibrium of game $\Gamma(P)$

Example

Example

$1 \backslash 2$	$\{a, b\}$	$\{c, d\}$
$\{\alpha, \beta\}$	1, 3	0, 0
$\{\gamma, \delta\}$	0, 0	3, 2

- A partition game is obtained by labeling subsets by a coarse frame:
  - $A_1^1 = \{\{a, b\}, \{c, d\}\}$
  - $A_2^2 = \{\{\alpha, \beta\}, \{\gamma, \delta\}\}$
- $(\{a, b\}, \{c, d\}) \in A_1^1 \times A_2^2$ is a Nash equilibrium. (Recall that it is locally dominant in the original game.)

Implications

Local dominance along with partitioning of language are applicable to the following contexts:

- Strategic Decisions – If local dominance is applicable successively, then the succession characterizes the legitimacy of successively focusing on finer details in a narrower range.
- Hierarchical Diversity in the Society – finer coordination is possible within the expected range in the smaller communities

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